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Bending of straight bars made of anisotropic materials

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Abstract

The paper aims to study the status of efforts and strains in structural bar elements made of anisotropic materials.
Because of the fiber structure of wood, it is classified as an anisotropic material with the physical and mechanical properties varying after three main directions (longitudinal, transverse, radial).
The present study establishes the general equations for bending of anisotropic bars and conducts a comparative study of efforts obtained by numerical and analytical methods.
Also, it compares the efforts and strains states considering physical and mechanical properties of anisotropic materials with the efforts and strains of isotropic and linear elastic materials.

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1. Introduction

Analysis of the behaviour of structures and elements and determination of their responses to static or mechanical actions can be evaluated more or less satisfactory compared to real behaviour.

Thus, if the structural elements are made from homogeneous, isotropic and linear elastic materials, the deformations that occur are small, changes of the geometric shape are not significant and the behaviour of

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thestructure can be determined with sufficient accuracy in terms of physical, mathematical and elastic characteristics.

If the material used in the structure is not homogeneous, is anisotropic and/or has a nonlinear elastic behaviour, the evaluation of structural response under static and especially dynamic actions presents immense difficulties, both in terms of physical-mathematical modelling and in experimental determining of most mechanical characteristics.

To reduce difficulties of mathematical modelling and of the behaviour of elastic bodies, it was necessary to introduce fundamental simplifying assumptions of which the most important are:

a) Fundamental assumptions:

- continuity and homogeneity assumption;
- ideal elasticity and linearity assumption that states the specific two-way physical stress and strain relations are linear;
- geometric linearity assumption (or small deformations assumption);
- isotropy hypothesis which admits that physico-mechanical properties of the material are identical in any direction around a point;

b) Simplifying assumptions, depending on the problem studied, such as:

- Bernoulli's hypothesis of flat sections for bar elements;
- surface normal median hypothesis introduced by Kirchoff in the study of flat and curved plates.

In practical engineering very few materials have a behaviour to match the fundamental assumptions stated above, many of them (such as wood or reinforced concrete) present both pronounced inhomogeneity and anisotropy.

This paper aims to highlight the methods of analysis of stresses and strains in structural bar elements considering the material anisotropy, while recognizing the rest of the fundamental assumptions of the theory of elasticity. For a few simple actions (centric tension, plane bending) it presents a comparative study considering both analytical and numerical methods.

It should be noted that analytical solutions that define stress and strain functions of the anisotropic bodies can be obtained in a reduced number of structure types and loads, while numerical methods based on finite element methods are more effective in solving problems.

2. Fundamental equations of anisotropic body elasticity theory

Elasticity Theory treats bodies in the static or dynamic linear elastic equilibrium, highlighting three categories of problem solving partial differential equations expressed in Cartesian coordinates, namely:

2.1. Equilibrium equations [1] [2]

$$\begin{aligned}
&\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}+X=0\\
&\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}+Y=0\\
&\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \sigma_z}{\partial z}+Z=0
\end{aligned} \tag{1}$$

2.2. Deformation equations [1] [2]

$$\begin{aligned}
&\varepsilon_x=\frac{\partial u}{\partial x}\,\varepsilon_y=\frac{\partial v}{\partial y}\,\varepsilon_z=\frac{\partial w}{\partial z}\\
&\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\,\gamma_{xz}=\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\,\gamma_{yz}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}
\end{aligned} \tag{2}$$

where u, v, w are the point displacement components in the direction of those coordinates.

Note that specific strains are not independent between them establishing the Saint-Venant compatibility