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Bayesian analysis for determination and uncertainty assessment of strength and crack growth parameters of brittle materials

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ABSTRACT

To determine the strength and the crack growth parameters of brittle materials, the common approach is to first evaluate the strength and then the crack growth parameters. When the parameters are needed for different temperatures, the procedure is repeated for each temperature of interest. Since the number of test specimens is generally small, the separated analysis of strength and crack growth parameters for each tested temperature leads to large parameters uncertainty. In order to improve the accuracy of the parameters, we propose a Bayesian method that allows to combine all strength and lifetime data obtained at different temperatures and determine the distribution of all material parameters in a single analysis. The results obtained from the analysis of measured skutterudite data show that in comparison to the standard approach the presented method significantly reduces the material parameters uncertainty and therefore is well adapted for a reduced number of samples.

1. Introduction

In component design, the strength and the crack growth parameters of brittle materials are relevant material properties. The strength parameters allow to forecast and avoid spontaneous failures whereas the crack growth parameters are used to predict the failure probability over the lifetime. Due to the inherent presence of statistically distributed flaws in brittle materials, these parameters are subjected to large uncertainty and required a large number of test specimens as well as statistical analysis in order to reduce their uncertainties and obtain accurate results. For the strength parameters, it is common to perform strength tests and derive the parameters from Weibull analysis. For the determination of crack growth parameters different methods can be used [1]. One of the common used approach, which we also used in this work is the Weibull analysis of lifetime data.

In many practical applications one is interested in the material properties over a wide temperature range. In such applications, the common procedure is to independently determine the material parameters for each temperature levels. This standard approach has two major drawbacks. First, it requires a large number of test specimens to get reliable results and second, it does not consider that some parameters are temperature independent according to the standard theory of fracture mechanics. For the strength analysis, pooling procedures based on regression models have been developed in order to take into account strength data from different temperature for the Weibull analysis and consequently reduce the required number of test specimens [2,3]. In the pooling approach, strength from different temperature levels are converted by a regression model to some corresponding strength at a reference temperature. This leads to loss of information since the temperature dependency of the strength cannot be exactly represented by an equation.

In this work, we present a novel method based on Bayesian networks, which allows the combined analysis of data from different temperature levels as well as the consideration of the temperature dependence of the parameters. The method does not require any kind of data transformation and can take into account strength as well lifetime parameters at the same time. Using Bayesian networks, the relationships between the material parameters and the measured data can easily be identified and based on probabilistic analysis, the distribution of all the parameters (strength and lifetime parameters) can be determined in only one step. The presented approach relies on fracture mechanic laws and on the Bayesian analysis.

In Section 2, theories about analysis of strength and lifetime data is presented. Section 3 gives a short introduction to Bayesian

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network. Using some measured strength data at RT and 500 °C as well as cyclic lifetime data at RT, the application of the novel Bayesian method is presented in Section 4. A comparison to the standard approach is also performed.

2. Mechanical properties of brittle material

2.1. Strength parameters

Fracture of brittle material results from large stress concentrations at microscopic flaws, which are unavoidably present in the form of pores, inclusions and precipitations. When a brittle specimen is loaded, stress singularities occur at crack tips. To describe these singularities, one defines the stress intensity factor K [1]:

$$K = \sigma\sqrt{\pi a}Y, \tag{1}$$

where the "geometry factor" Y depends on the crack size a, the geometry of crack, specimen and stress field. If the stress increases, the stress intensity factor also increases. This happens until K reaches a critical value K_c at which the specimen fails. This critical value is a material property called fracture toughness. The corresponding critical crack length is given by [4,5]:

$$a_c = \frac{1}{\pi}\left(\frac{K_c}{\sigma Y}\right)^2. \tag{2}$$

Since K_c is a material constant, the strength of a brittle component depends principally on the size of defects present in the component. The scatter of the flaws size results, therefore, in a scatter of the strength. Knowing that small flaws are not relevant for failure, the density function of flaws $f(a)$ can be approximated by a power law:

$$f(a) \propto \frac{1}{a^r}. \tag{3}$$

Based on this equation and on probabilistic derivation, it can be shown that the failure strength of a brittle component follows the Weibull distribution:

$$P_f = 1 - \exp\left[-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right], \tag{4}$$

where V_0 is a normalising volume. The parameters m and σ_0 denote the Weibull modulus and characteristic strength, respectively. m is a measure of the scatter of strength data and is related to the scatter of flaw size by the relation $m = 2(r - 1)$ [1,5].

To determine the Weibull parameters m and σ_0, the standard procedure consists in performing bending tests and evaluating the maximum likelihood of the parameters using Eq. (4). According to the product rule of probability, the likelihood function can be written as:

$$L(\sigma|m, \sigma_0) \propto \prod_{i=1}^{n} p_f(\sigma_i, m, \sigma_0) \tag{5}$$

where n is the number of tested specimens and $p_f(\sigma_i, m, \sigma_0)$ is the Weibull probability density function (PDF) of failure strength with parameters m and σ_0. For any set of parameters m and σ_0, the function L adopts a different value. The best set of parameters are the one giving the highest value of L.

2.2. Subcritical and fatigue crack growth parameters

Subcritical crack growth (SCG) is a time dependent phenomenon in which a crack progressively grows at load below the fracture load until reaching its critical size. The commonly used model to describe SCG rate is the power law also known as Paris law. The model covers low crack growth rates which are interesting for lifetime prediction and is given by the following equation:

$$v = \frac{da}{dt} = A_s\left(\frac{K}{K_c}\right)^{n_s} \tag{6}$$

where v is the crack growth rate and K and K_c are the actual and the critical stress intensity factor, respectively. A_s and n_s are the subcritical crack growth parameters, which depend on the temperature and the environmental medium [1,6]. Based on Eq. (6), it is possible to compute the time to failure t_f using the equation:

$$\int_0^{t_f} \sigma(t)^{n_s} dt = B_s\sigma_c^{n_s-2} \tag{7}$$

with

$$B_s = \frac{2K_c^{2-n_s}}{A_s Y^2(n_s - 2)} \tag{8}$$

It should be noted that this equation is only valid for high value of n_s ($n_s > 10$). The parameter B_s is generally determined and used for lifetime estimation instead of A_s. To determine the SCG parameters B_s and n_s various tests and analysis methods are available. We here only introduce the ones which are relevant for this paper. A good overview can be found in [1]. One of the commonly used approaches to determine SCG parameters for natural flaws, is to perform static bending tests at a single stress level and to analyze the resulting lifetimes. The time to failure is subjected to a large amount of scatter which is related to the scatter of the initial flaw size. Specimens which small flaws need a long time to fail, where as, those with large flaws just need a short time to reach the critical size and fails first. Using the fact that the time to failure depends on the initial flaws size, it has been derived that the time to failure is Weibull distributed given the distribution function [1]:

$$P_{t_f} = 1 - \exp[-(t_f/t_0)^{m^\star}] \tag{9}$$

with

$$m^\star = \frac{m}{n_s - 2} \tag{10}$$

$$t_0 = B\sigma_0^{n_s-2}\sigma^{-n_s} \tag{11}$$

The SCG parameters can be found in this case by means of a Weibull analysis, whereby, the strength parameters m and σ_0 need to be known.

A widespread alternative to the Weibull analysis of lifetime data is the linear fit of lifetime data obtained from static bending tests using different stress levels. Under static load, Eq. (7) can be integrated and write to the logarithmic form:

$$\log t_f = -n_s \log\sigma + \log[B_s\sigma_c^{n_s-2}]. \tag{12}$$

The parameters n and B are determined from the linear regression of $\log t_f$ versus $\log\sigma$. The best results of the regression are obtained by minimizing the error in the axis of t_f.

Slow crack propagation can be observed under static as well as under cyclic load. For some materials like porcelain and glass, the same mechanism is responsible for crack extension in both loading conditions, so that the lifetime prediction for cyclic loading can be made on basis of SCG [1]. However, cyclic effects also known as fatigue occur in many brittle materials and must be considered for corresponding applications. Similar as SCG, Fatigue crack growth rate can be described using Paris law [6,7]:

$$v = \frac{da}{dN} = A\left(\frac{\Delta K}{K_c}\right)^n. \tag{13}$$

$\Delta\sigma$ denotes the difference between the maximum and the minimum stress. The fatigue crack growth parameters A and n depend on the test conditions. For cyclic loading, the crack growth parameters can be determined using the same methods as for SCG. A