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Q-factor of optical delay-line based cavities and oscillators

S. Esmail Hosseini [a,*], Azadeh Karimi [a], Sajad Jahanbakht [b]

[a] Department of Communications and Electronics, School of Electrical and Computer Engineering, Shiraz University, Shiraz, Iran
[b] Department of Electrical and Computer Engineering, University of Kashan, Kashan, Iran

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ABSTRACT

In this paper a theoretical derivation of unloaded and loaded Q-factor of delay-line cavities, such as optical fiber delay-lines, and delay-line based oscillators, such as optoelectronic oscillators (OEOs), is presented based on three approaches: (I) second-order resonator approximation, (II) linear time-invariant phase-space model and (III) energy approach. Theoretical expressions for unloaded and loaded Q-factor of delay-line based cavities and oscillators are derived. We show that the Q-factor of a delay-line based cavity is a function of its round-trip time that is not equal to the energy decay-time of usual microwave or optical resonators. Hence, the behavior of the Q-factor of a delay-line based cavity will not be the same as that of the usual resonators. We show that the loaded Q-factor of a delay-line cavity is greater than its unloaded Q-factor!, besides we show that the Q-factor of a lossy delay-line cavity is the same as that of the lossless one! (in contrast to the behavior of the usual resonators). We also show that the Q-factor of a delay-line based oscillator is proportional to the half of the round-trip time of its delay line while the Q-factor of an oscillator based on a usual resonator is proportional to the energy decay time of its resonator.

1. Introduction

Oscillators are one of the most widely used components in the present day technology. Some instances are mechanical oscillators (such as pendulum), electromagnetic oscillators (such as electronic and cavity based oscillators), lasers and masers. In oscillators, phase noise of output oscillation depends on the energy storage capability of resonators. To achieve sustained oscillations in an oscillator the Barkhausen criterion must be satisfied as [1]

$$|A\beta(j\omega)| = 1 \qquad \arg A\beta(j\omega) = 2n\pi \quad n = 0, 1, 2, \ldots \tag{1}$$

where A and $\beta(j\omega)$ are transfer functions of amplifier and resonator in the oscillator loop, respectively, ω is oscillation angular frequency and $A\beta(j\omega)$ is the open loop transfer function of the oscillator.

One relevant group of oscillators is electronic oscillators, where the inductor–capacitor resonators are used. However, as a result of low Q-factor cavities, the output signal of the electronic oscillators does not exhibit low phase noise. As a promising approach to attain low phase noise oscillators, high Q-factor resonators are used. There are various types of resonators including mechanical resonators (such as quartz crystals), electromagnetic resonators (such as dielectric cavities) and acoustic resonators. These types of resonators have few modes with high Q-factor in certain frequencies. As a result, there is a limited tuning frequency range of the resonator where an increase in oscillation frequency and frequency tunability result in phase noise degradation.

To overcome the above limitations, optoelectronic oscillators (OEOs) have been introduced to generate high frequency, low phase noise oscillations with high frequency tuning range. The first optoelectronic oscillator was developed by Yao et al. in 1996 [2]. An optoelectronic oscillator, as shown in Fig. 1, benefits from an optical fiber delay-line as the storage element, which determines the oscillation frequency and its phase noise.

In an optoelectronic oscillator, the continuous light of a laser is modulated by a microwave signal and transmitted by a long optical fiber and then applied to a photodetector. As a result, this oscillator produces an ultra-pure microwave oscillation, therefore it is named microwave optoelectronic oscillator. The transparency of some optical materials such as silica provides large delays at 1550 nm with ultra-low loss. Consequently, an optical fiber delay line is a good choice to act as a high-Q storage element in oscillator loop for generating ultra-low phase noise microwave oscillation.

One of the solution to reach ultralow phase noise OEOs is to use a long fiber loop. However the long fiber loop causes equally spaced spurs in the output spectrum of the oscillator. Many techniques have

* Corresponding author.
E-mail address: se.hosseini@shirazu.ac.ir (S.E. Hosseini).

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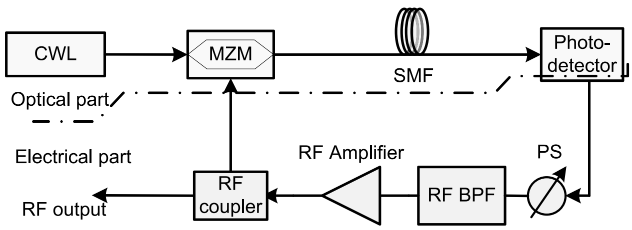

Fig. 1. The most basic architecture of a single-loop OEO. CWL: continuous wave laser, MZM: Mach–Zehnder modulator, SMF: single mode fiber, RF: radio frequency, BPF: bandpass filter, PS: phase shifter.

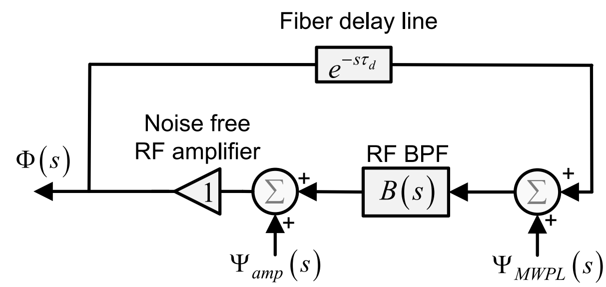

Fig. 2. Phase-space model for a single-loop OEO.

been proposed for suppressing unwanted spurs such as using multiple loops in an OEO [3–15], dual injection-locked OEO [16–19], coupled OEO [20–23] and transposed-frequency OEO [24].

Q-factor of delay-line cavities is one of the most important parameters that determine the phase noise of delay-line based oscillators. So, prediction and increasing this parameter is important to analysis and decreasing oscillator phase noise.

In this paper, Q-factor of delay-line cavities is studied in Section 2 using three approaches, (I) based on second-order resonator approximation, (II) linear time-invariant phase-space model and (III) energy approach. In Section 3, theoretical expressions for unloaded and loaded Q-factor of lossy and lossless stand-alone optical fiber delay-line cavities are derived and their behavior are compared to those of usual microwave and optical resonators. In Section 4, theoretical expressions for unloaded and loaded Q-factor of delay-line based oscillators, such as OEO, are derived and finally, in Section 5 we comment on a few highly cited previously published papers on OEOs that confuse the delay-line round-trip time and decay time of usual microwave and optical resonators.

2. Three approaches for calculating Q-factor of delay-line cavities

2.1. Q-factor calculation based on second-order resonator approximation

A large group of resonators including microwave resonators can be described with a second-order differential equation [1]

$$\frac{d^2}{dt^2}i(t) + \frac{\omega_0}{Q}\frac{d}{dt}i(t) + \omega_0^2 i(t) = 0 \tag{2}$$

where ω_n and Q are the natural frequency and Q-factor of the resonator, respectively and $i(t)$ is the current (or another variable that describes the specific system.
An optional variable. Transfer function of these resonators, which are in fact band-pass filters, can be defined as

$$\beta(j\omega) = \frac{\beta_0}{1 + jQ\left(\omega/\omega_0 - \omega_0/\omega\right)} \tag{3}$$

where, ω_0 is its center frequency and β_0 is the value of the transfer function at its center frequency. In the vicinity of the center frequency, where $\omega - \omega_0 \ll \omega_0/2Q$, the transfer function can be approximated as

$$\beta(j\omega) \simeq \frac{\beta_0}{1 + j2Q\left(\omega - \omega_0\right)/\omega_0} \simeq \beta_0 e^{-j2Q\frac{\omega-\omega_0}{\omega_0}} \tag{4}$$

Group delay of a transfer function can be calculated by

$$\tau_{GD} = -\frac{d}{d\omega}\left[\arg \beta(j\omega)\right] \tag{5}$$

Using (4) and (5), the group delay of a circuit that is described by a second-order differential equation is equal to

$$\tau_{GD} = 2Q/\omega_0 \tag{6}$$

or its Q-factor can be expressed as

$$Q = \frac{\omega_0 \tau_{GD}}{2} = -\frac{\omega_0}{2}\frac{d}{d\omega}\left[\arg \beta(j\omega)\right] \tag{7}$$

On the other hand, as mentioned before, in an OEO an optical fiber delay-line is used as a high Q-factor storage element in order to reduce phase noise of the output oscillation. The transfer function of a lossless delay-line cavity with total round-trip time delay of τ_d can be expressed as

$$\beta_d(j\omega) = e^{-j\omega\tau_d} \tag{8}$$

Using (5) and (8), group delay of a delay line is equal to its round-trip time delay (τ_d) which is

$$\tau_{GD} = -\frac{d}{d\omega}\left[\arg \beta_d(j\omega)\right] = \tau_d \tag{9}$$

For a fiber delay-line in a closed loop, an equivalent Q-factor, such as Q-factor of a resonator, can be defined. So, if a delay line that is used as the storage element in OEOs, is considered as an equivalent resonator with second-order differential equation, by using (6) and (9) we can write

$$\tau_d = 2Q/\omega_0 \tag{10}$$

So the unloaded Q-factor of a delay-line cavity can be calculated using (10) that is

$$Q_d = \omega_0\tau_d/2 \tag{11}$$

For example, a 10 km long optical delay-line with refractive index of 1.5 has an unloaded Q-factor of $(\pi/2)\times 10^6$ at 10 GHz.

2.2. Linear time-invariant phase-space model approach

In this section, a simplified phase noise analysis of OEOs is presented which is based on a linear time-invariant phase-space model that we have proposed in [24,25]. By comparing the proposed OEO phase noise model and the well-known Leeson's model, the equivalent loaded Q-factor of the optical delay-line cavity inside an OEO loop can be obtained.

A linear time-invariant phase-space model for a single-loop OEO is shown in Fig. 2, where $\Psi_{amp}(s)$ is the noise of the microwave amplifier that is referred to the amplifier input, $\Psi_{MWPL}(s)$ is the noise of the microwave photonic link (optical part of Fig. 1) that is referred to the output of the link and s is the Laplace variable. $\tau_d = nL/c$ is the time delay due to the fiber delay line, where L and n are the length and refractive index of the optical fiber, respectively and $B(s)$ is the phase transfer function of a general BPF.

Phase noise of the generated microwave signal can be written as a function of the noise of the loop components by applying basic control system theory as

$$\Phi(s) = H_1(s)\Psi_{amp}(s) + H_2(s)\Psi_{MWPL}(s) \tag{12}$$

where $H_1(s)$ and $H_2(s)$ are the phase transfer functions from the RF amplifier noise and microwave photonic link noise to the phase noise of the output signal, respectively,

$$H_1(s) = 1/\left[1 - B(s)\,e^{-s\tau_d}\right] \tag{13}$$

$$H_2(s) = B(s)/\left[1 - B(s)\,e^{-s\tau_d}\right] \tag{14}$$