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Propagation properties of the chirped Airy beams through the gradient-index medium

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A B S T R A C T

Through analytical derivation and numerical analysis, the propagation properties of the chirped Airy(CAi) beams in the gradient-index medium are investigated. The intensity and the phase distributions, the propagation trajectory and the Poynting vector of the CAi beams are demonstrated to investigate the propagation properties. Owing to the special and symmetrical refractive index profile of the gradient-index medium, the CAi beams propagate periodically. The effects of the distribution factor and the chirped parameter on the propagation of the CAi beams are analyzed. As the increasing of the distribution factor, the intensity distribution of the CAi beams is more scattering. However, with the chirped parameter increasing, the focusing property of the CAi beams strengthens. The variation of the chirped parameter can change the position of the peak intensity maximum, but it cannot alter the period of the peak intensity. The variations of the initial phase and the energy of the beams in the transverse plane expedite accordingly.

1. Introduction

Different from traditional materials, the gradient-index medium is a heterogeneous material with regular and continuous distribution of the refraction index. The fabrication of the gradient-index medium has already stridden forward to the practical stage. Results that characterize a fast-diffusing titania silicate gradient-index glass in a sodium for lithium ion exchange have been presented [1]. Both positive and negative axial and radial gradients have been fabricated in high index glasses by ion exchange [2]. With the self-focusing properties, the gradient-index medium has high value of application in fiber splicing, optical systems and wave propagation [3–5].

In 2007, it was first reported that the Airy beams with finite energy can retain their intensity features over several diffraction lengths and self-accelerate in both one- and two-dimensional configurations [6]. The Airy beams exhibit unusual features such as the ability to remain diffraction-free over long distances during propagation, the self-healing properties during propagation in spite of the severity of the imposed perturbations and the parabolic trajectories of the beams [7–9]. The interactions of two in-phase and out-of-phase Airy beams and nonlinear accelerating beams in Kerr and saturable nonlinear media have been investigated [10,11]. Based on the Airy beams, the chirped Airy(CAi) beams can be obtained through the phase modulation and the Fourier transform lens. Usually, the phenomenon of the shift of the central wavelength during pulse propagation is called chirp, which can be simply defined as the variation of signal frequency with time, described by the chirped parameter. The chirp has been applied to phase modulation while the beams propagate in different medium, and the technique of the chirped pulse amplification is used to generate ultrahigh peak power pulses [12]. The energy-chirped electron beam can be applied to high power femtosecond X-ray pulse generation [13].

For the propagation of the beams in the gradient-index medium, different kinds of beams including the Gaussian beams, the partially coherent flat-topped beams and the Airy-Gaussian vortex beams have been investigated [14–16]. The gradient-index medium has been demonstrated peculiar properties and amazing effects on the propagation of the beams. With the effect of the linear chirp, the transverse displacement of the finite energy Airy beams can be introduced at the phase transition point [17]. In the study of the initial frequency chirp on Airy pulse, it is found that the linear propagation of the pulse depends considerably on whether the second-order dispersion parameter β_2 and chirp c have the same or opposite signs [18]. Here we investigate the propagation

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properties of the CAi beams through the gradient index medium. Through deriving the analytic propagation equations and numerical calculations, we analyze the trajectory, the intensity and the phase distribution of the CAi beams. The effects of the distribution factor and the chirped parameter on the propagation properties of the CAi beams in the gradient-index medium are also investigated.

2. Analytical expression of the chirped Airy beams through the gradient-index medium

Considering the CAi beams propagate along the z-axis in Cartesian coordinate system through the gradient-index medium, the optical field distribution of the CAi beams in the initial plane can be written as

$$E_0(x_0,y_0,0) = A_0 Ai\left(\frac{x_0}{w_1}\right) Ai\left(\frac{y_0}{w_2}\right) \exp\left(\frac{ax_0}{w_1}+\frac{by_0}{w_2}\right) \times \exp\left(-ic\frac{x_0^2+y_0^2}{w_0^2}\right), \tag{1}$$

where $E_0(x_0,y_0,0)$ represents the electric field distribution in the input plane($z=0$), A_0 is the constant complex amplitude, $0 \le a < 1$ and $0 \le b < 1$ are the exponential truncation factors, $0 \le c < 1$ is the chirped parameter in the exponential function, w_0 is the beam waist size, w_1 and w_2 denote the arbitrary transverse scales in x and y directions and $Ai(\cdot)$ denotes the Airy function [19].

The gradient-index medium is organized into three categories: the radial distribution, the axial distribution and the spherical distribution. We focus on the radial gradient-index medium of which constant refractive-index surfaces are the cylindrical surfaces based on the symmetry axis of z-axis. The refractive-index distribution of the radial gradient-index medium can be written as

$$n = n_0\left(1-\frac{r^2}{2\beta^2}\right), \tag{2}$$

where n_0 denotes the refractive-index of the symmetry axis, β is the distribution factor of the radial gradient-index medium, r is the distance from the symmetry axis and can be expressed as $r=\sqrt{x^2+y^2}$.

Under the paraxial approximation, the $ABCD$ optical matrix of the gradient-index medium can be expressed as [20,21]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos(z/\beta) & \beta\sin(z/\beta) \\ -\sin(z/\beta)/\beta & \cos(z/\beta) \end{pmatrix}. \tag{3}$$

The paraxial propagation of the CAi beams through the gradient-index medium with the $ABCD$ optical matrix can be derived with the Huygens diffraction integral [22]

$$E(x,y,z) = \frac{ik}{2\pi B}\iint_{-\infty}^{\infty} E_0(x_0,y_0,0)\exp\left\{-\frac{ik}{2B}\left[A(x_0^2+y_0^2) - 2(x_0x+y_0y)+D(x^2+y^2)\right]\right\}dx_0dy_0, \tag{4}$$

where $k=2\pi/\lambda$ is the wave-number in the free space and λ is the wavelength of the incident light. A, B and D are elements of the transfer matrix. By substituting Eq. (1) into Eq. (4), the ultimate field distribution of the CAi beams through the gradient-index medium can be formulated as

$$E(x,y,z) = \frac{iA_0k}{2BM}\exp\left[H(x,y,z)\right]Ai\left[f(x)\right]Ai\left[g(y)\right], \tag{5}$$

where

$$H(x,y,z) = -\left(\frac{ikD}{2B}+\frac{k^2}{4B^2M}\right)(x^2+y^2)+\frac{ik}{8BM^2}\left(\frac{x}{w_1^3}+\frac{y}{w_2^3}\right) + \frac{ik}{2BM}\left(\frac{ax}{w_1}+\frac{by}{w_2}\right)+\frac{1}{96M^3}\left(\frac{1}{w_1^6}+\frac{1}{w_2^6}\right) + \frac{1}{8M^2}\left(\frac{a}{w_1^4}+\frac{b}{w_2^4}\right)+\frac{1}{4M}\left(\frac{a^2}{w_1^2}+\frac{b^2}{w_2^2}\right), \tag{6}$$

$$f(x) = \frac{ikx}{2BMw_1}+\frac{a}{2Mw_1^2}+\frac{1}{16M^2w_1^4}, \tag{7}$$

$$g(y) = \frac{iky}{2BMw_2}+\frac{b}{2Mw_2^2}+\frac{1}{16M^2w_2^4}, \tag{8}$$

$$M = \frac{ic}{w_0^2}+\frac{ikA}{2B}. \tag{9}$$

In the gradient-index medium, the ballistic trajectory of the CAi beams shows a certain regularity and can be formulated with the analytic propagation expression. In Eq. (5), by setting the imaginary part of the result to be zero, we obtain the ballistic trajectory of the CAi beams in the x–z plane and the y–z plane and it can be given by

$$s = \frac{\beta\sin(\frac{z}{\beta})}{8kw_1^3\left(\frac{c}{w_0^2}+\frac{k\cos(\frac{z}{\beta})}{2\beta\sin(\frac{z}{\beta})}\right)}, (s=x,y). \tag{10}$$

From Eq. (10), the ballistic trajectory of the CAi beams through the gradient-index exists uncertain points, the positions of which can be depicted as

$$\tan\left(\frac{z}{\beta}\right) = -\frac{kw_0^2}{2c\beta}, \tag{11}$$

$$z = \beta\arctan\left(\frac{-Z_R}{2c\beta}\right)+j\pi\beta, (j=0,1,2...), \tag{12}$$

where $Z_R=kw_0^2$ is the Rayleigh distance of the CAi beam. Thus, we can find out that the positions of this uncertain points present the periodic variations with $L/2=\pi\beta$ and be proportional to the chirped parameter. With the analytical expressions of these results, the further analysis of the propagation properties of the CAi beams will be discussed in the next part.

3. Numerical analysis and discussion for chirped Airy beams

To find out the regularity of the propagation of the CAi beams through the gradient-index medium, we mainly probe the intensity and the phase distributions of different planes, the ballistic trajectory in a certain distance and the beam center of the CAi beams. First, Fig. 1 shows the intensity and the phase distributions in the initial plane ($z=0$) from Eq. (1) with $A_0=1$, $w_0=w_1=w_2=0.1$ mm, $a=b=0.1$.

The initial intensity of the CAi beams remains steady and unchanged with different values of the chirped parameters. Fig. 1(b1)–(b2) demonstrate that the value of the initial phase changes abruptly from 2π to -2π with the absolute value of the negative chirped parameter increasing. Fig. 1(b3) shows the phase distribution of the Airy beams ($c=0$). Fig. 1(b4)–(b5) illustrate that the value of the initial phase changes abruptly from -2π to 2π with the absolute value of the positive chirped parameter increasing.

Next, with the obtained analytical results Eqs. (5)–(9), the numerical analysis of the CAi beams propagating in the gradient-index medium is showed in Fig. 2, considering the case of $\lambda=632.8$ nm, $c=-2$, $\beta=0.05$ and $n_0=1.56$. From Eq. (12), it can be clearly seen that the first uncertain point appears on $z=\beta\arctan(\frac{-Z_R}{2c\beta})$. In order to facilitate the analysis, we select 3/2 period of the trajectory starting from $z_0=\beta\arctan(\frac{-Z_R}{2c\beta})+L/4$.

In Fig. 2(b), along the z-axis in the observation plane $x=y$, the bright tracks perform the ballistic trajectory of the CAi beams. We can clearly see that there are three special positions in the distance of $3L/2$, corresponding to $z=z_0+L/4$, $z=z_0+3L/4$, $z=z_0+5L/4$. As the CAi beams tend to propagate in the direction of a bigger refractive index and the Airy beams have the self-healing feature, the CAi beams can propagate periodically in the gradient-index medium, which has special and symmetrical refractive index profile. In addition, different from the parabolic trajectory of the Airy beams in the free space, the ballistic trajectory of the CAi beams is shaped like trigonometric function. As is shown in Fig. 2, the ballistic trajectory of the CAi beams has divergent points and peculiar phenomena in the singularities. On the left of the