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Research paper

Multiplicities and thermal runaway of current leads for superconducting magnets

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ABSTRACT

The multiple solutions of conduction and vapor cooled copper leads modeling current delivery to a superconducting magnet have been numerically calculated. Both ideal convection and convection with a finite heat transfer coefficient for an imposed coolant mass flow rate have been considered. Because of the nonlinearities introduced by the temperature dependent material properties, two solutions exist, one stable and one unstable regardless of the cooling method. The limit points separating the stable form the unstable steady states form the blow-up threshold beyond which, any further increase in the operating current results in a thermal runway. An interesting finding is that the multiplicity persists even when the cold end temperature is raised above the liquid nitrogen temperature. The effect of various parameters such as the residual resistivity ratio, the overcurrent and the variable conductor cross section on the bifurcation structure and their stabilization effect on the blow-up threshold is also evaluated.

1. Introduction

Superconducting magnets maintained at cryogenic temperatures are powered from current leads at room temperature, Wilson [1], Iwasa [2]. The thermal connection between the room and cryogenic environments through the current leads introduces a significant heat leak to the cryostat and a substantial theoretical and experimental work has been carried out in the effort to minimize this heat leak to the cryogenic liquid and optimize the lead design with respect to material and geometry selection [3–8]. The basic configurations examined consist of vapor and conduction cooled (or 'dry') leads.

An interesting feature of the mechanism of heat generation produced by an applied current and its balance by conduction is the multiple steady states. Jones et al. [9] employed temperature and purity dependent material properties to determine burn-out limits for copper current leads cooled by helium vapor. During numerical overcurrent simulations two steady states were calculated, in certain cases depending on the initial conditions and on the rate the current was increased and reached its final value. Aharonian et al. [10] and Buchs and Hyman [11] demonstrated that for a fixed current, geometry and vapor flow rate two solutions exist. The stability of the two solutions was assessed numerically i.e. by introducing a small perturbation on the temperature profile and calculating the response in the time domain. A solution resulting in thermal runaway was considered as unstable. More recently Hanzelka [12] reported that for certain combinations of copper lead geometry and applied current operating in vacuum with radiative exchange, as many as three solutions were found.

As it is obvious from the literature cited in the preceding paragraphs most of the work has been focused on the design and optimization of current leads and only a few studies considered the multiple solutions and the stability aspects of the problem. As a matter of fact Hull [13] argued that the multiplicity may be an artifact of the numerical methods employed. Today, the field of numerical bifurcation analysis has become mature and bifurcation mechanisms are widely accepted as decisive phenomena for explaining and understanding stability and structural change [14–16]. Within this framework the aim of the present study is to numerically explore the multiplicity and blow-up (thermal runaway) features of copper current leads delivering current from a relatively warm environment to superconducting magnets at cryogenic temperatures. Multiple solutions exist not only for the vapor cooled leads but for conduction cooled leads as well. The solution structure is analyzed with sufficient bifurcation diagrams describing the effects of the residual resistivity ratio, the conductor geometry, the cold end temperature and the overcurrent on the multiplicity regions and the blow-up threshold.

2. Analysis

Consider a copper conductor of cylindrical geometry with a variable cross section $A(X)$, length L, thermal conductivity K, electrical resistivity $\hat{\rho}$ and specific heat C, as it is schematically

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Nomenclature

A	conductor cross sectional area [m^2]
c	$(C/C_{\rm ref})$ reduced specific heat capacity [–]
C	conductor specific heat capacity [J/(kg K)]
F	flow number, Eq. (9) [–]
G	generation number, Eq. (8) [–]
h	$(H/H_{\rm ref})$ reduced heat transfer coefficient [–]
H	heat transfer coefficient [W/(m^2 K)]
I	current through lead [A]
k	$(K/K_{\rm ref})$ reduced thermal conductivity [–]
K	conductor thermal conductivity [W/(m K)]
L	conductor length, Fig. 1 [m]
$\dot{m}$	coolant mass flow rate [kg/s]
p	profile tapper ratio, Eq. (3) [–]
P	wetted perimeter [m]
q	heat load, Eq. (14) [W]
Q	reduced heat load, Eq. (15) [–]
RRR	residual resistivity ratio [–]
t	time [sec]
T	temperature [K]
u	conduction-convection parameter (CCP) [–]
v	$(V/V_{\rm ref})$ dimensionless voltage [–]
V	voltage difference across the lead [V]
x	(X/L) dimensionless distance along conductor, Fig. 1 [–]
X	distance along conductor, Fig. 1 [m]
y	dimensionless transverse coordinate, Fig. 1 [–]
z	$(I/I_{\rm ref})^2$ current overload factor [–]

Greek symbols

α	thermal diffusivity [m^2/s]
β	$(\gamma c_p A)_g/(\gamma C_{\rm ref} A_H)$ time scaling factor [–]
γ	density [kg/m^3]
Θ	$(T/\Delta T_{\rm ref})$ dimensionless temperature [–]
λ	eigenvalue [–]
ρ	$(\hat{\rho}/\hat{\rho}_{\rm ref})$ reduced conductor electrical resistivity [–]
$\hat{\rho}$	conductor electrical resistivity [Ω m]
τ	$(\alpha t/L^2)$ dimensionless time [–]

Subscripts

g	gas coolant
H	warm end $(x = 1)$ of lead
L	cold end $(x = 0)$ of lead
LP	reference to limit points
ref	reference value
s	reference to steady state

Superscript

$(\prime)$	derivative with respect to x

depicted in Fig. 1. The warm end is maintained at ambient temperature, say $T_H = 300$ K and the cold end at liquid helium temperature $T_L = 4.2$ K. A helium gas stream of constant mass flow rate $\dot{m}$ is used to cool the conductor. Assuming that the conductor is thermally thin so that transverse temperature gradients may be neglected the energy balance for the lead and the cooling gas take the form:

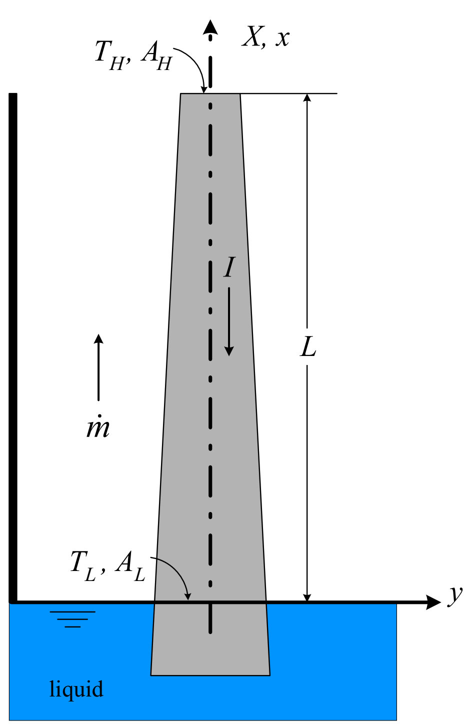

Fig. 1. Physical model and coordinate system.

$$\gamma C(T) A \frac{\partial T}{\partial t} = \frac{\partial}{\partial X}\left[K(T) A \frac{\partial T}{\partial X}\right] - HP(T - T_g) + I^2 \frac{\hat{\rho}(T)}{A}, \tag{1}$$

$$[\gamma(T_g) c_p(T_g) A]_g \frac{\partial T_g}{\partial t} = \dot{m} c_p(T_g) \frac{\partial T_g}{\partial X} - HP(T - T_g), \tag{2}$$

where T is the conductor temperature, γ its density, c_p is the coolant specific heat capacity, T_g its temperature, H is the convective heat transfer coefficient and I is the applied current. The design of the conductor cross section A has been a subject of optimization as well, either with simple geometries, Eckert et al. [17], Jiahui et al. [18] or as a variational problem, Okolotin and Bol'shakov [19]. For the purposes of the present study and in order to reduce the number of the parameters involved in the analysis a simple linear profile has been considered

$$y(X) = p + (1 - p)(X/L), \quad p \geqslant 1. \tag{3}$$

In terms of the above profile the cross sectional area and the wetted perimeter may be expressed as

$$A(X) = A_H y^2(X), \quad P(X) = P_H y(X). \tag{4}$$

Introducing dimensionless variables

$$\begin{matrix} x = X/L, & \Theta = T/\Delta T_{\rm ref}, & \Theta_g = T_g/\Delta T_{\rm ref}, & \tau = \alpha t/L^2, & y^2 = A/A_H \\ h = H/H_{\rm ref}, & k = K/K_{\rm ref}, & \rho = \hat{\rho}/\hat{\rho}_{\rm ref}, & c = C/C_{\rm ref}, & z = (I/I_{\rm ref})^2 \end{matrix}, \tag{5}$$

the partial differential equations describing the temperature distribution of the conductor and the cooling gas take the form:

$$c y^2 \frac{\partial \Theta}{\partial \tau} = \frac{\partial}{\partial x}\left(k y^2 \frac{\partial \Theta}{\partial x}\right) - u^2 y h(\Theta - \Theta_g) + \frac{zG\rho}{y^2}, \tag{6}$$

$$\beta \frac{\partial \Theta_g}{\partial \tau} = u^2 h(\Theta - \Theta_g) - F \frac{\partial \Theta_g}{\partial x}, \tag{7}$$

where $\beta = (\gamma c_p A)_g/(\gamma C_{\rm ref} A_H)$ is a time scaling factor. The current overload factor z is introduced since the leads may not always