Optics Communications 402 (2017) 97–103

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

A dispersion-balanced Discrete Fourier Transform of repetitive pulse sequences using temporal Talbot effect

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ARTICLE INFO

Keywords:
Talbot effect
Optical pulses
Optical discrete signal processing

ABSTRACT

We propose a processor based on the concatenation of two fractional temporal Talbot dispersive lines with balanced dispersion to perform the DFT of a repetitive electrical sequence, for its use as a controlled source of optical pulse sequences. The electrical sequence is used to impart the amplitude and phase of a coherent train of optical pulses by use of a modulator placed between the two Talbot lines. The proposal has been built on a representation of the action of fractional Talbot effect on repetitive pulse sequences and a comparison with related results and proposals. It is shown that the proposed system is reconfigurable within a few repetition periods, has the same processing rate as the input optical pulse train, and requires the same technical complexity in terms of dispersion and pulse width as the standard, passive pulse-repetition rate multipliers based on fractional Talbot effect.

1. Introduction

Fractional Talbot effect [1] in the temporal domain [2] represents a successful framework for the coherent manipulation and control of picosecond pulse trains using linear optics. The range of demonstrated applications includes the multiplication of the repetition rate of the intensity [3] with optical fiber [4–7], chirped fiber Bragg gratings [8,9], or spectral line-by-line pulse shaping [10], and also at switchable [11], tunable [12], and programmable [10,13–16] repetition rates. It has also led to systems capable of factorizing prime numbers [17], amplifying a subsequence of pulses by the coherent addition of the remaining pulses in the train [18], or mitigating nonlinear impairments in pulse transmission by divided pulse amplification [19]. The effect has been used for implementing programmable spectral OCDMA codes [20], and also to perform fractional averages of pulse trains [21]. Being a collective phenomenon, it shows inherent capacities to smooth imperfections in the input periodic pattern, such as amplitude of timing errors [22–24] and, for the same reason, it can produce clock signals even when the train is on–off keyed [25–28]. Finally, self-imagined pulse trains can be compactly generated by use of injection-locked frequency-shifted feedback lasers [29,30].

Some of these demonstrations are based on self-imagined pulse trains where the unit cell or input repetitive pattern, rather than a single pulse, is a pulse sequence [13–21]. The basic output of this type of processors is a repetitive sequence of L optical pulses with a total sequence duration T. Such an optical train is described by an electric field of the form:

$$\mathcal{E}(t) = e(t)e^{j\omega_0 t} = e^{j\omega_0 t} \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{L-1} c_n\, p(t - nT/L - mT)\,, \tag{1}$$

where $e(t)$ is the complex optical envelope, ω_0 the optical central frequency, $p(t)$ is the pulse envelope, and c_n a complex sequence of length L that accounts for the amplitude and phase of the pulses in the fundamental period of the train. Restricting to direct-detection schemes, control of coefficients c_n allows for several applications of the train. On the one hand, direct detection of (1) leads to a signal composed of the repetition of the sequence $|c_n|^2$, finding immediate application as a programmable electrical waveform generator [13]. On the other, sequence $|c_n|^2$ can be employed to encode a binary or multilevel unipolar code. The resulting train defines a repetitive optical code of use in time-spread OCDMA [31] or incoherent pulse-compression systems [32].

Previous studies [33,34] were focused on systems where the control of the output sequence c_n is performed through a repetitive set of voltage levels x_n driving a modulator, so that c_n is proportional, up to a phase, to the Discrete Fourier Transform (DFT) of sequence x_n. In our examples above, this means that electrical waveforms or optical codes can be designed by Fourier synthesis. Moreover, if x_n represents an unknown data sequence that is periodically addressed to the modulator, direct detection of the train provides an estimate of its DFT power spectrum, with the sole latency associated to optical propagation and detection.

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http://dx.doi.org/10.1016/j.optcom.2017.05.071
Received 17 March 2017; Received in revised form 26 May 2017; Accepted 26 May 2017

The architecture analyzed in [33] is somewhat demanding, as it requires at least an additional phase modulation stage. The second proposal [34] is simpler, as this stage is avoided, but shows tighter requirements on the maximum allowable pulse width. In this case, the basic scheme is based on a $1/N^2$ Talbot line, which has been used to demonstrate programmable pulse repetition rate multiplication (PRRM) [13,16] and electrical waveform generation [13,35] in a fixed dispersive line.

In this paper, we introduce an architecture for the manipulation of optical pulse trains composed of periodic pulse sequences through the DFT of a control sequence, with the objective of providing a simpler implementation of the concept. The proposal is based on an optical pulse source, two Talbot dispersive lines with matched dispersion, and an intermediate modulation stage. Related schemes have been used to perform analog processing of optical or electrical, continuous-time signals within a certain temporal aperture, such as optical [36] and electrical [37] waveform magnification using time lenses and time stretch, respectively, or temporal Vander Lugt filters using temporal holograms [38]. In our proposal, however, both dispersive sections are fractional Talbot lines, so the system operates upon pulse-to-pulse multi-interference. After dispersion in the first fractional Talbot line, the optical pulses are modulated by the control levels x_n using electro-optic modulation, and are subsequently dispersed in a second fractional Talbot line. In contrast to previous approaches, this architecture shows the same technical requirements with regard to dispersion and pulse width as the standard, passive, PRRM systems, while keeping advantages such as a DFT processing rate equal to the input pulse repetition rate and absence of the initial, compensating phase modulation stage.

Our presentation is based on a compact description of fractional Talbot effect of pulse sequences by use of discrete-signal formalism. In particular, general fractional Talbot effect is here described, for the first time to the best of our knowledge, by use of a linear, discrete-signal transform mapping the complex amplitudes of input and output pulse sequences. This formalism reflects a close similarity with standard, continuous-time Fourier optics transformations. The transform is introduced in Section 2, and its construction is based on recent results regarding the structure of the quadratic Gauss sums that underlie fractional Talbot effect [39–41]. Our proposal is presented in Section 3, together with a brief account of previous developments. In Section 4, we discuss the practical differences between them and present our conclusions.

2. Discrete signal transform induced by fractional Talbot effect

The temporal Talbot effect can be introduced as follows [1]. Let us consider an optical periodic train described by an electric field whose envelope $e(t)$ consists of the repetition of a waveform $w(t)$, with $w(t) \neq 0$ in $0 \leq t < T$:

$$e(t) = \sum_{m=-\infty}^{+\infty} w\,(t - mT)\,. \tag{2}$$

This field feeds a delay line with lowest-order dispersion $\phi = d\tau/d\omega$, which is the derivative of group delay τ with respect to the optical central frequency ω_0. Fractional Talbot effect refers to the structure of the periodic output at concrete values of ϕ, determined by period T and two positive and mutually prime integers, p and q:

$$2\pi|\phi| = \frac{p}{q}T^2. \tag{3}$$

Ratio p/q defines the order of the fractional temporal Talbot effect. In diffractive optics, this corresponds to propagation lengths $z = (p/q)z_T$ with $z_T = \ell^2/\lambda$, ℓ being the period of the grating and λ the wavelength. At these values of dispersion, the output envelope is:

$$e'(t) = \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{q-1} \frac{e^{j\sigma_\phi \xi_n}}{\sqrt{q}}\, w\left(t - n\frac{T}{q} - mT - e_{pq}\frac{T}{2}\right), \tag{4}$$

where $\sigma_\phi = \pm 1$ is the sign of dispersion ϕ and e_x represents the parity of integer x, so that $e_x = 0$ when x is even and $e_x = 1$ when x is odd.

The output periodic waveform, of the same period T, is composed of the coherent sum of q replicas of the input waveform $w(t)$, mutually shifted by T/q, weighted by phase and amplitude factors, and with an additional half-period shift when the product pq is odd. The Talbot weighting factors are of constant amplitude and quadratic phase:

$$\frac{1}{\sqrt{q}} e^{j\xi_n} = \frac{1}{\sqrt{q}} e^{j\xi_0} e^{j\pi s n^2/q} \equiv t_n, \qquad (n = 0, \ldots, q-1) \tag{5}$$

where both the integer $s(p,q)$ and the phase $\exp[j\xi_0(p,q)]$ are functions of p and q enclosing the result of the quadratic Gauss sums that underlie fractional Talbot effect. This set of weighting factors are periodic with period q, and thus (5) constitutes a chirp sequence with chirp rate s, which here will be simply referred to as the *Talbot sequence* and denoted by t_n. Explicit expressions and properties of s and $\exp(j\xi_0)$ are described in [39–41]. Apart from concrete values of s and $\exp(j\xi_0)$, in our development we will only need that s is a positive integer coprime with and of opposite parity to q, so that the product sq is always even.

These formulas can be specialized to input waveforms composed of optical pulses. Following Fig. 1, let us consider an input waveform $w(t)$ which comprises q pulses of width $\Delta t < T/q$, separated by T/q, and with complex amplitudes a_k:

$$w(t) = \sum_{k=0}^{q-1} a_k\, p\,(t - kT/q)\,. \tag{6}$$

The set of q complex amplitudes a_k will be referred to as the *input sequence*. Using this expression in (4), and taking for simplicity $\sigma_\phi = 1$, we get

$$e'(t) = \frac{e^{j\xi_0}}{\sqrt{q}} \sum_{m=-\infty}^{+\infty} \sum_{k,r=0}^{q-1} e^{j\pi s r^2/q}\, a_k\, p\left(t - (r+k)\frac{T}{q} - mT - e_{pq}\frac{T}{2}\right). \tag{7}$$

The inner sum can be separated as $\sum_{r=0}^{k-1} + \sum_{r=k}^{q-1}$, and changing the variable to $r' = r + q$ in the first of these sums we are led to:

$$\sum_{m=-\infty}^{+\infty} \sum_{k=0}^{q-1} \Big[\sum_{r'=q}^{q-1+k} e^{j\pi s(r'-q)^2/q}\, a_k\, p\left(t - (r'+k-q)\frac{T}{q} - mT - e_{pq}\frac{T}{2}\right) + \sum_{r=k}^{q-1} e^{j\pi s r^2/q}\, a_k\, p\left(t - (r+k)\frac{T}{q} - mT - e_{pq}\frac{T}{2}\right)\Big]. \tag{8}$$

The phase is periodic with period q, since the product sq is even, and using a second change of variable $m' = m - 1$ in the first sum we may present (8) as:

$$\sum_{m=-\infty}^{+\infty} \sum_{k=0}^{q-1} \sum_{r=k}^{q-1+k} e^{j\pi s r^2/q}\, a_k\, p\left(t - (r+k)\frac{T}{q} - mT - e_{pq}\frac{T}{2}\right), \tag{9}$$

Finally, with the change $n = r + k$ we get a train of the form:

$$e'(t) = \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{q-1} b_n\, p\left(t - n\frac{T}{q} - mT - e_{pq}\frac{T}{2}\right), \tag{10}$$

where the *output sequence* b_n of pulse amplitudes is given by a linear transform $\mathcal{T}$, here referred to as the Talbot transform, of the input sequence a_k:

$$b_n = \mathcal{T}(\{a_k\})_n = \frac{e^{j\xi_0}}{\sqrt{q}} \sum_{k=0}^{q-1} e^{j\pi s(n-k)^2/q}\, a_k = t_n \circledcirc a_n. \tag{11}$$

As shown in the final part of the equation, the Talbot transform $\mathcal{T}$ can also be expressed as a q-point circular (or periodic) convolution $\circledcirc$ [42, ch. 5] between input and Talbot sequences. In fact, (11) defines a series of transforms acting on a_k sequences of length q, which depends on integer $s = s(p,q)$. For any s, the transform is unitary and circulant, and its inverse can be physically implemented by a Talbot line of the same order and opposite dispersion.

We stress that, in general, (11) can be used to describe the output optical pulse trains at arbitrary Talbot planes when the input has the form (6). To gain further insight into this formalism, we briefly describe some