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Geometry of power flows and convex-relaxed power flows in distribution networks with high penetration of renewables

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Abstract

Renewable energies are increasingly integrated in electric distribution networks and will cause severe overvoltage issues. Smart grid technologies make it possible to use coordinated control to mitigate the overvoltage issues and the optimal power flow (OPF) method is proven to be efficient in the applications such as curtailment management and reactive power control. Nonconvex nature of the OPF makes it difficult to solve and convex relaxation is a promising method to solve the OPF very efficiently. This paper investigates the geometry of the power flows and the convex-relaxed power flows when high penetration level of renewables is present in the distribution networks. The geometry study helps understand the fundamental nature of the OPF and its convex-relaxed problem, such as the second-order cone programming (SOCP) problem. A case study based on a three-node system is used to illustrate the geometry profile of the feasible sub-injection (injection of nodes excluding the root/substation node) region.

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1. Introduction

The integration of more and more renewable energies, such as wind power (WP) and solar power (SP), into distribution networks becomes a big challenge to distribution system operations. The impacts on the distribution networks due to high penetration of these distributed generators (DG) include overvoltage and overloading issues. Extensive research has been carried out to deal with these issues. In [1], a local voltage control method based on voltage sensitivity to reactive power injection was proposed. The development of information communication technologies for smart grid enables the voltage control methods based on centralized coordination. In [2], two coordinated control methods, i.e. the rule based method and the optimal power flow (OPF) based method, were proposed. The OPF based method has better economic efficiency because it tries to minimize the active power curtailment of the DGs and the power losses of the network. The OPF problems are difficult to solve to the global

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optimum due to the non-convexity. In order to solve the OPF problem for optimal curtailment of DGs, linear approximations were made based on sensitivity analysis in [3].

More accurate methods are needed for solving the OPF problems formed for applications of energy management in distribution networks, such as the congestion management due to DGs and/or flexible demands including electric vehicles (EV) and heat pumps (HP). The convex relaxation method for solving the AC OPF was first presented in [4] as a second-order cone programming (SOCP) for radial networks and in [5] as a semidefinite programming (SDP) for meshed networks. In [6], a sufficient condition which has requirements on the upper limit of the active and reactive power injections was proposed for the convex relaxation of the OPF problem to be exact. Another sufficient condition proposed in [7] ensures the exactness of the convex relaxation and the convexity of the feasible sub-injection (the injection of the nodes excluding the root node of the network) region when the active reverse power flow is not heavy. However, neither of these sufficient conditions is valid for the applications discussed in [1]–[3] where heavy active reverse power flows are present.

In this paper, the OPF for applications with heavy active reverse power flows will be investigated. The main contributions of this paper include: (a) Visualize the geometry boundary of the feasible sub-injection of the OPF through a case study based on a three-node system; (b) Visualize the geometry boundary of the feasible sub-injection region of the convex-relaxed OPF; (c) Show that the sub-injection region is nonconvex when the reverse power flow is heavy.

The paper is organized as follows. Section 2 introduces the formulation of the OPF problem and its convex relaxation. Section 3 presents the methodology for visualizing the geometry boundary of the sub-injection for both the original OPF and the convex-relaxed one. A case study based on a three-node system is described in Section 4, followed by conclusions.

2. Optimal power flow and concept of sub-injection region

2.1. Optimal power flow based on branch flow model

OPF problems can be employed for applications such as minimizing the curtailment of the renewables or equivalently maximizing the sub-injection. An OPF based on the branch flow mode [8] is written as (1)-(7). Notice that the distribution network operates in tree configuration. The substation is deemed as the root node, denoted as 0, of the tree. An edge, denoted as (i, j) or $i \to j$, of the tree is a segment of the feeder and the direction is pointing to the root, implying that i is a child node of j.

OPF:

$$\max_{s,S,\mathbf{v},\mathbf{i},s_0} \sum_{i\in\mathcal{N}^+} c_i \operatorname{Re}(s_i)\,, \tag{1}$$

s.t.

$$S_{ij} = s_i + \sum_{h:h\to i} (S_{hi} - z_{hi}\mathbf{i}_{hi}), \forall (i,j)\in\mathcal{E}\,, \tag{2}$$

$$0 = s_0 + \sum_{h:h\to 0} (S_{h0} - z_{h0}\mathbf{i}_{h0})\,, \tag{3}$$

$$\mathbf{v}_i - \mathbf{v}_j = 2\operatorname{Re}(\overline{z}_{ij}S_{ij}) - \left|z_{ij}\right|^2 \mathbf{i}_{ij}, \forall (i,j)\in\mathcal{E}\,, \tag{4}$$

$$\mathbf{i}_{ij} = \frac{\left|S_{ij}\right|^2}{\mathbf{v}_i}, \forall (i,j)\in\mathcal{E}\,, \tag{5}$$

$$\underline{s}_i \le s_i \le \overline{s}_i, \forall i\in\mathcal{N}^+\,, \tag{6}$$

$$\underline{\mathbf{v}}_i \le \mathbf{v}_i \le \overline{\mathbf{v}}_i, \forall i\in\mathcal{N}^+\,, \tag{7}$$