Optical Materials xxx (2016) 1–9

Contents lists available at ScienceDirect

Optical Materials

journal homepage: www.elsevier.com/locate/optmat

Octonacci photonic crystals with negative refraction index materials

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A R T I C L E I N F O

Article history:
Received 19 October 2016
Accepted 10 November 2016
Available online xxx

Keywords:
Photonic crystals
Metamaterials
Transmission
Multilayers

A B S T R A C T

We investigate the optical transmission spectra for *s*-polarized (TE) and *p*-polarized (TM) waves in one-dimensional photonic quasicrystals on a quasiperiodic multilayer structure made up by alternate layers of SiO_2 and *metamaterials*, organized by following the Octonacci sequence. Maxwell's equations and the transfer-matrix technique are used to derive the transmission spectra for the propagation of normally and obliquely incident optical fields. We assume Drude-Lorentz-type dispersive response for the dielectric permittivity and magnetic permeability of the metamaterials. For normally incident waves, we observe that the spectra does not have self-similar behavior or mirror symmetry and it also features the absence of optical band gap. Also for normally incident waves, we show regions of full transmittance when the incident angle $\theta_C = 0^\circ$ in a particular frequency range.

1. Introduction

Recently, the idea of complex materials in which both the permittivity and the permeability possess negative real values at certain frequencies has received considerable attention. In 1967, Veselago theoretically investigated plane-wave propagation in a material whose permittivity and permeability were assumed to be simultaneously negative [1]. His theoretical study showed that for a monochromatic uniform plane wave in such a medium the direction of the Poynting vector is antiparallel to the direction of the phase velocity, contrary to the case of plane-wave propagation in conventional simple media. In recent years, Smith, Schultz, and their group constructed such a composite medium for the microwave regime and demonstrated experimentally the presence of anomalous refraction in this medium [2,3]. For metamaterials with negative permittivity and permeability, several names and terminologies have been suggested, such as “left-handed” media [1–7]; media with negative refractive index (NRI) [1–4,6]; “backward-wave media” (BW media) [8]; and “double-negative” (DNG) metamaterials [9], to name a few. Many research groups all over the world are now studying various aspects and applications of these materials which have been proposed.

Also, since the discovery of quasicrystals by Shechtman et al. in 1984 [10], many efforts have been conducted to understand the physical properties of these aperiodic materials, which possess a long range order without having a translational symmetry. Hence, quasicrystals are regarded to have a degree of order intermediate between crystals and disordered systems. Quasicrystalline systems have been extensively studied, not only with respect to their structure, which shows uncommon rotational symmetries [10–12], and their electronic states, which were found to show a Cantor-set spectrum in one dimension [13–16], but also phonons and magnetic properties of these materials have been investigated [17–21]. Although the term quasicrystal is more appropriate when applied to natural compounds or artificial alloys, in one dimension (1D) there is no difference between this and the quasiperiodic structures formed by the incommensurate arrangement of periodic unit cells. An appealing motivation for studying such structures is that they exhibit a highly fragmented energy spectrum displaying a self-similar pattern.

Further, the photonic properties of those quasiperiodic structures are of special interest because the complex symmetries in quasicrystals make them suitable for the application in several optical devices such as single-mode light-emitting diodes, polarization switching and microelectronic devices that are based on photons rather than on electrons, which potentially can be the electromagnetic analogue to semiconductors [22–26]. The theoretical study of photonic properties of one-dimensional systems is based on the transfer matrix method and the concept of aperiodic mathematical sequences, as the Fibonacci sequence, the Thue-Morse sequence, and Cantor sequences [23,27–29]. Such one-dimensional systems can be relatively easily produced in reality

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http://dx.doi.org/10.1016/j.optmat.2016.11.013
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Please cite this article in press as: E.R. Brandão, et al., Octonacci photonic crystals with negative refraction index materials, Optical Materials (2016), http://dx.doi.org/10.1016/j.optmat.2016.11.013

and a comparison of the theoretical and the experimental results shows a good agreement [30,31]. For the theoretical study of quasiperiodic systems one often applies the concept of aperiodic mathematical sequences/tilings. Especially, the photonic properties of one-dimensional systems have been extensively analyzed with this approach, where transfer matrix methods can be applied. There are examples based upon the Fibonacci, Thue-Morse and Cantor sequences [23,27,28,32–34], and also systems with negative refractive indices have been studied [35–38,40,41].

It is the aim of this work to study the propagation of light waves in multilayer photonic structures composed of SiO_2/metamaterial (labeled *A* and *B* on this work) layers stacked alternately following the Octonacci sequence, which describes the arrangement of spacing of the Ammann quasilattice (8-grid), namely, the octagonal Ammann-Becker tiling [42]. The quasiperiodic structure follows the Octonacci sequence, and the multilayer photonic structure can be grown by juxtaposing the two building blocks *A* and *B*, where the *n*th-stage of the superlattice S_N is given iteratively by the rule $S_N = S_{N-1}S_{N-2}S_{N-1}$, for $N \geq 3$, with $S_1 = A$ and $S_2 = B$. The number of the building blocks increases according to the Pell number, $P_N = 2P_{N-1}+P_{N-2}$ (with $P_1 = 1$ e $P_2 = 1$). This structure can also be grown by following a recurrence rule, namely: $A \rightarrow B$, $B \rightarrow BAB$. Recently, we have studied the transmission spectra in one-dimensional photonic quasicrystals, made up of SiO_2 (A) and TiO_2 (B) materials, organized following the Octonacci sequence [43]. In that work we report, for normally incident waves and for a same generation, that the transmission spectra for transverse electric (TE) and transverse magnetic (TM) waves presents a perfect scaling property where a self-similar behavior is obtained, as an evidence that these spectra are fractals. Also we show regions where the omnidirectional band gaps emerges for specific generations of Octonacci photonic structure, except for TM waves. On the other hand, M.-R. Wu et al. [44], have studied theoretically the photonic bandgap structure for a polaritonic photonic crystal containing lithium tantalate ($LiTaO_3$) in the NRI region. They have concluded that in NRI region we have a multi band gap structure and that the gap falling in the anomalous dispersion region can be treated as a zero-index gap which is further shown to be omnidirectional. However in both works, the authors do not explore the NRI region in order to study the transmission spectra. It is one of our aims to fill this gap, by studying the optical transmission in this region in order to search perfect, or almost perfect, transmission peaks.

Specifically, in this paper we want to investigate the behavior of the light when it pass through an Octonacci photonic layered system, considering the central wavelength $\lambda_0 = 700$ nm [45]. We also intend to investigate the influence of the oblique incidence at the system, by searching for frequency regions where the band gaps are independent from polarization and the incident angle θ_C.

The plan of this work is as follows. In Sec. 2, we present the method of calculation employed here, which is based on the transfer-matrix approach, together with a discussion of the transfer matrices for the quasiperiodic structure presented here. In Sec. 3 we present our results and discuss them. This is followed by a brief conclusion in Sec. 4.

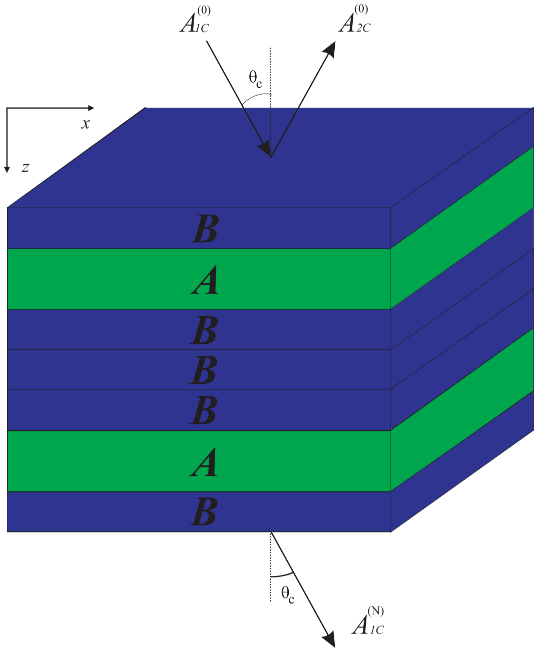

Fig. 1. Schematic representation showing the geometry of the Octonacci quasiperiodic multilayer system considered in this work. More precisely, for sequence $S_4 = [B|A|B|B|B|A|B]$, with $P_4 = 7$. *L* is the size of the whole superlattice structure.

2. Theoretical model

In the present work we make use of a theoretical model based on a transfer-matrix treatment (for a review see Refs. [38,40]). Consider a *s*-polarized (TE wave) light of frequency ω, normally incident from a transparent medium *C* at an arbitrary angle θ_C with respect to the normal direction of the layered system (see Fig. 1). The layered system is formed from an array of slabs of different materials (*A* or *B*). The reflectance and the transmittance coefficients are simply given by

$$R = \left|\frac{M_{21}}{M_{11}}\right|^2 \quad \text{and} \quad T = \left|\frac{1}{M_{11}}\right|^2, \tag{1}$$

where M_{ij} $(i,j = 1,2)$ are the elements of the optical transfer-matrix M_N, which links the coefficients of the electromagnetic fields in the region $z < 0$ to the coefficients of the electromagnetic fields in the region $z > L$, L being the size of the quasi-periodic structure.

Let us consider first, to illustrate our method, the optical transfer-matrix calculation for the quasiperiodic multilayer which is characterized by having two dielectric media *A* and *B* with thicknesses d_A and d_B, and refractive indexes n_A and n_B, respectively, organized in accordance to the Octonacci sequence. It is surrounded by the transparent medium *C* with refractive index n_C (see Fig. 1). The transmission of an obliquely incident light wave across the interfaces $\alpha \rightarrow \beta$ (i.e., $C \rightarrow A, A \rightarrow B, \ldots, B \rightarrow C$) is represented by the matrix

$$M_{\alpha\beta} = \frac{1}{2}\begin{bmatrix} 1 + k_{z\alpha}\big/k_{z\beta} & 1 - k_{z\alpha}\big/k_{z\beta} \\ 1 - k_{z\alpha}\big/k_{z\beta} & 1 + k_{z\alpha}\big/k_{z\beta} \end{bmatrix}. \tag{2}$$

with

$$k_{z\alpha} = \left[(n_\alpha \omega/c)^2 - k_x^2\right]^{1/2} \tag{3}$$

and

$$k_x = n_c(\omega/c)\sin(\theta_C). \tag{4}$$

The propagation of the light wave within one of the layers γ ($\gamma = A$ or B) is characterized by the propagation matrices

$$M_\gamma = \begin{bmatrix} \exp(-ik_\gamma d_\gamma) & 0 \\ 0 & \exp(ik_\gamma d_\gamma) \end{bmatrix}, \tag{5}$$

We assume that, in each layer, the electrical field is given by

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