Optics Communications 407 (2018) 392–397

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Propagation of radially polarized multi-cosine Gaussian Schell-model beams in non-Kolmogorov turbulence

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ARTICLE INFO

Keywords:
Radially polarized multi-cosine Gaussian Shell-model beams
Propagation
Non-Kolmogorov turbulence
Statistical properties

ABSTRACT

Recently, we introduced a new class of radially polarized beams with multi-cosine Gaussian Schell-model(MCGSM) correlation function based on the partially coherent theory (Tang et al., 2017). In this manuscript, we extend the work to study the statistical properties such as the spectral density, the degree of coherence, the degree of polarization, and the state of polarization of the beam propagating in isotropic turbulence with a non-Kolmogorov power spectrum. Analytical formulas for the cross-spectral density matrix elements of a radially polarized MCGSM beam in non-Kolmogorov turbulence are derived. Numerical results show that lattice-like intensity pattern of the beam, which keeps propagation-invariant in free space, is destroyed by the turbulence when it passes at sufficiently large distances from the source. It is also shown that the polarization properties are mainly affected by the source correlation functions, and change in the turbulent statistics plays a relatively small effect. In addition, the polarization state exhibits self-splitting property and each beamlet evolves into radially polarized structure upon propagation.

1. Introduction

Over the past decades, there has been a growing interest in the evolution of partially coherent beams, either in free space or in turbulent atmosphere [1–6]. This interest is motivated by a multitude of potential applications, briefly to free-space optical communications, remote sensing and tracking [7,8]. The random fluctuations in the index of refraction of atmosphere cause spreading of the beam beyond that due to pure diffraction, beam wander, loss of spatial coherence, and random fluctuations in the irradiance and phase. These effects can seriously degrade the signal-to-noise ratio of an optical heterodyne receiver. Therefore, much work has been devoted to a reliable theory for predicting the propagation properties of light beams in turbulent medium. It is demonstrated that the atmosphere exhibits homogeneous and nearly isotropic under the atmospheric boundary layer, which is roughly 1–2 km above the Earth's surface. Therefore, the isotropic Kolmogorov power spectrum model of refractive index is generally correct within this inertial sub-range. However, in portions of the troposphere and stratosphere, theoretical and experimental results have shown that the Kolmogorov power spectrum does not properly describe the real turbulence behavior. Consequently, a variety of different power spectrum models and extended turbulence models have been proposed [9–12]. Among them, the most popular one is the non-Kolmogorov spectrum which is proposed by I. Toselli et al. [11]. It was assumed that instead of classic power law11/3 the power spectrum has a generalized law, defined by parameter α, in the range $3 < \alpha < 5$, as the one-dimensional fractal distribution stipulates. Since the atmosphere was shown to be layered in terms of the power spectra at different altitudes, many studies were carried out on the modeling of the non-Kolmogorov spectrum specifically for up/down/slant path propagation.

Due to the constraint of non-negative definiteness of the spatial correlation functions, a Gaussian correlated Schell-model (GSM) beam has always been chosen as a typical example of partially coherent beams in the early years. Recently, a new sufficient condition for devising genuine correlation functions of light beams was established by Gori et al. [13,14], a variety of partially coherent beam models with special correlation functions have been proposed in succession. It is shown that these new classes of partially coherent beams exhibit many novel propagation properties. Such as non-uniformly correlated beams lead to self-focus and laterally shifted in their maximum intensity [15,16], multi-Gaussian Schell-model beams and sinc-Schell model beams acquire flat-top profile in the far-field [17,18], cosine-Gaussian Schell beams with circular symmetry possess dark-hollow profile [19].

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https://doi.org/10.1016/j.optcom.2017.09.067
Received 13 July 2017; Received in revised form 17 September 2017; Accepted 21 September 2017
0030-4018/

Gaussian Schell-model array beams can radiate desirable lattice-like average intensity in far-field [20]. In addition, the propagation factors of several partially coherent beams have been investigated, one can see that beams with special correlation functions are less affected by turbulence than conventional GSM beams [21–23].

On the other hand, cylindrical vector (CV) beams with non-uniform state of polarization (e.g., radially polarized beams, azimuthally polarized beams) have attracted a wealth of attention due to their unique and interesting properties [24,25], and have widely applications in many research fields, for example, polarization information encryption, optical data storage, and optical tweezers [26]. Partially coherent radially polarized beams with Gaussian Schell-model correlations have been widely investigated both theoretically and experimentally [27–29]. In the past several years, generation and propagation of partially coherent radially polarized beams with peculiar correlations have become a hot topic [30,31]. It has been revealed that under the influence of non-conventional coherence properties, these novel beams exhibit distinctive propagating characteristics. Quite recently, we introduced a new class of partially coherent radially polarized beam with multi-cosine Gaussian Schell-model (MCGSM) correlations, termed as radially polarized MCGSM beams [32]. It is shown that the statistical properties of radially polarized MCGSM beams in the far field can be flexible modulated by varying the source coherence parameters. Moreover, unlike deterministic arrays, once the pattern is formed in the far field it remains structurally invariant on further propagation. Such feature makes the beam of particular importance for certain applications in which a far field with tunable lattice structure must be formed, such as optical trapping, material processing, and free-space and atmospheric optical communications.

In this manuscript, we explore the behavior of the statistical properties for radially polarized MGSM beams propagating in atmospheric turbulence by using a non-Kolmogorov power spectrum. The impacts arising from the source correlation functions and the turbulence parameters on the spectral density, the spectral degree of coherence and the polarization properties are emphasized.

2. Analytic solutions for radially polarized MCGSM beams in non-Kolmogorov turbulence

The elements of the cross-spectral density (CSD) matrix of a radially polarized MCGSM beam in the source plane are described as [32]

$$W_{\alpha\beta}^{(0)}\left(\rho_1',\rho_2',0\right)=\frac{\alpha_1'\beta_2'}{N^2}\exp\left(-\frac{\rho_1'^2+\rho_2'^2}{4\sigma^2}\right)\exp\left[-\frac{\left(x_1'-x_2'\right)^2}{2\delta_0^2}\right]\times\exp\left[-\frac{\left(y_1'-y_2'\right)^2}{2\delta_0^2}\right]\times\sum_{n=-(N-1)/2}^{(N-1)/2}\cos\left[\frac{2\pi n}{\delta_0}\left(x_1'-x_2'\right)\right]\times\sum_{n=-(N-1)/2}^{(N-1)/2}\cos\left[\frac{2\pi n}{\delta_0}\left(y_1'-y_2'\right)\right],\quad(1)$$

where $(\alpha,\beta=x,y)$, ρ_1' and ρ_2' are two-dimensional position vectors in the source plane, σ denotes its source width , δ_0 represents spatial coherence width, and N is the positive integer. When $N=1$, the radially polarized MCGSM beam reduces to a conventional radially polarized Gaussian Schell-model beam. The realizability conditions for a radially polarized MCGSM source and corresponding beam conditions are derived in our recent work [32].

The paraxial form of the extended Huygens–Fresnel principle which describes the interaction of waves with random medium implies the elements of the CSD matrix at two points $\mathbf{r}_1=\left(\rho_1,z\right)$ and $\mathbf{r}_2=\left(\rho_2,z\right)$ in the same transverse plane of the half-space $z>0$ are related to those in the source plane as [33]

$$W_{\alpha\beta}\left(\rho_1,\rho_2,z\right)=\left(\frac{k}{2\pi z}\right)^2\iiiint W_{\alpha\beta}^{(0)}\left(\rho_1',\rho_2',0\right)\times\exp\left[-ik\frac{\left(\rho_1-\rho_1'\right)^2-\left(\rho_2-\rho_2'\right)}{2z}\right]\times\left\langle\exp\left[\psi^*\left(\rho_1,\rho_1',z\right)+\psi\left(\rho_2,\rho_2',z\right)\right]\right\rangle_m d^2\rho_1' d^2\rho_2',\quad(2)$$

here $k=2\pi/\lambda$ is the wave number with λ being the wavelength of the light, ψ denotes the complex phase perturbation due to the random medium, and $\langle\cdot\cdots\cdot\rangle_m$ implies averaging over the ensemble of statistical realizations of the turbulent medium. For points located sufficiently close to the optical axis, the term in the sharp brackets with the subscript m in Eq. (2) can be written as

$$\left\langle\exp\left[\psi^*\left(\rho_1,\rho_1',z\right)+\psi\left(\rho_2,\rho_2',z\right)\right]\right\rangle_m=\exp\left\{-\frac{\pi^2k^2z}{3}\left[\left(\rho_1-\rho_2\right)^2+\left(\rho_1-\rho_2\right)\left(\rho_1'-\rho_2'\right)+\left(\rho_1'-\rho_2'\right)^2\right]\times\int_0^\infty\kappa^3\Phi_n(\kappa)\,d\kappa\right\},\quad(3)$$

where $\Phi_n(\kappa)$ is the one-dimensional spatial power spectrum of the refractive-index fluctuations of random medium, κ being spatial frequency. For the non-Kolmogorov case, the spatial power spectrum of the refractive index fluctuations of the turbulent atmosphere is known to have form [33]

$$\Phi_n(\kappa)=A(\alpha)\,\widetilde{C}_n^2\frac{\exp\left[-\kappa^2/\kappa_m^2\right]}{\left(\kappa^2+\kappa_0^2\right)^{\alpha/2}},\quad 0\le\kappa<\infty,\quad(4)$$

where $3<\alpha<4$ and the term $\widetilde{C}_n^2$ is a generalized refractive-index structure parameter with units $m^{3-\alpha}$,

$$\kappa_0=\frac{2\pi}{L_o},\quad\kappa_m=\frac{c(\alpha)}{l_o}\quad(5)$$

$$c(\alpha)=\left[\frac{2\pi}{3}\Gamma\left(5-\frac{\alpha}{2}\right)A(\alpha)\right]^{\frac{1}{\alpha-5}},\quad(6)$$

$$A(\alpha)=\frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left(\frac{\alpha\pi}{2}\right),\quad(7)$$

L_0 and l_0 are the outer and inner scales of turbulence, respectively, and $\Gamma(\cdot)$ is the Gamma function. With the power spectrum in Eq. (4), the integral in Eq. (3) becomes

$$I=\int_0^\infty\kappa^3\Phi_n(\kappa)\,d\kappa=\frac{A(\alpha)}{2(\alpha-2)}\widetilde{C}_n^2\kappa_m^{2-\alpha}\beta\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)\times\Gamma\left(2-\frac{\alpha}{2},\frac{\kappa_0^2}{\kappa_m^2}\right)-2\kappa_0^{4-\alpha},\quad(8)$$

where $\beta=2\kappa_0^2-2\kappa_m^2+\alpha\kappa_m^2$ and $\Gamma(\cdot,\cdot)$ denotes the incomplete Gamma function.

Substituting Eqs. (1) and (3) into Eq. (2) and calculating the resulting integral we arrive at the formulas

$$W_{xx}\left(\rho_1,\rho_2,z\right)=\Gamma\left(\rho_1,\rho_2\right)\times\sum_{n=-(N-1)/2}^{(N-1)/2}\left[\exp\left(\frac{\gamma_{y1-}^2}{4M}+\frac{\Omega_{y21}^2}{4\Pi}\right)+\exp\left(\frac{\gamma_{y1+}^2}{4M}+\frac{\Omega_{y22}^2}{4\Pi}\right)\right]\times\sum_{n=-(N-1)/2}^{(N-1)/2}\left\{\left(\Delta+\gamma_{x1+}\Omega_{x22}+\frac{\Delta\Omega_{x22}^2}{2\Pi}\right)\exp\left(\frac{\gamma_{x1+}^2}{4M}+\frac{\Omega_{x22}^2}{4\Pi}\right)+\left(\Delta+\gamma_{x1-}\Omega_{x21}+\frac{\Delta\Omega_{x21}^2}{2\Pi}\right)\exp\left(\frac{\gamma_{x12}^2}{4M}+\frac{\Omega_{x21}^2}{4\Pi}\right)\right\},\quad(9)$$

$$W_{yy}\left(\rho_1,\rho_2,z\right)=\Gamma\left(\rho_1,\rho_2,\right)\times\sum_{n=-(N-1)/2}^{(N-1)/2}\left[\exp\left(\frac{\gamma_{x1-}^2}{4M}+\frac{\Omega_{x21}^2}{4\Pi}\right)+\exp\left(\frac{\gamma_{x1+}^2}{4M}+\frac{\Omega_{x22}^2}{4\Pi}\right)\right]\times\sum_{n=-(N-1)/2}^{(N-1)/2}\left\{\left(\Delta+\gamma_{y1+}\Omega_{y22}+\frac{\Delta\Omega_{y22}^2}{2\Pi}\right)\exp\left(\frac{\gamma_{y1+}^2}{4M}+\frac{\Omega_{y22}^2}{4\Pi}\right)+\left(\Delta+\gamma_{y1-}\Omega_{y21}+\frac{\Delta\Omega_{y21}^2}{2\Pi}\right)\exp\left(\frac{\gamma_{y12}^2}{4M}+\frac{\Omega_{y21}^2}{4\Pi}\right)\right\},\quad(10)$$