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A micromechanical derivation of the macroscopic strength statistics for pristine or corroded/abraded float glass

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ABSTRACT

The macroscopic strength of float glass is governed by the presence of micro-cracks, whose size, orientation and distribution affects the corresponding statistics. A micro-mechanically motivated model is here proposed, which spells out the connection between crack population and strength statistics, leading to generalized distributions of the Weibull type. Aging in the form of corrosion or abrasion can produce a variation of the defectiveness scenario originally present on the pristine glass surface, and we discuss how such a modification can statistically affect the macroscopic strength. A practical application is made to justify the change in strength experimentally observed passing from the “air” to the “tin” side of float glass. Assuming that the contact with the tin bath and the rollers produce a damage equivalent to the abrasion of the glass surface, we theoretically derive a bimodal Weibull statistics that agrees with the experimental evidence.

1. Introduction

The role of glass in architecture is constantly changing and its application have expanded from that of simple window panes to large load-bearing structures. The mechanical response of glass, linear elastic up to brittle failure, is governed by the presence of micro-defects on its surface, which can be modelled as thumbnail cracks opening in mode I. The macroscopic glass fracture is thus a consequence of the flaw microstructure, since it is associated with the unstable propagation of a crack and the statistical expectation of reaching critical conditions. Due to this inhomogeneity at the microscopic level, glass response is characterized by a considerable spread in the experimentally-measured fracture stress.

Micro-cracks arise on the surfaces of soda-lime glass plates during the *float production process*. In this industrial method, patented by Alastair Pilkington, glass paste is poured on a bed of molten tin forming a floating panel. Very smooth surfaces are obtained while temperature is gradually reduced from 1100 °C down to 600 °C; then, the glass sheet is pulled off by rollers and passes through a lehr where it is gradually cooled. Consequently, the defectiveness scenarios on the side exposed to air (*air-side*) and on the face in contact with the tin bath (*tin-side*) are different. In general, the tin-side is more damaged than the air side, especially because of the abrasion produced by the rollers [1].

Glass cutting represents another aspect of primary importance because, even though the technique has been improving during the years, non-negligible additional defects are usually introduced at the borders, which can be schematized as semi elliptical surface flaws and/or quarter elliptical corner cracks. The edge finishing (clean cut, seamed or polished) is certainly a discriminant issue for the evaluation of macroscopic strength of structural glass elements when maximum tensile stresses act at the borders. It should also be mentioned that cracks can grow over time even at stress levels much lower than the critical limit [2], due to a phenomenon usually referred to as *static fatigue* or *subcritical crack growth*, which makes the gross material strength dependent upon time and thermo-hygrometric conditions.

A strict factory production control is provided for marketed glass plates, which rejects those panes with defects in transparency. Since the optical and aesthetic properties of glass are determined by the amount of existing flaws, another consequence of the factory production control is that it eliminates those elements that presents cracks whose size is above a certain limit. From a statistical point of view, this is equivalent to a lower truncation in the population of glass strength, i.e., there is a lower bound of glass strength that has been confirmed by experiments [1].

Anyway, the defectiveness scenario that is present on the glass surfaces after the acceptance phase (pristine glass) could

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be modified by corrosion and/or abrasion phenomena. The attack of alkali solutions leads to the dissolution of external surfaces, producing a leveling of the preexisting defects, whereas an increasing in the porosity could be due to acid corrosion or corrosion by water. On the other hand, abrasion consists in adding new cracks to the pre-existing ones. The typical scenario is that of sand blasting, a common situation for buildings in desert areas. Various experimental campaigns on abraded glass specimens are recorded in the technical literature [3–5], which provide the conclusion that the effect of ageing is that of producing an additional flaws distribution, whose maximum depth can be considered much less than that due to production process. Obviously the sharp grooves produced by a diamond bit, which are much deeper than the largest preexisting cracks so to drive a precise cut, should not be associated with a natural degradation of the material.

Structural strength assumes strongly non-deterministic values because of the random nature of shape, size and distribution of surface flaws. Also the state of stress can influence the macroscopic glass strength. To this respect, the uniform equibiaxial state of stress represents the most severe condition for a glass specimen, since in such case there is the 100% probability that the crack axis is perpendicular to the maximum principal direction of stress. The opposite is true for what concerns uniaxial stress state, because the existing crack may not be oriented at right angle to the maximum tensile direction. The macroscopic strength is also size-dependent, because the larger the loaded area is, the higher is the probability of finding a defect of critical size [6]. Furthermore, the annealing treatment at the end of the float production process leads to an initial thermal healing of the cracks previously generated. As shown in [7], the effects of thermal healing are more pronounced on the edges of the glass panes than in the central parts of their surface.

The Weibull statistics, based upon the weakest-link-in-the-chain concept [8], is generally chosen for interpreting the variability of failure stress values with reference to structural glass specimens. In fact, it is sufficient to reach the critical condition at one crack to produce the catastrophic failure of the specimen. The 2-parameter Weibull distribution is certainly the statistical model most widely used for brittle materials. However, its inability to interpret the experimental data associated with small failure probabilities has been substantiated by many researchers [9–11]. In a previous work [1], the authors have justified the experimental finding by postulating the existence of a lower bound for glass strength due to the aforementioned production control phase, and have consequently proposed to use a left-truncated Weibull distribution to interpret the experimental results. Arguments were presented which supported this hypothesis and, moreover, it was shown that such minimal strength can be reduced, but not annihilated, by natural degradation due to ageing and in-service-related damage.

Many works, in which the variability of material strength due to aging is discussed, are available in technical literature, but just a very few of these correlate the gross material response with the underlying micro-crack scenario. Indeed, a limited knowledge exists about crack density, size, shape and orientation, which instead represent relevant parameters to understand the macroscopic mechanical response of glass. Assuming that exhisting cracks do not interact one another, the connection between flaw and macroscopic-strength distributions was inferred by *Freudenthal* [12] and, more recently, by *Batdorf and Crose* [13] and *Le et al.* for silicon mems [14]. The work by *Freudenthal* [12], in particular, could be considered a milestone for what concerns the micro-mechanical motivation of the 2-parameter Weibull statistics.

Here, we present a modified microstructurally-motivated model, able to describe the strong connection between the distribution of crack length, assumed to be power-law shaped, and generalized Weibull statistics. In particular, an upper-truncation of the crack lengths distribution gives rise to a left-truncated Weibull distribution for macroscopic glass strength. Moreover, we discuss in detail how variations of the defectiveness scenario, due to corrosion and abrasion phenomena, can affect upon the macroscopic strength. Our approach necessarily does not cover all complex phenomena that accompany the change in the crack distribution, such as the crack healing consequent to thermal treatments, or the appearance of residual stresses due to the generation of new micro-cracks, or the crack blunting associated with corrosion. Although the proposed approach could be directly extended to cover such effects, the lack of appropriate experimental data suggests us to postpone further discussion on future works. As a practical application, manipulating the results of a large experimental campaign recorded in the technical literature [15], we use the proposed rationale to interpret the observed difference between the experimentally-measured population of strengths for the air- and tin-side of float glass.

2. Cumulative probability distribution for the size of micro-defects in float glass

The macroscopic strength of glass is governed by the distribution of micro-defects on its surface. Such defects can be modelled as thumbnail micro-cracks, whose plane is orthogonal to the glass surface. Failure occurs when the stress intensity factor associated with the dominant micro-crack reaches the critical value.

The first assumption in the present theory is that there is a *Representative Area Element* (RAE) on the glass surface, say ΔA, whose diameter is comparable with the average size of the micro-cracks. The main property of the RAE is that it can host one crack, so that the number of cracks that are contained in a specimen of area A is $A/\Delta A$. Inspecting the glass surface with a microscope, it is possible, at least in principle, to divide the area in sub-areas ΔA, and measure in each of them the size δ of the micro-crack there located. One can thus calculate the corresponding statistics, i.e., the probability of finding, in a specific area ΔA, a microcrack of size δ. We expect that as the number of elements in the population tends to become very large, the statistics tends to a definite probability function.

It is reasonable to expect a highly right-skewed distribution of the crack size, meaning that while the bulk of the distribution occurs for fairly small size, there is a small number of critical cracks, of size much higher than the average value, which leads to a very long right-hand-side tail. One of the functions able to interpret this kind of variability is certainly the *power law distribution*, whose most relevant analytical attribute is that of *scale invariance*.

We assume that, right after the production process, in a material that could be considered "pristine", the probability density function for the crack size δ in an area ΔA can be written in the form

$$p_{\Delta A,\delta}(\delta) = C\ \delta^{-\alpha}, \tag{2.1}$$

where α is the *scaling parameter* and C is a normalization constant.

Observe that the function (2.1) diverges as $\delta \to 0$, so that a lower bound $\delta_{\min}$ shall be imposed. The parameter $\delta_{\min}$ might be physically interpreted as the size of the *physiological* defects in glass, i.e., the size of cracks that are naturally present in any glass produced with an industrial process. This definition, somehow artificial at this point since it is introduced to make consistent the power-law assumption (2.1), will be discussed in detail later on. One should observe, however, that strength of glass is governed by large cracks, i.e., $\delta \gg \delta_{\min}$, so that what is really important is the shape of the probability function on the right-hand-side tail of the distribution. Consequently, $\delta_{\min}$ should rather be considered as a material parameter, not necessarily associated directly with the minimum flaw size, whose importance consists in the fact that, in the expression (2.1), it analytically characterizes the statistics of large cracks in the asymptotic limit $\delta \to \infty$.