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Photonic band gap spectra in Octonacci metamaterial quasicrystals

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ABSTRACT

In this work we study theoretically the photonic band gap spectra for a one-dimensional quasicrystal made up of SiO_2 (layer *A*) and a metamaterial (layer *B*) organized following the Octonacci sequence, where its *n*th-stage S_n is given by the inflation rule $S_n = S_{n-1}S_{n-2}S_{n-1}$ for $n \geq 3$, with initial conditions $S_1 = A$ and $S_2 = B$. The metamaterial is characterized by a frequency dependent electric permittivity $\varepsilon(\omega)$ and magnetic permeability $\mu(\omega)$. The polariton dispersion relation is obtained analytically by employing a theoretical calculation based on a transfer-matrix approach. A quantitative analysis of the spectra is then discussed, stressing the distribution of the allowed photonic band widths for high generations of the Octonacci structure, which depict a self-similar scaling property behavior, with a power law depending on the common in-plane wavevector k_x.

1. Introduction

The discovery of quasicrystals in 1982 by Shechtman et al. [1] has started a new field in condensed matter physics. They define a new class of neither amorphous nor crystalline structure exhibiting non translational symmetry. Besides, they can be generated by a substitution rule based on two or more building blocks with long-range order [2–4], exhibiting properties of self-similarity in their spectra with an undoubtedly fractal behavior, with a distinct appearance for each chain [5], even for different excitations [6–8]. As a consequence, many theoretical and experimental works have been reported on this subject (for reviews see Refs. [9–11]).

Quasicrystals are a special class of deterministic aperiodic structures [12]. A recent precise definition of quasicrystals with dimensionality *d* ($d = 1$, 2 or 3), is that in addition to their possible generation by a substitution process, they can also be formed from a projection of an appropriate periodic structure in a higher dimensional space *mD*, where $m > d$ [13]. In contrast, structures that are part of other deterministic structures cannot be built in such way, as by instance quasicrystalline structures of Fibonacci type and their generalizations [14–16], as well as systems that obey the Thue-Morse, double-period and Rudin-Shapiro sequences [17,18]. In this context, the one-dimensional Octonacci structure can be considered as a quasi-crystal because it can be formed from a projection of a 2D periodic lattice in a straight line with an irrational slope $\sigma = 1 + \sqrt{2}$ [19].

On the other hand, the idea of complex materials in which both the electrical permittivity and the magnetic permeability possess negative real values at certain frequencies has received considerable attention nowadays due to their potential technological application. This idea was born in 1967 when Veselago theoretically investigated the electromagnetic plane-wave propagation in a material whose permittivity and permeability were assumed to be simultaneous negative [20]. In his theoretical study, Velesago showed that for a monochromatic electromagnetic uniform plane wave in such a medium, the direction of the Poynting vector is antiparallel to the direction of the phase velocity, contrary to the electromagnetic plane-wave propagation in conventional simple media. This theoretical complex medium was setting up by Smith, Pendry, and collaborators [21–24]. They have constructed a composite medium that exhibit the anomalous refraction in microwave regime, demonstrating experimentally the negative refraction studied by Veselago. Many research groups all over the world are now studying their various aspects looking for technological applications [25].

Bulk and surface plasmon-polariton have been

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experimentally and theoretically studied for many years due to their possible use in novel photonic and sensing applications (for a review see Ref. [26]). In quasiperiodic structures they exhibit collective properties due to the appearance of long range correlations, which are reflected in their fractal spectra, defining a novel description of disorder [27]. The study of the fractal spectra generated by these quasiperiodic structure can help us to understand the global order and the rules that these systems obey at high generation order. By instance, their spectra in Fibonacci quasiperiodic photonic crystal composed by metamaterials, were already the subject of intense research works [28–30].

The aim of this work is twofold: first, we want to extend our previous work on the transmission spectra in Octonacci photonic quasicrystals [31] by considering the photonic band gap spectra arising from the propagation of a plasmon-polariton excitation in these quasiperiodic multilayer structure. Second, we intend to present a quantitative analysis of the results, mainly those related to the allowed photonic band widths, looking for information about their localization and power laws.

This paper is organized as follows: in Section 2, we present the theoretical model based on the transfer matrix approach to set up analitically the plasmon-polariton dispersion relation (bulk and surface modes). The discussion of this dispersion relation for the Octonacci quasiperiodic structure is then depicted in Section 3, together with their localization profiles through the scaling law of their photonic bandwidth spectra. The conclusions of this work are presented in Section 4.

2. Theoretical model

The Octonacci sequence, also known as Pell sequence, can be built from the Ammann-Beenker tiling, which is an octagonal tiling obtained by using a strip projection method (see Fig. 1.18 in Ref. [32]). The name Octonacci comes from *Octo* for orthogonal and *nacci* from the Fibonacci sequence, the oldest example of a quasiperiodic chain. Its quasi-periodicity can be of the type so-called substitutional sequences, and is characterized by the dense pure point nature of its Fourier spectrum, being described in terms of a series of generations that obey peculiar recursion relations. It can also be defined by the growth, by juxtaposition, of two building blocks A (here considered to be SiO_2) and B (a metamaterial), where the nth-stage of the multilayer S_n is given iteratively by the rule [33]:

$$S_n = S_{n-1}S_{n-2}S_{n-1}, \tag{1}$$

for $n \geq 3$, with $S_1 = A$ and $S_2 = B$. The number of the building blocks increases according to the Pell number $P_n = 2P_{n-1} + P_{n-2}$, for $n \geq 3$, with $P_1 = P_2 = 1$. The number of building blocks B divided by the number of building blocks A, in the limit $n \to \infty$, is $\sigma = 1 + \sqrt{2}$. Another way to obtain this sequence is by using the following inflation rule: $A \to B$, $B \to BAB$. Note that this sequence is classified as Pisot-Vijayaraghavan (PV), when we take the negative eigenvalue of the substitution matrix, i e., $\sigma^- = 1 - \sqrt{2}$ [34,35].

Let us consider first the periodic photonic crystal case. The bulk plasmon-polariton dispersion relation is obtained by solving the electromagnetic wave equation for a p-polarized electromagnetic mode, within the layers A and B of the nth unit cell of the layered photonic crystal (see Fig. 1), yielding:

$$\cos(QL) = (1/2)\text{Tr}(T), \tag{2}$$

where Tr(T) means the trace of a transfer matrix T. The details of

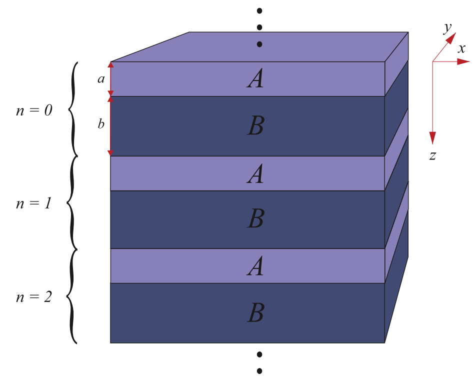

Fig. 1. Schematic representation of a periodic photonic structure $ABAB\cdots$. Here a and b are the thicknesses of the layers A and B, respectively.

this calculation can be found elsewhere [27]. Using these equations, we can show that for the periodic case this dispersion relation is a function of sines and cosines of the wavevectors k_{zA}, k_{zB} and Q, the Bloch wavevector, and the size $L = a + b$ of the unit cell.

To set up the dispersion relation for the surface plasmon-polariton modes, we consider the multilayers structure truncated at $z = 0$, with the region $z < 0$ filled by a transparent medium C, whose dielectric constant is denoted by ε_C. This semi-infinite structure does not present translational symmetry in the z-direction and therefore the Bloch theorem is not valid in this case. Its implicit dispersion relation is [27].

$$T_{11} + T_{12}\lambda = T_{22} + T_{21}\lambda^{-1}, \tag{3}$$

were $T_{ij}\,(i,j = 1,2)$ are the elements of the transfer matrix T, and λ is a surface dependent parameter given by

$$\lambda = (\xi_A + \xi_C)/(\xi_A - \xi_C), \tag{4}$$

$$\xi_j = \varepsilon_j / k_{zj}\,, \tag{5}$$

with $j = C$ or A. Now we extend this method to obtain the plasmon polariton dispersion relation for the Octonacci photonic structure by determining the appropriated transfer matrices. It is easy to prove, by induction method, that the transfer matrices for any Octonacci n-generation (with $n \geq 3$) is given by

$$T_{S_n} = T_{S_{n-1}}T_{S_{n-2}}T_{S_{n-1}}, \tag{6}$$

with the initial conditions

$$T_{S_1} = N_A^{-1}M_A; \quad T_{S_2} = N_B^{-1}M_B. \tag{7}$$

The matrices M_J and N_J ($J = A$ or B) are defined elsewhere [27]. Therefore, from the knowledge of the transfer matrices T_{S_1} and T_{S_2}, we can determine the transfer matrix of any other Octonacci generation.

3. Numerical results

Now we present some numerical results related to the photonic band gap spectra due to the plasmon-polariton excitation (bulk and surface modes) that can propagate in the Octonacci structure considered here. Medium B (A) is a metamaterial (SiO_2) with a frequency dependent (constant) electric permittivity $\varepsilon_B(\omega)$ ($\varepsilon_A = 12.3$) and magnetic permeability $\mu_B(\omega)$ ($\mu_A = 1$) in the