Available online at www.sciencedirect.com

ScienceDirect

Energy Procedia 94 (2016) 497 – 503

Energy Procedia

13th Deep Sea Offshore Wind R&D Conference, EERA DeepWind'2016, 20-22 January 2016, Trondheim, Norway

A continuously differentiable turbine layout optimization model for offshore wind farms

Arne Klein[a,*]

[a]*University of Bergen, P.B. 7803, 5020 Bergen, Norway*

Abstract

This article presents a continuously differentiable modification of the commonly used Jensen wake model. This property is conserved in the wake combination model and, by formulating the turbine's power curve as a set of constraints, also in the computation of a turbine's power output. The resulting objective function, maximizing the total power production, and optimizing the turbine positions, is thus continuously differentiable, and gradient based solution methods can be applied. Numerical experiments are conducted with simulated wind data of six offshore wind farm locations.

Peer-review under responsibility of SINTEF Energi AS

Keywords: wind energy, turbine layout, non-linear optimization, wake model

1. Introduction

With the trend of the annually installed offshore wind energy continuously growing in the last 10 years, and offshore wind farms becoming bigger than ever before [1], the importance of optimizing the turbine locations within a wind farm for an optimal power production are increasing. While there was only a total installed capacity of approximately $2GW$ in 2009 in Europe, an additional $2.3GW$ were installed in the first half of 2015, which was an increase of 200% compared to the same period of the previous year. An additional $100GW$ of installed capacity are currently in the planning phase [2].

Several wake models calculating the wind velocity deficits created by wind turbines, the so called turbine wake, are available in different levels of detail. They range from kinetic wake models, which have a closed formulation and allow a fast computation, to computationally expensive computational fluid dynamics (CFD) models which are able to model the wind flow on a small scale.

The wake effect of a turbine reduces the power generated by turbines further downstream. This effect increases with increasing wind farm size, as there can be more upstream turbines reducing the wind velocity. The optimization of the turbine layout, i.e. the optimization of the single turbine locations within a wind farm, to obtain a maximum annual energy production (AEP) thus becomes more relevant for larger wind farms. The main input for this optimization

* Corresponding author

E-mail address: arne.klein@uib.no

Peer-review under responsibility of SINTEF Energi AS

doi:10.1016/j.egypro.2016.09.221

problem, when only including the wake effects in the optimization process, is the wind rose. It is possible to use more complex cost models which possibly include foundation costs, operation and maintenance, structural loads and more. This is for example done in TOPFARM [3].

Advanced wake models have the disadvantage of no usable or unknown mathematical properties, and thus only allow the application of metaheuristics or black-box optimization. These methods generally require a higher number of function evaluations than gradient based optimization methods. This can lead to computationally infeasible problems, especially if combined with detailed, computationally expensive wake models.

As optimization requires a high number of function evaluations, computationally cheap wake models are most suitable. It is in addition desirable to use models with known mathematical properties like smoothness and differentiability. The kinetic wake models fulfil these desired properties well. While they only provide an approximation of the wake deficits on the scale of turbine diameters, this is sufficient for an approximate calculation of a turbine's power generation.

The wake model by Jensen and Katic has become the most widely used model for wind farm layout optimization in recent years [4]. While the model is non-smooth, several modifications have been suggested to make it suitable for gradient-based optimization methods. Haugland [5] and Park and Law [6] suggest modifications to the Jensen model which increase its area of definition to the downwind half-plane by applying a Gaussian function normal to the wind direction.

In this article we introduce an additional extension of the wake model. This yields a smooth wake function on $\mathbb{R}^2$ which we use to formulate a differentiable optimization model, optimizing the turbine locations for a maximum annual energy production. We conduct numerical experiments with the wind data of six real wind farm locations in northern Europe.

Section 2 starts with an explanation of the extended wake model, continues with the wake combination model, the turbines' power curve and the computation of the annual energy production, and finally states the full optimization model in a concise form in 2.5. In section 3 the implementation, turbine specification and wind data is explained before showing our numerical experimental results in 3.4.

2. Optimization model

The optimization model in this work is set up to maximize the energy production of the wind farm, using the local wind data and valid turbine placement area as parameters and optimizing the turbines' position vectors.

A wake model as well as a wake combination model are required for the computation of the wind velocities at the turbine locations. The so called power curve of the turbines is used to determine the power production of a turbine as a function of the wind velocity at its location.

The combined power productions of all turbines for each wind vector are weighted with the frequency of occurrence of the corresponding wind vector in the objective function.

The variables defined in the following will be used in the next sections. Assume that a set of wind data W is given with wind velocities v_w and wind angles ϕ_w, $w \in W$. The set of turbines T has turbine coordinates $\mathbf{r}_t = (x_t, y_t) \in \mathbb{R}^2$, $t \in T$. The coordinates of all turbines $t \in T$ are bounded by the same polyhedral constraint

$$\mathbf{A}\mathbf{r}_t \leq \mathbf{b} \tag{1}$$

with $\mathbf{A} \in \mathbb{R}^{n\times|T|}$, $\mathbf{b} \in \mathbb{R}^n$, $n \in \mathbb{N}^+$. A minimum inter-turbine distance is enforced by the constraint

$$(x_i - x_j)^2 + (y_i - y_j)^2 \geq (6R)^2 \tag{2}$$

for $i, j \in T$, $i > j$ with turbine radius R, ensuring a minimal distance of 3 turbine diameters between turbines. This is a requirement due to safety regulations.

2.1. Wake model

The Jensen wake model is widely used in science and industry. In addition to having a simple formulation and being efficient to calculate, it performs well in predicting wake effects. It is, however, only defined in the wake region,