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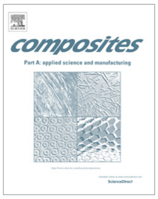

Mean-field based micro-mechanical modelling of short wavy fiber reinforced composites

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ABSTRACT

Eshelby based Mean-Field homogenization is an effective method for modelling the mechanical response of short fiber reinforced composites. These models and especially the Mori–Tanaka model have been successfully used in previous studies for predicting the overall composite mechanical response. The present work describes a method for extending Mean-Field methods to discontinuous wavy fiber reinforced composites, including calculating local stresses in the fibers. The method involves discretization of a wavy fiber into smaller segments and replacing the original segments with an equivalent ellipsoids system which can be solved with Eshelby concept. The focus of this work is the validation of the local stress fields in fibers using Finite Element benchmarks of original Volume Elements (VEs) of wavy fibers. This validation is an essential basis for further accurate modelling of the damage behavior (i.e. debonding, fiber fracture) of discontinuous wavy fiber composites.

1. Introduction

Short steel fiber reinforced polymer composites (SSFC) produced with injection molding are a new class of materials combining outstanding properties of high stiffness and ductility. Nevertheless, conditions of ductility together with high aspect ratios of fibers leads to a highly curved geometry or waviness of the steel fiber embedded in the polymeric matrix that varies throughout the composite.

In a previous paper [1], the complex micro-structure of short steel fiber composites was described. A methodology for the characterization of the geometry of short steel fiber composites using micro-computed tomography was presented and a geometrical model was developed for the generation of representative volume elements of the short random steel fiber composites.

The focus of the present work is the prediction of the mechanical behavior of short wavy steel fiber composites. The goal is to investigate a modelling approach that can be used to assess the effect of the fiber waviness on the behavior of SSFCs and to accurately predict the local composite response.

Mean-field homogenization approaches, most common among them is the Mori–Tanaka (M–T) formulation, are analytical methods which provide a very cost-effective way of predicting the effect of micro-structure, volume fraction, aspect ratio and orientation of inclusions on overall composite properties [2]. These models are based on the dilute Eshelby's solution [3] for single ellipsoidal inclusions and hence, have been typically applied in literature on simple geometries of straight fibers.

Several approaches were used in previous studies for the application of mean-field models to wavy fiber composites. The first line of approaches was proposed by Fisher et al. [4] who combined Finite Element Analysis (FEA) and M–T approach for modelling the effective properties of composites reinforced with wavy carbon nanotubes (CNT). The authors performed FE analysis on VEs of single nanotubes with a given waviness to calculate the reduced effective reinforcing moduli E_{wavy} of each NT with different magnitude of waviness, are utilized in the multi-phase M–T model to calculate the elastic properties of the composite with different wavy nanotubes. In a similar way, Bradshaw et al. [5] performed 3D FEA to compute the dilute strain concentration tensor of wavy nanotubes to be used in M–T model.

Another line of approaches was suggested by Gommers et al. [6] for knitted fabric composites where curved yarns were subdivided in small segments and each segment was replaced by an infinitely long straight inclusion. The same method was used e.g. in [7] for wavy carbon nanotubes composites. Huysmans et al. [8,9] pro-

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posed a so-called Poly-Inclusion (P-I) model in which they extended the methodology of Gommers et al. by taking into account the effect of curvature of the yarn. This is realized by decreasing the aspect ratio of equivalent inclusion depending on the local segment curvature. The method was later used and showed good predictions of the elastic properties of different textile composites [10–13].

The above studies were primarily focused on the prediction and validation of the effective mechanical properties of the wavy fiber composites. For predictions of damage of composites using mean-field models, reliable estimates of the average local stress fields in the inclusions are needed. To date the accuracy of predictions of local stresses in inclusions have not been validated.

It is clear that performing FEA which requires meshing of the fibers and the matrix is computationally expensive, even if less heavy FE formulations, e.g. embedded elements are used. While it may be reasonable for composites with smooth and prescribed wavy fiber geometries, it significantly reduces the efficiency of the mean-field techniques for composites with random waviness. In this respect the methods developed by Gommers et al. and Huysmans et al. are attractive alternatives for micro-mechanical modelling of wavy fiber reinforced composites.

In this paper, the P-I model of Huysmans et al. [8] is further developed for short wavy steel fiber composites. The aim of the study is the validation of the predictions of the average local stress state in the equivalent inclusions by comparison of the stress state in original wavy segments obtained from full FE calculations. The good prediction of the effective properties by this scheme has already been shown by Huysmans et al. In this paper, the average local stress is studied, the correct prediction of the local stresses is a more stringent requirement of the accuracy of the poly-inclusion scheme and also tests the feasibility of the PI scheme to also include damage modelling which are dependent on the local stresses. A number of models with different short wavy fiber architectures are considered in order to investigate the validity domains of the model.

2. The Mori–Tanaka approach

The effective stiffness tensor of a two-phase composite is calculated as follows:

$$\boldsymbol{C}^{eff} = \boldsymbol{C}^{m} + \langle c_\alpha(\boldsymbol{C}^{\alpha} - \boldsymbol{C}^{m})\boldsymbol{A}^{\alpha}\rangle \tag{1}$$

Subscripts α and m denote the inclusion and matrix respectively; where $\boldsymbol{C}^{eff}$ is the homogenized composite stiffness, $\boldsymbol{C}^{m}$, $\boldsymbol{C}^{\alpha}$ are the stiffness matrix of the matrix and inclusion respectively, c_α is the volume fraction of individual inclusion, and A^{α} is the so-called inclusion strain concentration tensor relating the average strain in individual inclusions to far field applied strain. The brackets $\langle\cdot\rangle$ denote the ensemble average over the total number of inclusions.

According to the Mori–Tanaka formulation [2,14] the inclusion strain concentration tensor A^{α} is given by:

$$\boldsymbol{A}^{\alpha} = \boldsymbol{A}^{\alpha}_{m}[c_m\mathbf{I} + c_\alpha\langle\boldsymbol{A}^{\alpha}_{m}\rangle]^{-1} \tag{2a}$$

$$\boldsymbol{A}^{\alpha}_{m} = [\mathbf{I} + \mathbf{S}^{\alpha\alpha}(\boldsymbol{C}^{m})^{-1}(\boldsymbol{C}^{\alpha} - \boldsymbol{C}^{m})]^{-1} \tag{2b}$$

where $\boldsymbol{A}^{\alpha}_{m}$ is the dilute strain concentration tensor relating the average strain in individual inclusions to the matrix strains. c_m is the matrix volume fraction, $\boldsymbol{I}$ is the identity matrix, and $\mathbf{S}^{\alpha\alpha}$ is the Eshelby tensor.

3. The Poly-Inclusion (P-I) model

The above mentioned formulations of the Eshelby solution and the Mori–Tanaka mean-field approximation deal with straight ellipsoidal inclusions for which the dilute Eshelby solution is known either analytically or numerically. Nevertheless, in more complex composite structures such as short random steel fiber composites the fibers are curved and the evaluation of the Eshelby tensor in Eq. (2b) is not possible. For this reason, a modelling approach is needed for the transformation of the original wavy composite into an equivalent system with ellipsoidal inclusions.

In early attempts, Gommers et al. [6] developed a methodology for modelling the effective properties of knitted fabric composites, which depict inherent curvatures of the knitting loops, based on the Mori–Tanaka homogenization method. Each repeating knitted loop is subdivided into straight fiber segments which are then replaced by ellipsoidal inclusions in the homogenization model. The equivalent ellipsoids retain the same orientation, cross-sectional shape and volume fraction as the corresponding segment. Gommers et al. considered an aspect ratio of equivalent inclusion equal to infinity.

Huysmans et al. [8] showed that this assumption leads to a strong overestimation of the predicted equivalent elastic properties in case of curved yarns. To overcome this shortcoming, they proposed taking into account the reduction of the load carrying capability of the curved segments by the so-called Poly-Inclusion (P-I) model. In the P-I model, a curved fiber is divided into a sufficiently large number of smaller segments. The effect of segment curvature is taken into account by assuming a simple inversely-proportional relationship between the equivalent inclusion's length and the original segment's curvature as follows:

$$a_r = \beta * \frac{R}{d} \tag{3}$$

where a_r is the aspect ratio of equivalent inclusion, β is the efficiency (proportionality) factor, R is the radius of curvature of the original segment as shown in Fig. 1, and d is the segment diameter. As indicated by Eq. (3), segments with higher local curvatures are modelled with equivalent inclusions with lower aspect ratios, reflecting lower efficiency of curved segment.

Huysmans et al. [8] performed an evaluation of β factor for the highly curved yarns in knitted fabric composites and found that the best correlation with the experimentally measured stiffness of the composite is obtained with β "lying around 3", and the chosen value was $\beta = \pi$. It should be noted that in the range $\beta = 1.5\ldots3$, the influence of the choice of β value on the stiffness of the composite is weak (difference in the stiffness values below 5%).

4. Problem statement and methods

The P-I model as described by Huysmans et al. [8] has been only validated for prediction of the overall macroscopic elastic constants by comparison to experimental values. While the model is based on the assumption that the average stress states in the curved fiber segments are correctly represented by the stress states in the equivalent ellipsoids, this assumption has not yet been validated. The predictions of average stresses in individual inclusions are moreover essential for modelling damage events such as fiber matrix debonding and fiber failure [15].

In the present paper, the P-I model is applied to discontinuous wavy fibers, with the main application to short steel fibers reinforcing a thermoplastic polypropylene matrix. The average stresses in equivalent inclusions, predicted by the P-I model, are compared with full-scale FE results of the average stresses in the original

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