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Short communication

Influence of the scatter of strength and of measurement uncertainties on the determination of the subcritical crack growth exponent in ceramics and glasses

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ABSTRACT

Subcritical crack growth (SCCG) in glasses and ceramics causes a decrease of strength with time under applied load. The dependence of the crack growth rate with the stress intensity factor can be approximated by a power law, with n being the SCCG exponent. In this work the effect of the inherent scatter of strength as well as measurement uncertainties on the determination of n is analysed. Constant stress rate tests as proposed in the EN 843-3 standard are simulated using Monte Carlo techniques. It is shown that the large scatter of strength causes large uncertainties on the determination of n, yielding a significant influence of the sampling procedure especially for small samples. Measurement uncertainties have a surprisingly low influence on the determination of n with exception of a special situation that may arise during long time testing campaigns.

1. Introduction

Ceramics and glasses are brittle materials. In general, they can be modelled as ideal elastic continua and their fracture can be described in the frame work of the linear elastic fracture mechanics. Fracture is caused by the most severe (the critical) flaw that is contained in the bulk material or at the surface of the tested specimen. Flaws are described as cracks. Accordingly, under external applied stress, fracture occurs if the stress intensity factor K at the crack tip equals or exceeds a critical value (the fracture toughness, K_c) [1,2]:

$$K \geq K_c. \tag{1}$$

For a given applied stress, σ, the stress intensity factor can be expressed as:

$$K = \sigma \cdot Y \cdot \sqrt{\pi \cdot a}, \tag{2}$$

where a is the crack length and Y is a geometric factor that depends on the geometry of the crack, the specimen and the shape of stress field. The strength of the material, σ_f, is the applied stress under which fracture occurs. It depends on the size of the greatest flaw, the so-called critical crack, a_c, in the specimen (more precisely: on the size of the flaw having the largest stress intensity factor, as derived from Eqs. (1) and (2)). It follows from the referred equations that $\sigma_f \propto {a_c}^{-1/2}$: i.e. large critical flaws cause a low strength and small critical flaws a high strength. If the frequency density of flaws decreases with the flaw size, this behaviour is well described by the Weibull theory (more precisely this happens if the frequency density decreases with an inverse power of the flaw size) and the material is considered to be a Weibull material [3–6]. A consequence of the brittle behaviour is the "size effect of strength", i.e., it is more probable to find a large flaw in a large specimen than in a small specimen. This behaviour is also well described by the Weibull theory [7,8]; a cumulative failure probability, F, can be defined as:

$$F = 1 - \exp\left[-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right], \tag{1}$$

where σ_0 is the characteristic strength (related to a reference volume V_0) of the distribution (i.e. for specimens of volume $V = V_0$, the failure probability is: $F(\sigma_0, V_0) = 1 - \exp(-1) = 63\ \%$). The Weibull modulus m (the shape parameter) describes the scatter of the strength data, which is as larger as smaller is m. Common values of m for ceramic materials are between 5 and 30.

In many glasses and ceramics cracks can also (stably) grow even if the stress intensity factor is lower than the fracture toughness.

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This damage mechanism is called subcritical crack growth (SCCG). In the early work of Wiederhorn it was detected that the crack growth rate in glass, $v = da/dt$, strongly depends on the stress intensity factor K. This effect is thermally activated and can be enhanced by humidity in the environment [9]. Wiederhorn did also recognise that in a so-called v-K curve (i.e. double-log plot of crack growth rates versus stress intensity factors), "regions" having different slopes exist, which can be ascribed to different damage mechanisms (see Ref. [10,11] for more details). Interestingly, SCCG may also be evidenced when testing in vacuum or inert atmospheres, yet crack growth rates are much higher when testing in humid atmospheres (more general: in environments containing polar molecules, e.g. water [10,11]). Possible reasons for crack growth are thermally activated and/or environmentally activated bond breaking [12]. The diffusion of active molecules (e.g. water) along the crack at the surface, the chemical reaction of the molecule with the bonds at the crack tip, or again thermally activated breaking of the weakened bonds at the tip have also been recognised as rate controlling mechanisms [13]. Also other reasons for SCCG have been proposed. It should be recognised that there is still much work to do to get a common sense on the physical mechanisms of damage of SCCG.

From the technical point of view SCCG causes a reduction of strength with time under load: even at a static load cracks may grow. This causes a monotonic increase of the stress intensity factor (see Eq. (2)) until it reaches the fracture toughness (Eq. (1)) and instantaneous brittle fracture occurs. At relatively low loads the crack growth rate may be very low and the life time may become extremely long, e.g. tens or hundreds of years. At high loads the stress intensity factor is only a little smaller than the fracture toughness and the growth rate may become very high, e.g. mm up to m per second and even more. In this case the life time may be only a little fraction of a second. For moderate loads, ranging between these extreme cases, the life time may be several hours, months or few years. This is the region of technical relevance (this corresponds to growth rates from 10^{-13} ms^{-1} to 10^{-6} ms^{-1}). In this region, so-called region I, SCCG in an air atmosphere is often caused by water assisted bond breaking (i.e. it depends on the humidity in the air).

The crack growth rate v in region I can empirically be described by a power law:

$$v = da/dt = v_0 \cdot \left(\frac{K}{K_c}\right)^n \tag{3}$$

The SCCG parameters v_0 and n are the crack growth parameters; n is called the SCCG exponent. Note that n is – in general – a relatively high number. For instance, values around $n = 30$ have been reported in alumina and alumina-glass composites tested in ambient air at room temperature [14–17]. In most cases values for n are between 10 and more than 100.

In this work possible measurement uncertainties on the determination of the SCCG exponent are investigated using Monte Carlo simulations. The influence of the scatter of material properties (scatter of strength expressed by the Weibull modulus and the inherent crack growth exponent), experimental parameters such as the sample size, experimental errors in the determination of strength as well as details of the data fitting procedure are analysed.

2. State of the art

The change of strength of specimens with time caused by SCCG or their life time can be calculated using Eq. (3) [18]. Separation of the variables time, t, and crack length, a (note: the stress amplitude may depend on time) yields:

$$\frac{da}{dt} = v_0\left(\frac{\sigma Y\sqrt{\pi a}}{K_c}\right)^n \rightarrow \sigma^n \mathrm{d}t = \frac{1}{v_0}\left(\frac{K_c}{Y\sqrt{\pi a}}\right)^n \mathrm{d}a. \tag{4}$$

Integration gives (under the assumption that $n > 2$):

$$\int_0^{t_f} \sigma\left(t'\right)^n dt' = \frac{1}{v_0}\left(\frac{K_c}{Y\sqrt{\pi}}\right)^n \left(\frac{-2}{n-2}\right) a^{1-\frac{n}{2}}\Big|_{a_0}^{a_c}, \tag{5}$$

where a_0 is the size of the critical flaw at the time $t = 0$ and a is the critical crack size at time t after some crack growth.

To predict the lifetime (i.e. the time to reach a critical crack length $a_c = (1/\pi)\cdot(K_c/Y\sigma)^2$), the SCCG parameters need to be known. The right side of Eq. (5) is a constant number. The left side increases with time until it gets the same value than the right side. The upper integration limit is the life time, t_f. It is important to mention that since n enters the equation as an exponent, small uncertainties in the determination of its value may cause large errors in life time estimation.

In order to determine the SCCG parameters v_0 and n, several techniques have been developed in the past (a thorough summary can be found in [18]). The so-called "direct" measurement techniques use a specimen with a long crack, for instance double torsion (DT) or compact tension (CT) specimens. During these experiments the crack is loaded in a controlled configuration resulting in a well-defined stress intensity factor. The crack length is measured in time intervals e.g. using a travelling microscope. Results of direct measurements are easy to interpret, but they have some drawbacks: (i) Measuring slow growth rates requires very long observation times (e.g. a growth rate about 10^{-13} ms^{-1} needs an observation time of about one year), which is not comfortable in practice. (ii) The microscopic observation of growing cracks over long time periods demands great expertise. (iii) Additionally, the relatively long cracks as used in direct experiments may behave different to typically critical cracks in components, which have a length of some ten μm or less.

Based upon the above arguments, there has been a search for alternative, cheaper, less time consuming and less demanding testing techniques, where the growth of small (natural) cracks may be assessed. This is done in the so-called "indirect" measurement techniques. Investigated are – in general – rectangular bar-shaped specimens in a three- or four-point bending configuration. The simplest type of test is the loading with a constant stress, where the time to failure is recorded. Since the size of the critical flaws in the specimens is distributed, the life time is also distributed. From the frequency distribution of life times the SCCG parameters can be deduced. However, since the distribution is very wide, the experimental time can hardly be planed, which is also not a comfortable situation.

In our lab, tests are preferably done with a constant loading rate. For specimens tested at a low rate, there is more time for SCCG than for specimens tested at a higher rate. Therefore, the cracks can grow (subcritical) to a greater length and the strength results lower. To determine the SCCG exponent, specimens are tested with different stress rates, $\dot{\sigma}$. The procedure for that type of data evaluation is described in detail in EN 843-3 and ASTM 1368[19,20].

Let us evaluate Eq. (5) for constant strain rate tests. Here it holds: $\sigma = \dot{\sigma}t$, with $\dot{\sigma}$ being the stress rate and t being the time. In the following, it is assumed that the geometric factor does not depend on the crack length. This holds true, if the crack length is small compared to the specimen dimensions. In modern ceramics, the size of typical flaws is in the range of some μm or of some ten μm. Compared to the height of a typical bending specimen (several mm) the assumption is therefore valid. Note that this assumption will become invalid if specimens contain large cracks. This happens in

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