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Chaotic spiking induced by variable delayed optoelectronic feedback in a model of class B laser

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ABSTRACT

We analyze the dynamics of a class B laser with optoelectronic delayed feedback and periodic modulation of the delay time. For the delay-differential model, we describe asymptotically spike regimes with various properties, namely, with inter-spikes intervals greater than delay, less delay, and a mixed type. Sets of initial conditions in an infinite-dimensional phase space are determined for each type of pulsed solutions. The dynamics of each regime is described by the dynamics of a certain finite-dimensional mapping. By computing the maps we obtain the modulation parameters at which the chaotic spiking of desired structure is realized. We demonstrate bistability of chaotic spiking, their intermittence and merge.

1. Introduction

The dynamics of nonlinear systems with delayed feedback (FB) is permanently in the focus of many researches. Recently, the possibility of stabilizing the stationary state in such systems by the method of high-frequency periodic modulation delay time has been discussed [1,2]. In particular, this effect has been studied in the laser model with incoherent optical FB [3].

From the other side, both delayed FB itself and periodic modulation of a parameter can cause oscillating dynamics in lasers. In [4] self-sustained picosecond pulse generation in a GaAlAs laser has been obtained at an electrically tunable repetition rate by optoelectronic FB. In [5] it was shown that the generation is possible in different modes of short pulses following in period correlated with the delay time. In [6] chaotic dynamics induced by FB in CO_2 laser with the short-delay regime was discussed. A lot of efforts have been paid to study nonlinear phenomena in laser diodes induced by coherent optical FB such as multistability, coherent collapse, low-frequency fluctuations, as well as to developing control methods, for example, see reviews [7,8]. Direct external periodic modulation of such laser parameters as pumping rate or intra-cavity loss can also lead to coexistence of oscillating modes, their bifurcations, annihilation of attractors, intermittency and dynamical chaos, see [9,10] and the references therein.

Thus, it can be expected that with appropriate modulation frequency of the delay time one can observe secondary instabilities of oscillating modes. Since nonlinear systems with delayed argument have an infinite-dimensional phase space, it is possible multistability of periodic attractors, as well as the formation of high-dimensional chaotic attractors [11]. Such a complex dynamics may be useful in various applications. In instance, variation of FB characteristics within the range ensuring chaotic lasing regime has been proposed for information encoding [12–16]. Peculiarities of oscillations resulted from FB variations may also be useful for the aims of optical vibrometry [17]. Instabilities can occur in oscillatory networks with time-varying delay connections [18]. That motivates us to study the dynamics of strongly nonlinear spiking created by the periodically varied delay.

In this paper we consider the model of a laser with FB circuit controlling pumping current. The scheme was first applied in CO_2 lasers [6] and in laser diodes [5]. With some parameters the optoelectronic FB can induce the generation of short pulses following in period correlated with the delay time. In accordance with experimental observations, the simple model proposed in [5] describes relaxation oscillations in form of regenerative large-amplitude spikes. Their dynamics was analyzed theoretically in paper [19] in the case of constant delay. Later, detailed theoretical studies of similar devices were conducted in the cases of positive and negative feedback [20], short and long feedback [21].

Here, our aim is to demonstrate that by applying a periodic modulation of the delay it is possible to observe chaotic spiking with special properties. We distinguish oscillations with interpulse intervals greater

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than the delay time or with several spikes in the delay interval, or of a mixed type. We describe the initial conditions and find asymptotic representation of the pulsed solution, as well as the domain of parameters for the desired regime. It will be shown that the periodic state can bifurcate to quasiperiodic and chaotic oscillations. Spiking of different structures can coexist (multistability of spiking). With increasing of modulation amplitude, intermittent chaotic modes and merge of attractors can be observed, thus resulting in monostability.

The paper is organized as follows. In Section 2 the non-autonomous delay-differential model and the asymptotic method of investigation are briefly discussed. In Section 3 we describe analytically slowly oscillating solutions in form short spikes following in intervals longer than the delay time. Two-dimensional map is derived which is responsible for the dynamics of such pulsing. Based on the dynamics of the map, we give numerical example of chaotic spiking. In Section 4 we find analytically fast oscillating solutions with inter-spike intervals less than the delay time, i.e. solutions with n spikes on the delay interval. Their dynamics are determined by $(2n+2)D$-dimensional maps. Bistability of slowly and fast oscillating chaotic spiking is shown numerically. In Section 5 mixed slowly and fast oscillating solutions are analyzed. Based on the dynamics of the obtained map, we give a numerical example of a chaotic regime in which spikes follow in strict sequence at intervals greater and less than the delay time.

2. Model

The mathematical model proposed in [5] describes the dynamics of the normalized light intensity $u(t)$ and the population inversion $y(t)$, and takes into account the delay in the light passing over FB:

$$\frac{du}{dt} = \upsilon u(y-1), \tag{1}$$

$$\frac{dy}{dt} = q + \gamma u\big(t-\tau(t)\big) - y - yu, \tag{2}$$

where q is the pumping rate; υ is the ratio of the rate of decay photons in the cavity to the rate of relaxation of populations; t and τ are current time and the delay time in the external FB circuit normalized at the time of relaxation of the population inversion; γ is the feedback factor (FB level). In this paper we additionally suppose that the delay time is periodically modulated by optoelectronics means,

$$\tau(t) = \tau_0 + B\cos\Phi(t),\ \ \Phi(t) = \omega t + \varphi,\ \ mod\ 2\pi,$$

where τ_0 is the constant part of the delay, B and ω are the modulation amplitude and the modulation frequency, respectively, $\Phi(t)$ and φ are the phase of modulation at current and the initial moment. For further analysis it is useful to introduce the function which describes the retarded argument,

$$g(t) = t - \tau_0 - B\cos\Phi(t),$$

so that $u(t-\tau) = u(g(t))$. The delay modulation amplitude and frequency should be limited by the inequalities which ensures positive values of the delay $\tau(t) > 0$ and positive derivation $g'(t) > 0$ (ensures pulsed structure of the solution). Further we study the system with the assumptions

$$B < \tau_0,\ \ B\omega < 1. \tag{3}$$

On variables and parameters of the system. According to the physical meaning of the variables, only positive solutions of the system, $u(t) > 0$ and $y(t) > 0$, are considered. For class B lasers including semiconductor lasers, some solid-state lasers and CO_2 gas lasers, typical values of normalized parameters are as follows. The pumping rate $q >\sim 1$, the FB level is naturally limited in the range $\gamma \in [0,1)$, the time delay τ_0 can be varied from 0 to $\sim$10. It is important that for class B lasers the parameter $\upsilon \sim> 10^3$ can be considered as a large one.

The presence of the large parameter $\upsilon \gg 1$ opens a way to get asymptotic solutions of the system (1)–(2) under $\upsilon \to \infty$. The solutions take the form of spikes. Let us note the advantages of the proposed method. First, the method is applied to the model taking into account physical processes whose characteristic decay rates differ by several orders. Second, analytical description of nonlinear oscillations enables to understand dynamic trends when the parameters vary in multiparametric systems. Third, our approach allows to study the mathematical model in infinite-dimensional phase space. Numerical analysis of corresponding DDEs, even using advanced computers, cannot provide comprehensive results. In addition, analytical information on the initial conditions of coexisting attractors can be used to develop methods for quickly switching between them.

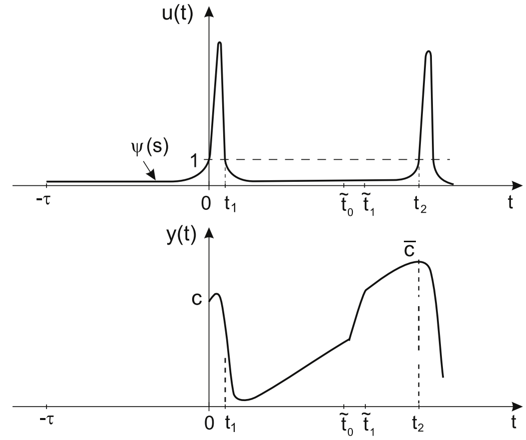

Fig. 1. Sketch of a slowly oscillating solution. The initial function $\psi(s) \in S_0$ is shown on the segment $s \in [-\tau_0 - B, 0]$, the initial value $y(0) = c$, $y(t_2) = \bar{c} + o(1)$.

On the method of investigation. The phase space of system (1)–(2) is the direct product of the Banach space $C_{[-(\tau_0+B),0]}$ of continues functions by the domain in R^2, i.e., the functions from $C_{[-\tau_0-B,0]}$ and the values $y(0) \in R^1$ and $\varphi \in R^1$ are given as initial conditions. In this space, we shall distinguish a fairly wide set $S(\xi)$ dependent on the vector parameter ξ and consider the solutions with initial conditions from this set. It is possible to construct uniform asymptotic approximations of all such solutions and show that after a certain time these solutions again fall within $S(\xi)$. Thus, the operator of the shifting along the trajectories, which makes a function from S corresponds to a function also from S, is naturally determined. The properties of this operator are mainly determined by the finite-dimensional map $\bar{\xi} = f(\xi)$. To a fixed point of the map there corresponds the fixed point of the operator, and to the later point there corresponds a periodic solution of system (1)–(2) of the same stability. Details of the method were considered in book [22] by the example of other problems.

Solutions in form of spikes can be classified as slowly oscillating (SO) solutions with inter-spikes intervals longer than the delay time, fast oscillating (FO^n) solutions with inter-spikes intervals shorter than the delay, mixed slowly and fast (MSF) oscillating solutions. For each type of solutions we will find asymptotically (at $\upsilon \to \infty$) the maps responsible for the dynamics of spikes.

3. Slowly oscillating solution

For SO solutions any interval between spikes greater than the time delay. Solutions of this structure are the most simple.

Fix a time point when the radiation spike begins as the starting point, so that radiation intensity $u(0) = 1$ whereas before that, in the delay interval, the radiation intensity is of noise level, Fig. 1. The initial conditions for SO solutions

$$y(0) = c,\ \Phi(0) = \varphi,\ u(s) = \psi(s),\ s \in [-\tau_0 - B, 0] \tag{4}$$

are given from the set

$$S(c,\varphi) = \{c \in (1,q],\ \varphi \in [0,2\pi],\ \psi(s) \in S_0\},$$