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Numerical predictions of the anisotropic viscoelastic response of uni-directional fibre composites

M.V. Pathan [a], V.L. Tagarielli [a,*], S. Patsias [b]

[a] Department of Aeronautics, Imperial College London, South Kensington Campus, SW7 2AZ, UK
[b] Rolls-Royce plc, PO Box 31, DE24 8BJ Derby, UK

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Finite Element (FE) simulations are conducted to predict the viscoelastic properties of uni-directional (UD) fibre composites. The response of both periodic unit cells and random stochastic volume elements (SVEs) is analysed; the fibres are assumed to behave as linear elastic isotropic solids while the matrix is taken as a linear viscoelastic solid. Monte Carlo analyses are conducted to determine the probability distributions of all viscoelastic properties. Simulations are conducted on SVEs of increasing size in order to determine the suitable size of a representative volume element (RVE). The predictions of the FE simulations are compared to those of existing theories and it is found that the Mori-Tanaka (1973) and Lielens (1999) models are the most effective in predicting the anisotropic viscoelastic response of the RVE.

1. Introduction

Fibre-reinforced polymers (FRPs) are widely used in industry due to their excellent specific strength and stiffness and also display relatively high material damping compared to metals of similar stiffness. Knowledge of their mechanical properties is essential to achieve optimal designs with FRPs; while the anisotropic stiffness and strength of FRPs have received great attention from the research community, less studies exist on their damping properties, which are particularly important in aerospace applications. The damping of FRPs is strongly anisotropic and depends on the imposed frequency and temperature; experimental investigations are therefore time-consuming and require specialist equipment. For these reasons, effective numerical and theoretical predictions of the damping properties need to be developed and validated.

Numerous theoretical models exist to predict the elastic response of UD composites; these can be easily extended to the case of viscoelastic materials via the elastic-viscoelastic correspondence principle. In addition to the upper and lower bounds given by the Voigt [3] and Reuss model [4], respectively, Hashin [5] and Hill [6] derived narrower bounds for transversely isotropic composites with isotropic constituents. Hashin and Rosen [7] later derived a predictive model based on a Composite Cylinder Assemblage (CCA). Several predictive models are based on mean-field homogenisation, in which the microfields within each constituent of an inhomogeneous material are approximated by their phase averages by using Eshelby's model [8]. Examples include the Mori and Tanaka [1] model, the Self-Consistent Method (Hill [9]) and Lielens model [2]. Other theoretical models have focused on predictions of viscoelastic properties via extension of previously developed elastic models, the most popular being such as Hashin [10–12], Christensen [13] and Saravanos and Chamis [14].

Several studies attempted validation of the above analytical models via numerical analysis; for example, Chandra et al. [15] and Brinson et al. [16,17] considered the viscoelastic response of square or hexagonal periodic unit cells; Tsai and Chi [18] pointed out that the damping properties predicted by simulations on unit cell are strongly dependent on the choice of unit cell. Such studies were either limited to a few selected loading cases or they analysed only damping properties but not the elastic response.

Since the spatial distribution of fibres in a UD composite is closer to being random than periodic, it is intuitive to expect that an analysis of a random microstructure should yield more realistic results than the analysis of a periodic unit cell. Several authors have analysed numerically random microstructures; for example Arnold et al. [19] analysed stiffness and strength of UD fibre composites and compared the predictions of periodic unit cells and random microstructures; Gusev et al. [20] analysed random distributions of spherical particles in a continuous matrix to extract its effective elastic properties. Several researchers have focused on the dependence of numerical predictions upon the size of the material volume investigated and gave guidelines for the choice of an effective minimum size. For the case of composites with

* Corresponding author.
E-mail address: v.tagarielli@imperial.ac.uk (V.L. Tagarielli).

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spherical filler particles, Drugan and Willis [21] found that the elastic properties could be effectively predicted using Representative Volume Elements (RVEs) of size 4*R*, where *R* is the radius of the spherical particle. Trias et al. [22] examined elasticity of UD carbon/epoxy composites and suggested an RVE size greater 50*R*.

In the present work, we present a comprehensive numerical analysis of the anisotropic viscoelastic response of a UD fibre composite lamina, simulating both periodic unit cells and random microstructures. For the case of random microstructures we analyse the size-dependence of the FE predictions and their scatter, determining an effective RVE size. Predictions are also compared to existing theoretical approaches with the objective of ranking the effectiveness of different models in predicting the viscoelastic properties.

The outline of the paper is as follows: in Section 2 we review the constitutive models assumed for the composite and its constituents; the FE simulations are described in detail in Section 3 and the corresponding numerical results are presented in Section 4. In Section 5 we present and discuss a comparison of numerical and theoretical predictions.

2. Review of viscoelastic constitutive models

2.1. Response of the constituent materials

Damping in FRPs is primarily due to the viscoelastic nature of the polymeric matrix, since the most commonly used reinforcing fibres are inorganic (e.g. carbon, glass) and their damping properties are negligible. Accordingly, in this work we shall assume a linear elastic response of the fibres.

In normal operating conditions composites experience small deformations; this justifies modelling the polymeric matrix as a linear viscoelastic material. Assuming an isotropic response of the matrix, the constitutive equations of viscoelasticity are given as

$$s_{ij}(t) = \int_{-\infty}^{t} 2G(t-\tau)\frac{de_{ij}}{d\tau}d\tau \tag{1}$$

$$p_{ii}(t) = \int_{-\infty}^{t} 3K(t-\tau)\frac{d\phi_{ii}}{d\tau}d\tau \tag{2}$$

where s_{ij} and p_{ii} are the components of the deviatoric and hydrostatic stress tensor, respectively, e_{ij} and ϕ_{ij} are the corresponding deviatoric and dilatational strains, $G(t)$ and $K(t)$ are time dependent shear and bulk moduli, respectively [16]. Taking a Fourier Transform of Eqs. (1) and (2) gives

$$s_{ij}(i\omega t) = 2G^*(\omega)e_{ij}(\omega) \tag{3}$$

$$p_{ii}(i\omega t) = 3K^*(\omega)\phi_{ii}(\omega). \tag{4}$$

The above equations are analogous to those governing isotropic elasticity but are expressed in the Fourier domain; this correspondence is referred to as the elastic-viscoelastic correspondence principle. $G^*(\omega)$ and $K^*(\omega)$ are Fourier transforms of $G(t)$ and $K(t)$ and can be decomposed in their real and imaginary parts

$$G^*(\omega) = G'(\omega) + iG''(\omega) \tag{5}$$

$$K^*(\omega) = K'(\omega) + iK''(\omega) \tag{6}$$

The real parts $G'(\omega)$ and $K'(\omega)$ are defined as storage moduli, while $G''(\omega)$ and $K''(\omega)$ are the corresponding loss moduli. Loss factors are defined as ratios of the loss modulus to the corresponding storage modulus, i.e.

$$\eta_G = G''/G'; \quad \eta_K = K''/K'. \tag{7}$$

For typical polymers it is typically $\eta_K \ll \eta_G$, due to the fact that dissipative mechanisms are more pronounced in presence of deviatoric strains. Existing predictive models of the effective elastic properties of fibre composites can be extended to the case of viscoelastic composites by using the elastic-viscoelastic correspondence principle.

The viscoelastic materials can be modelled using normalised Prony series based on the generalised Maxwell model [23] as follows:

$$\frac{G(t)}{G_0} = 1 - \sum_{i=1}^{N} g_i\left(1 - e^{(-t/\tau_i)}\right) \tag{8}$$

$$\frac{K(t)}{K_0} = 1 - \sum_{i=1}^{N} k_i\left(1 - e^{(-t/\tau_i)}\right) \tag{9}$$

where G_0 and K_0 are instantaneous shear and bulk moduli, and g_i, k_i and τ_i are the normalised shear and bulk moduli and relaxation time constant of the *i*-th arm of the generalised Maxwell model.

2.2. Response of a transversely isotropic lamina

For a transversely isotropic, uni-directional composite lamina, viscoelasticity can be expressed, in terms of complex engineering constants, as

$$\begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} 1/E_{11}^* & -\nu_{21}/E_{22}^* & -\nu_{21}/E_{22}^* & 0 & 0 & 0 \\ -\nu_{12}/E_{11}^* & 1/E_{22}^* & -\nu_{23}/E_{22}^* & 0 & 0 & 0 \\ -\nu_{12}/E_{11}^* & -\nu_{23}/E_{22}^* & 1/E_{22}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{12}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12}^* \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{pmatrix} \tag{10}$$

Five independent loss factors can also be defined for such transversely isotropic material, as

$$\eta_{11} = \frac{E_{11}''}{E_{11}'}; \quad \eta_{22} = \frac{E_{22}''}{E_{22}'}; \quad \eta_{12} = \frac{G_{12}''}{G_{12}'}; \quad \eta_{23} = \frac{G_{23}''}{G_{23}'};$$
$$\eta_{\nu_{12}} = \frac{\nu_{12}''}{\nu_{12}'} \tag{11}$$

where a prime indicates storage properties and a double prime refers to loss properties.

The engineering constants in Eqs. (10) and (11) have to be determined experimentally or predicted numerically. Several analytical models exist to predict the values of such engineering constants and loss factors. We shall compare our numerical predictions to those of selected analytical models, namely: direct and inverse rule of mixture [3,4], Hashin's upper and lower bounds [5,6,24], Saravanos and Chamis model [14,25], Composite Cylinder Assemblage model [7], Mori-Tanaka model [1] and Lielens interpolative model [2]. These analytical models and the corresponding predictions are presented in Appendix A.

3. Numerical methods

We employed the Finite Element (FE) method to simulate the viscoelastic response of a transversely isotropic lamina and to compare the numerical predictions to those of the existing theoretical models mentioned in the previous section. We conducted comprehensive numerical analyses aimed at determining a homogenised viscoelastic tensor for a composite lamina. This was done by analysing the response of three-dimensional random arrays of cylindrical fibres, mimicking the microstructure of a unidirectional fibre composite.