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Impact of 3rd-order dispersion on photonic time-stretch system

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A B S T R A C T

In this paper, we present a particular theoretical analysis of the impact of 3rd-order dispersion on the photonic time-stretch (PTS) system. Analytical expressions of all harmonics with time-dependent attenuation caused by 3rd-order dispersion of the PTS system are derived, for the first time to the best of our knowledge. The theoretical analysis is based on strict derivation rather than small signal modulation. The effectiveness of the analytical model is verified by a simulated experiment. Our proposed theoretical analysis has guidance role for PTS system design.

1. Introduction

Photonic analog-to-digital conversion has attracted lots of research interest in recent decades due to its potential in achieving extremely high sampling rate [1–4]. Of all the techniques employed to improve the performance of ADC, photonic time-stretch (PTS) has been extensively investigated due to its capability of scaling the sampling rate [5–10]. In a PTS system, an optical pulse emitted from a mode-locked laser propagates through two highly dispersive mediums to get dispersed. The radio-frequency signal is modulated on the optical pulse after the first dispersive medium to perform the time-to-wavelength mapping and the signal is stretched in the time as the pulse further chirped after the second dispersive medium. Previous works have addressed on theoretical model of the PTS system with second-order group-velocity dispersion (GVD) which cause dispersion penalty and harmonic distortion [5–10].

Generally, optical fibers are usually used as the dispersion medium in the PTS system and the electro-optic intensity modulator is Mach–Zehnder modulator (MZM). It is well known that the optical fiber possesses nonlinear GVD characterized by the higher order derivative terms of propagation constant β with respect to angular frequency, such as β_3, etc. If pulse width is less than 1 ps, the impact of term caused by3rd-order dispersion β_3 should not be ignored [11,12]. In PTS system, β_3 brings about residual phase error to the stretched signal. The effect of this phase error will cause the amplitude of demodulated stretched signal being both time and frequency dependent, so it can hardly be corrected through post-signal processing. In [7], theoretical derivation of time-dependent attenuation under the assumption of linear modulation are presented and the influence of β_3 is summed up as making β_2 wavelength sensitive and causing a time-dependent attenuation via wavelength-to-time mapping.

In this paper, we present a detailed analytical model of the PTS system with 3rd-order dispersion. Analytical expressions of all harmonics with time-dependent attenuation caused by 3rd-order dispersion of the PTS system are derived, for the first time to the best of our knowledge. The theoretical analysis is based on strict derivation rather than small signal modulation. To normalize the impact of 3rd-order dispersion, a ratio factor to measure the extent of time-dependent attenuation is introduced. Numerical simulations are presented to verify the proposed theoretical model.

2. Principle

Schematic illustration of the PTS system is shown in Fig. 1. An optical pulse generated by the super-continuum source is chirped and broadened after propagating through the first spool of fiber which induces the effects of GVD. When the chirped optical pulse is modulated with an input radio-frequency (RF) by a MZM, the time-domain RF signal is mapped into wavelength-domain. As a result of propagating through the second spool of fiber, the RF signal is stretched in the time as the pulse further chirped.

Assuming the ultrashort optical pulse emitted from the super-continuum source is Gaussian-shaped, the electric field can be expressed as

$$e_1(t) = E_0 \exp\left(\frac{-t^2}{2\tau_0^2}\right) \tag{1}$$

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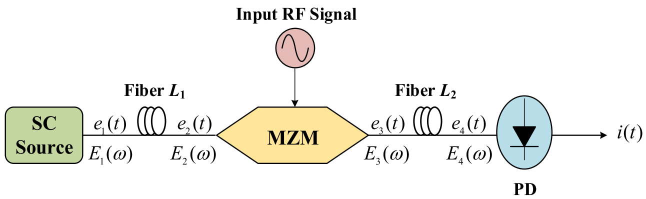

Fig. 1. Schematic illustration of the PTS system. (SC source: super-continuum source, MZM: Mach–Zehnder modulator, PD: photo-detector).

where E_0 represents the amplitude, τ_0 denotes the half-width at 1/e maximum of the pulse. Its representation in frequency domain is as

$$E_1(\omega) = E_0(2\pi\tau_0^2)^{1/2} \exp\left(\frac{-\omega^2\tau_0^2}{2}\right) \tag{2}$$

where ω denotes the angular frequency deviation from the optical carrier angular frequency.

After propagating through the first spool of fiber, the electric field becomes

$$E_2(\omega) = E_1(\omega) \exp\left(\frac{jL_1\beta_2\omega^2}{2} + \frac{jL_1\beta_3\omega^3}{6}\right) \tag{3}$$

where L_1 is the length of the first spool of fiber and β_2, β_3 are the second-order and the third-order dispersion coefficient, respectively.

Using a single-arm MZM with half-wave voltage V_π biased at the negative quadrature point, after modulation with the sinusoid RF signal of angular frequency ω_{RF}, the electric field can be represented as

$$\begin{aligned} e_3(t) &= \frac{e_2(t)}{2}\left\{1 + \exp\left[-j\frac{\pi}{2} + jm\cos(\omega_{RF}t)\right]\right\} \\ &= \frac{e_2(t)}{2}\left[1 - \sum_{n=-\infty}^{+\infty} j^{n+1}J_n(m)\exp(jn\omega_{RF}t)\right] \end{aligned} \tag{4}$$

where $e_2(t)$ is the inverse Fourier transform of $E_2(\omega)$, $m = \pi V_{RF}/V_\pi$ is the modulation index of the MZM and $J_n(\cdot)$ denotes the first kind of Bessel function of order n.

After propagating through the second spool of fiber, the electric field in frequency domain becomes

$$\begin{aligned} E_4(\omega) &= E_3(\omega)\exp\left(\frac{jL_2\beta_2\omega^2}{2} + \frac{jL_2\beta_3\omega^3}{6}\right) \\ &= E_{env}(\omega) * \left\{2\pi\delta(\omega) - \sum_{n=-\infty}^{+\infty} j^{n+1}J_n(m)\right. \\ &\quad \left.\times \exp\left[jn^2\left(\ddot{\Phi} + \frac{\beta_3L_2}{2M}\omega\omega_{RF}^2\right) - jn^3\dddot{\Phi}\right] \times 2\pi\delta\left(\omega - \frac{n\omega_{RF}}{M}\right)\right\} \end{aligned} \tag{5}$$

where $*$ represents the convolution operation, $\delta(\cdot)$ is the Dirac delta function, $E_3(\omega)$ is the Fourier transform of $e_3(t)$, L_2 is the length of the second spool of fiber, $E_{env}(\omega)$ is the spectrum of the pulse envelope, $M = \frac{1-j\beta_2(L_1+L_2)\tau_0^{-2}}{1-j\beta_2L_1\tau_0^{-2}}$ is a complex value related to the stretch factor and $\ddot{\Phi} = \frac{\beta_2L_2(1-j\beta_2L_1\tau_0^{-2})}{2[1-j\beta_2(L_1+L_2)\tau_0^{-2}]}\omega_{RF}^2$ is the phase shift induced by the second-order dispersion term [9]. The presence of 3rd-order dispersion brings about additional phase shift $\dddot{\Phi} = \frac{\beta_3L_2(M+1)\omega_{RF}^3}{6M^2}$ and $\frac{\beta_3L_2\omega\omega_{RF}^2}{2M}$ to the PTS system. The item $\frac{\beta_3L_2\omega\omega_{RF}^2}{2M}$ is wavelength related and it causes a time-dependent attenuation via wavelength-to-time mapping [7].

Since the effect of $\dddot{\Phi}$ has several orders of magnitude less than that of $\ddot{\Phi}$ for practical parameters, the Φ term in Eq. (5) can be ignored. We can get

$$\begin{aligned} E_4(\omega) &= E_{env}(\omega) * \left[2\pi\delta(\omega) - \sum_{n=-\infty}^{+\infty} j^{n+1}J_n(m)\exp(jn^2\ddot{\Phi})\right. \\ &\quad \left.\times \left(1 + jn^2\frac{\beta_3L_2}{2M}\omega\omega_{RF}^2\right) \times 2\pi\delta\left(\omega - \frac{n\omega_{RF}}{M}\right)\right]. \end{aligned} \tag{6}$$

The spectrum of the pulse envelope $E_{env}(\omega)$ is expressed as

$$E_{env}(\omega) = \sqrt{2\pi}\tau_0E_0\exp\left[-\frac{\omega^2\tau_0^2}{2} + j\frac{(3\beta_2 + \beta_3\omega)\left(L_1 + L_2\right)\omega^2}{6}\right]. \tag{7}$$

Ignoring the third-order dispersion term in the envelope and by inverse Fourier transform, we obtain the pulse envelope

$$e_{env}(t) = \frac{\tau_0E_0}{\sqrt{2\tau_0^2 - j2\beta_2\left(L_1 + L_2\right)}}\exp\left[-\frac{t^2}{2\tau_0^2 - j2\beta_2\left(L_1 + L_2\right)}\right]. \tag{8}$$

In general, the PTS system has several kilometers of the first fiber and sub-picosecond pulse, therefore the dispersion amount of the first fiber and the pulse width of the pulse satisfy the condition $\beta_2L_1 \gg \tau_0^2$.

The time domain expression of the electric field after the second fiber is achieved through inverse Fourier transform of Eq. (6)

$$\begin{aligned} e_4(t) &= e_{env}(t) \times \left\{1 - \sum_{n=-\infty}^{+\infty} j^{n+1}J_n(m)\exp(jn^2\ddot{\Phi})\exp\left(j\frac{n\omega_{RF}}{M}t\right)\right. \\ &\quad \left.\times\left[1 + \frac{n^2\beta_3L_2\omega_{RF}^2t}{j2M\beta_2(L_1+L_2)}\right]\right\} \\ &= e_{env}(t) \times \left[1 - \sum_{n=-\infty}^{+\infty} T_n \cdot \exp\left(j\frac{n\omega_{RF}}{M}t\right) - \frac{\beta_3L_2\omega_{RF}^2t}{j2M\beta_2(L_1+L_2)}\right. \\ &\quad \left.\times \sum_{n=-\infty}^{+\infty} n^2T_n \cdot \exp\left(j\frac{n\omega_{RF}}{M}t\right)\right] \end{aligned} \tag{9}$$

where $T_n = j^{n+1}J_n(m)\exp(jn^2\ddot{\Phi})$.

After photo-detecting, the photocurrent is given by

$$i(t) = r_d \cdot e_4(t)e_4^*(t) \tag{10}$$

where r_d is the responsivity of the PD and the superscript $*$ denotes the complex conjugate.

For simplifying the expression of $i(t)$, we use the Graf's addition theorem which is given as [13]

$$J_k(R)e^{jk\Omega} = \sum_{n=-\infty}^{+\infty}[J_{n+k}(r)e^{j(n+k)\theta}][J_n(r_0)e^{-jn\theta_0}] \tag{11}$$

where R and Ω denote the amplitude and phase angle of $re^{j\theta} - r_0e^{j\theta_0}$, respectively.

Combining Eq. (10) with (9), the kth order harmonic of the photocurrent can be written as

$$\begin{aligned} i_k(t) &= r_d \cdot i_{env}(t)\exp\left(j\frac{k\omega_{RF}}{M}t\right) \\ &\quad \times \left\{-j^{k+1}J_k(m)\exp(jk^2\ddot{\Phi}) - j^{k-1}J_{-k}(m)\exp(-jk^2\ddot{\Phi})\right. \\ &\quad + (-1)^kJ_k[2m\sin(k\ddot{\Phi})] + \frac{\beta_3L_2\omega_{RF}^2t}{j2M\beta_2\left(L_1+L_2\right)} \\ &\quad \left.\times\left[k^2(T_{-k}^* - T_k) + \sum_{n=-\infty}^{+\infty}(2nk + k^2)T_{n+k} \times T_n^*\right]\right\} \end{aligned} \tag{12}$$

where $i_{env}(t) = e_{env}(t)e_{env}^*(t)$ represents the photocurrent envelope.

$$i_{env}(t) = \frac{\tau_0^2E_0^2}{2\sqrt{\tau_0^4 + \beta_2^2\left(L_1+L_2\right)^2}}\exp\left[-\frac{t^2}{\tau_0^2 + \beta_2^2\left(L_1+L_2\right)^2\tau_0^{-2}}\right]. \tag{13}$$

We derive Eq. (12) without assumption of small signal modulation and it is suitable for the system with a large modulation index. The fundamental component is given by

$$\begin{aligned} i_1(t) &= r_d \cdot i_{env}(t)\exp\left(j\frac{\omega_{RF}}{M}t\right)\left\{2J_1(m)\cos(\ddot{\Phi}) - J_1[2m\sin(\ddot{\Phi})]\right. \\ &\quad \left.+ \frac{\beta_3L_2\omega_{RF}^2t}{j2M\beta_2\left(L_1+L_2\right)}\left[T_{-1}^* - T_1 + \sum_{n=-\infty}^{+\infty}(2n+1)T_{n+1} \times T_n^*\right]\right\}. \end{aligned} \tag{14}$$