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Denoising analysis of spatial pixel multiplex coded spectrometer with Hadamard H-matrix

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ABSTRACT

We present a multiplex coded Hadamard transform spectrometer with which Hadamard H-matrix is employed to encode the entrance pixels. Specifically, the Hadamard H-matrix is divided into two coding matrices, $\mathbf{H}_+$ and $\mathbf{H}_-$; here $\mathbf{H}_- = (\mathbf{H} - \mathbf{J})/2$ and $\mathbf{H}_+ = (\mathbf{H} + \mathbf{J})/2$, where each element of $\mathbf{J}$ equals 1. We analyzed its denoising capability and compared with the ordinary Hadamard spectrometer with cyclic-S matrix coded and slit spectrometers. The simulation and experiment show that the proposed Hadamard spectrometer with H-matrix outperforms Hadamard transform spectrometer with cyclic-S matrix, gaining about 2 dB of the signal-to-noise ratio.

1. Introduction

A traditional spectrometer cannot obtain a good result unless the source is in a good condition. Moreover, high spectral resolution and high signal-to-noise ratio (SNR) are required, but the narrow slit limits the throughput and results in unexpected spectrum data. Several instruments have been purposed to address this problem, including the Fourier transform spectrometer (FTS) [1,2] and Hadamard transform spectrometer (HTS) [3,4]. The FTS records interference patterns to estimate the spectrum of the source; normally, a scanning mechanism is needed. The HTS measures the spectrum in a suitable combination instead of separately. A weighing matrix encodes the spectrum, which makes the measurement exhibit superior throughput. However, high throughput does not guarantee a high SNR of the measured result. The weighting design is the key to the denoising capability of HTS. Moreover, the improvement of the SNR depends on not only the type of noise, but also the structure of the signal [5–8].

Golay's creative work presents the concept of coding aperture [9], which greatly improves throughput by turning spectral estimation into a weighing design problem. Furthermore, under independent Gaussian noise conditions, a good estimate of spectrum can be accessible by optimizing the weighing designs [3,10]. Even so, the denoising capability of a multiplex spectrometer with coding matrix has been questioned [11,12]. Our previous work analyzed the SNR improvement of HTS [6]. The study shows that the HTS with cyclic-S matrix will significantly outperform the traditional slit-based spectrometer when the detector noise dominates. But the HTS will decrease the SNR of results slightly when photon noise dominates [13]. In particular, if the signal is sparse using non-linear estimators, a coding spectrometer based on cyclic-S matrix could improve the SNR even when photon noise dominates [5,7].

Hadamard H-matrix is seldom used in multiplex coding measurement because "−1" cannot be realized in the optical path, that is, the photons can only be added, but cannot counteract each other. However, encoding the illumination of the samples with H-matrix to improve the SNR of the spectral image is proposed [13]. In this article, a spatial pixel coding Hadamard transform spectrometer to realize Hadamard H-matrix multiplex coding is presented. Moreover, the denoising capability of HTS with H-matrix is analyzed, and a simulation and experiment are performed.

To realize "–1" coding, the Hadamard H-matrix is divided into two different matrices $\mathbf{H}_+$ and $\mathbf{H}_-$, $\mathbf{H} = \mathbf{H}_+ - \mathbf{H}_-$. Specifically, the two matrices can be written as follows

$$\mathbf{H}_+ = (\mathbf{H} + \mathbf{J})/2, \quad \mathbf{H}_- = (\mathbf{H} - \mathbf{J})/2 \tag{1}$$

where every element of $\mathbf{J}$equals 1, and both $\mathbf{H}_+$ and $\mathbf{H}_-$ are composed only "1" and "0". The schematic diagram is shown in Fig. 1. The digital micromirror device (DMD) is used to encode the pixels of the image at entrance. Each of DMD's mirror pixels can be electrically driven: when turned on, mirrors are tilted about their diagonal hinge at an angle of +12°, and when turned off, they are tilted at −12°. According to the shift invariant [14], the distance of pixels at entrance is linear

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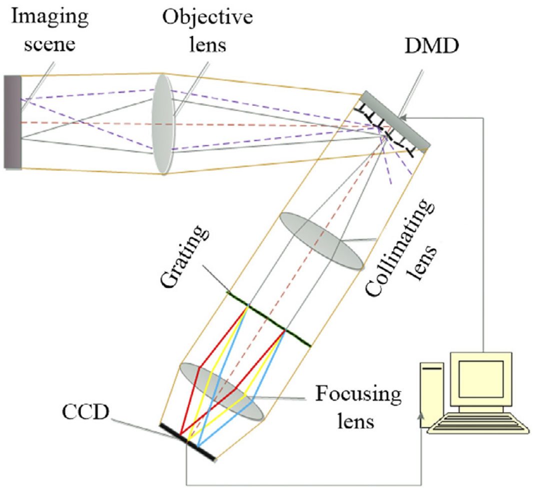

Fig. 1. Schematic drawing of spatial pixel coded spectrometer.

with the distance of related spectrum. The system thus creates the high-throughput by overlapping spectra from different pixels of the entrance image rather than different channels of the same spectrum. This is much more efficient than traditional HTS in spectral imaging.

Because the H-matrix is divided into two different matrices, which consist of "1" and "0" only, the H-matrix coding can be realized by DMD as same as the cyclic-S matrix. Taking 8 × 8 $\mathbf{H}_+$ matrix as an example, the procedure of coding spatial pixels is described in Fig. 2.

Unlike the traditional HTS [4], the high throughput of the system is realized by multiplexing spectra of different pixels, which makes the measurement faster and more efficient. From Fig. 2, we can see that the overlapping occurs between pixels in the same row at entrance; the spectrum of pixels from different rows is irrelevant. For spectral imaging, the system is comparable with slit-based push broom scanning, which is time consuming. Beyond the high throughput and efficiency, the H-matrix gains significant SNR improvement even with double measurements for $\mathbf{H}_-$ and $\mathbf{H}_+$ matrices.

2. Theoretical model

The multiplex coding measurement can be defined as a linear model, which is stated as [5,15]

$$\mathbf{g} = \mathbf{S}\mathbf{f} + \mathbf{n}_{\mathrm{HTS}} \tag{2}$$

where $\mathbf{g}$ is the measured coded data, and $\mathbf{f}$ is original signal to be recovered, $\mathbf{S}$ is the coding matrix, and $\mathbf{n}$ is noise. The coding H-matrix is divided into two different matrices, so the coding mathematic problem can be written as two separate equations:

$$\mathbf{g}_+ = \mathbf{H}_+\mathbf{f} + \mathbf{n}_+, \mathbf{g}_- = \mathbf{H}_-\mathbf{f} + \mathbf{n}_- \tag{3}$$

Here, both $\mathbf{g}_+$ and $\mathbf{g}_-$ are the measured coded data related with $\mathbf{H}_+$ and $\mathbf{H}_-$, respectively. The $\mathbf{n}_+$ and $\mathbf{n}_-$ are irrelevant noise in those two measurements. To evaluate the denoising performance of H-matrix weighting, the widely used Euclidean distance is employed to quantify the distance between two measured spectra $\mathbf{f_a}$ and $\mathbf{f_b}$, which is written as

$$D_E = \left|(\mathbf{f_a} - \mathbf{f_b})^{\mathrm{T}}(\mathbf{f_a} - \mathbf{f_b})\right|^{1/2} = \left|\Delta\mathbf{f}^{\mathrm{T}}\Delta\mathbf{f}\right|^{1/2} \tag{4}$$

Furthermore, the SNR of the distance D_E is defined as

$$SNR^{DE} = 10 \times \log_{10}\left(\left|\frac{\Delta\mathbf{f}^{\mathrm{T}}\Delta\mathbf{f}}{\Delta\mathbf{n}^{\mathrm{T}}\Delta\mathbf{n}}\right|^{1/2}\right) \tag{5}$$

where $\Delta\mathbf{f} = \mathbf{f_a} - \mathbf{f_b}$, and $\Delta\mathbf{n} = \mathbf{n_a} - \mathbf{n_b}$. For multiplex measurement, the original spectrum is not obtained directly, but is demodulated from the measured coded data, $\hat{\mathbf{f}} = \mathbf{H}^{-1}(\mathbf{g}_+ - \mathbf{g}_-)$. So the SNR of the distance can be written as

$$SNR^{DE}_{\mathbf{H}} = 10 \times \log_{10}\left(\left|\frac{\Delta\mathbf{f}^{\mathrm{T}}\Delta\mathbf{f}}{(\mathbf{H}^{-1}\Delta\mathbf{n_H})^{\mathrm{T}}(\mathbf{H}^{-1}\Delta\mathbf{n_H})}\right|^{1/2}\right) \tag{6}$$

According to the mathematical characteristic of H-matrix, the right of Eq. (6) can be simplified as

$$\begin{aligned}(\mathbf{H}^{-1}\Delta\mathbf{n_H})^{\mathrm{T}}(\mathbf{H}^{-1}\Delta\mathbf{n_H}) &= \Delta\mathbf{n}_\mathbf{H}^{\mathrm{T}}(\mathbf{H}^{-1})^{\mathrm{T}}\mathbf{H}^{-1}\Delta\mathbf{n_H} \\ &= \frac{1}{n}\Delta\mathbf{n}_\mathbf{H}^{\mathrm{T}}\mathbf{I_n}\Delta\mathbf{n_H}\end{aligned} \tag{7}$$

Substituting Eq. (7) into Eq. (6), the SNR of distance in HTS with H-matrix coding is stated as

$$\begin{aligned}SNR^{DE}_{\mathbf{H}} &= 10 \times \log_{10}\left(\left|\frac{\Delta\mathbf{f}^{\mathrm{T}}\Delta\mathbf{f}}{\frac{1}{n}\Delta\mathbf{n}_\mathbf{H}^{\mathrm{T}}\mathbf{I_n}\Delta\mathbf{n_H}}\right|^{1/2}\right) \\ &= 10 \times \log_{10}\left(\left|\frac{\Delta\mathbf{f}^{\mathrm{T}}\Delta\mathbf{f}}{\Delta\mathbf{n}_\mathbf{H}^{\mathrm{T}}\Delta\mathbf{n_H}}\right|^{1/2}\right) + 5 \times \log_{10} n\end{aligned} \tag{8}$$

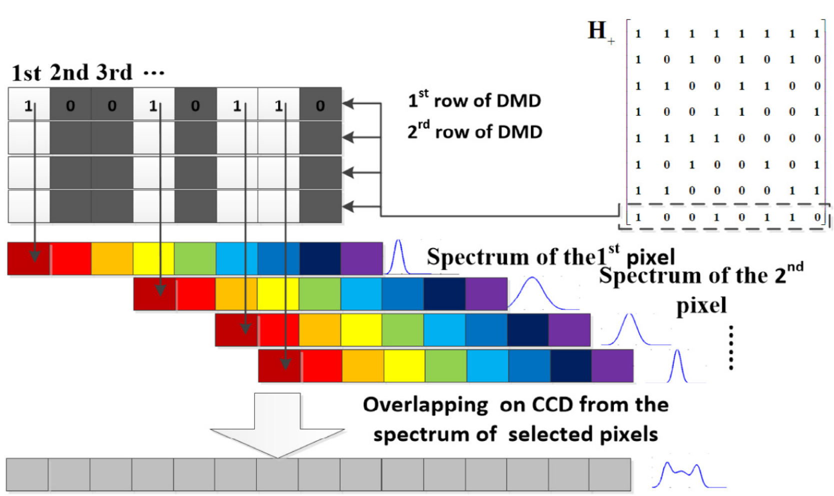

Fig. 2. Encoding the pixels at entrance with $\mathbf{H}_+$matrix.