Optics Communications 402 (2017) 296–301

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Coupled mode theory of microtoroidal resonators with a one-dimensional waveguide

Thi Phuc Tan Nguyen [a,*], Leonid Krivitsky [d], Leong Chuan Kwek [b,c,e,f,g]

[a] SyMMES/INAC/CEA, 17 Rue des Martyrs, F-38054 Grenoble Cedex 9, Grenoble, France
[b] Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543, Singapore
[c] Physics Department, National University of Singapore, 2 Science Drive 3, Singapore 117551, Singapore
[d] Data Storage Institute, Agency for Science, Technology and Research (A*STAR), 2 Fusionopolis Way, #08-01 Innovis, Singapore 138634, Singapore
[e] Institute of Advanced Studies, Nanyang Technological University, 60 Nanyang View, Singapore 639673, Singapore
[f] National Institute of Education, Nanyang Technological University, 1 Nanyang Walk, Singapore 637616, Singapore
[g] MajuLab, CNRS–UNS–NUS–NTU International Joint Research Unit, UMI 3654, Singapore

ARTICLE INFO

Keywords:
Microtoroidal resonator
Coupled-mode theory
Transmission spectrum

ABSTRACT

We study the transmission of light through a system consisting of an arbitrary number N of microtoroidal resonators coupled to a one-dimensional (1D) waveguide. The transmission T through such a system and its full-width at half-maximum (FWHM) are calculated for various values of N and mutual-mode coupling coefficients. We found that at small mutual-mode coupling, the minimum transmission vanishes exponentially with N while the FWHM is proportional to $\sqrt{N}$. At big mutual-mode coupling, as the number of resonators increases, the mode-splitting is reduced. Our findings contribute to better understanding of novel interfaces between quantum emitters and resonant photonic structures for quantum information processing.

1. Introduction

The atom–photon coupling can be greatly enhanced by using a cavity [1]. The strong coupling between atoms and photons is one of the main building blocks for quantum-state transfer [2] and leads to a number of interesting quantum effects [3]. Some examples of these effects are the vacuum-Rabi oscillation [4,5] and tunable photon transmission in 1D waveguides [6].

Nevertheless, attaining the strong coupling regime remains technically challenging due in part to the need for high quality-factor (Q-factor) cavities [7]. One of the several promising approaches for fabricating high Q-factor cavities is based on the use of integrated silicon photonics platforms. These platforms combine the benefits of intrinsically stable operation, CMOS compatible fabrication and compact footprint. A considerable amount of efforts has been made to achieve the highest possible enhancement of atom–photon interaction by optimizing various designs for silicon-based cavities. Photonic crystal cavities (PCCs) reaching experimental values of Q-factors at the order 10^3 have been reported [8,9]. In some cases, a Q-factor as high as 10^5 has also been achieved [10]. Other cavity designs such as microtoroidal cavities can reach very high values of Q-factor ($> 10^5$) [11,12]. They are potentially useful for achieving strong coupling between quantum emitters and cavity modes [13].

The quantum-mechanical description of the coupling between an emitter and a photonics platform can be studied using an input–output theory [14]. If there is no quantum emitter and the input light is coherent, a classical calculations can be employed. These calculations are typically done with the coupled-mode theory (CMT) [15]. Using the CMT, the transmission spectrum for one or two microtoroidal resonators coupled to four input–output ports has been reported [16]. In the presence of fabrication and dielectric defects for instance, the two counter-propagating modes inside a ring resonator can couple to each other [17–19]. When mutual-mode coupling is strong, it has been shown that the mode splitting occurs in the transmission spectrum T [20]. This mode splitting reduces the coupling efficiency between the cavity modes and the quantum emitters at the resonant frequency of the cavity.

In this paper, we present a theoretical study of the transmission of light in a 1D waveguide via a system of N microtoroidal resonators with mutual-mode coupling. There are two main motivations behind our research. On the one hand, we would like to understand the effects that an array of N microtoroidal resonators has on the mode-splitting.

* Corresponding author.
E-mail address: phuctan3108@gmail.com (T.P.T. Nguyen).

http://dx.doi.org/10.1016/j.optcom.2017.06.035
Received 14 April 2017; Received in revised form 8 June 2017; Accepted 10 June 2017
0030-4018/

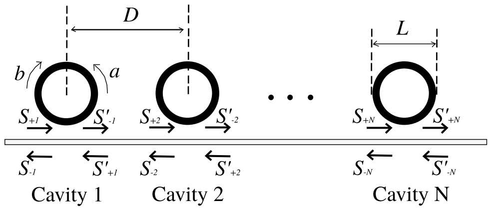

Fig. 1. Arrangement of N microtoroidal resonators (black circles) along a waveguide (transparent horizontal line); a, b, the two modes of a resonator; D, distance between two adjacent resonators; L, radius of a resonator; $S_{+/-i}(S'_{+/-i})$, incoming/outgoing light field from the left (right).

We discovered that the problem of mode-splitting is reduced for large N. We discuss in Section 3 the dependence of the depth and the width of the transmission spectrum T on N. On the other hand, the research on various systems of microtoroidal resonators has been mainly experimental or numerical [21–23]. Theoretical work has been mostly focused on one or two resonators [24,16,20] or with only one mode present in each ring [25]. A theory for the case of an arbitrary number of ring resonators that have mutual-mode coupling and that are coupled to a one-dimensional (1D) waveguide has not been investigated to the best of our knowledge. A more systematic framework, which simplifies the study for any number of resonators, is useful for future progress of the field.

The paper is organized as follows. In Section 2, we describe the general framework that allow us to study the problem of arbitrary N microtoroidal resonators and derive the N-ring transfer matrix. After giving the formal expression for the transmission spectrum in Section 3, we show the numerical results for T and discuss its dependence on N. We summarize our results in Section 4 with some remarks regarding the future directions.

2. Methods

We begin our analysis by considering N identical microtoroidal resonators with radii L coupled to a 1D waveguide as depicted in Fig. 1. Let D be the distance between the ith and $(i+1)$th rings. For simplicity, we assume that all the resonators have the same radii and that they are placed along the waveguide such that the distances between any two adjacent rings are equal. The case in which D varies among the resonators is discussed in Section 3.

Let us consider the ith resonator. Let $S_{+/-i}$ and $S'_{+/-i}$ denote the incoming/outgoing light field from the left and the right respectively. The two counter-propagating modes that are denoted a_i and b_i oscillate at frequencies ω_{a_i} and ω_{b_i}, respectively. a_i and b_i are coupled to the waveguide with coupling coefficients κ_{a_i} and κ_{b_i} given below

$$\kappa_{a_i} = \sqrt{\frac{\omega_{a_i}}{Q_{i,e}}},\ \kappa_{b_i} = \sqrt{\frac{\omega_{b_i}}{Q_{i,e}}}, \tag{1}$$

where $Q_{i,e}$ are the quality factors of the resonators. The coupling of the resonators to the waveguide gives rise to the decay rates $\Gamma_{a_i,e} = \frac{\omega_{a_i}}{2Q_{i,e}}$ and $\Gamma_{b_i,e} = \frac{\omega_{b_i}}{2Q_{i,e}}$ of the cavity modes. The coupling to other lossy channels leads to the intrinsic decay of a_i and b_i at the rate $\Gamma_{a_i,i}$ and $\Gamma_{b_i,i}$, respectively. Thus, the total decay rates are

$$\Gamma_{a_i} = \Gamma_{a_i,e} + \Gamma_{a_i,i},\ \Gamma_{b_i} = \Gamma_{b_i,e} + \Gamma_{b_i,i}. \tag{2}$$

We also take into account the coupling between a_i and b_i. This coupling is characterized by the coefficients u_i that are usually taken to be real [20,26] since the coupling is essentially the energy transfer between a_i and b_i without losses. One of the most common reasons for u_i to be non-zero is dielectric defects. We further let u_i to be frequency-independent as we work in linear optics regime, though the current framework can be extended to ω-dependent coupling.

2.1. Coupled-mode analysis

In this subsection, we solve for a_i and b_i using the CMT [15] in the frequency domain. On the one hand, this approach gives the same results as the steady-state solution from the time-domain analysis for the case in which $N = 1$ and the system is pumped from the left with $S_0 e^{i\omega t}$. On the other hand, $a_i(\omega)$ and $b_i(\omega)$ can be obtained by solving algebraic equations while $a_i(t)$ and $b_i(t)$ are the Fourier transforms of the former.

By making Fourier transform the coupled-mode equations for the ith resonator [20], the set of equations for a_i and b_i in the ω-domain is

$$\begin{cases} \left[i(\omega-\omega_{a_i}) + \Gamma_{a_i}\right] a_i(\omega) + iu_i b_i(\omega) = -i\kappa_{a_i} S_{+i}(\omega) \\ \left[i(\omega-\omega_{b_i}) + \Gamma_{b_i}\right] b_i(\omega) + iu_i a_i(\omega) = -i\kappa_{b_i} S'_{+i}(\omega). \end{cases} \tag{3}$$

To simplify the subsequent analysis, we employ the following shorthand notations

$$A_i(\omega) = i(\omega-\omega_{a_i}) + \Gamma_{a_i},\ B_i(\omega) = i(\omega-\omega_{b_i}) + \Gamma_{b_i}, \tag{4}$$

$$D_i(\omega) = A_i(\omega)B_i(\omega) + u_i^2, \tag{5}$$

and

$$t_{A_i} = 1 - \frac{|\kappa_{b_i}|^2 A_i(\omega)}{D(\omega)},\ t_{B_i} = 1 - \frac{|\kappa_{a_i}|^2 B_i(\omega)}{D(\omega)}. \tag{6}$$

It is straightforward to show that the solutions $a_i(\omega)$ and $b_i(\omega)$ with respect to $S_{+i}(\omega)$ and $S'_{+i}(\omega)$ are

$$a_i(\omega) = \frac{-i\kappa_{a_i} B_i(\omega)}{D_i(\omega)} S_{+i}(\omega) - \frac{u_i \kappa_{b_i}}{D_i(\omega)} S'_{+i}(\omega) \tag{7}$$

$$b_i(\omega) = \frac{-i\kappa_{b_i} A_i(\omega)}{D_i(\omega)} S'_{+i}(\omega) - \frac{u_i \kappa_{a_i}}{D_i(\omega)} S_{+i}(\omega). \tag{8}$$

2.2. Input–output relation

In this subsection, we relate the signal on the left most $S_{+/-1}$ to the right most $S'_{+/-N}$ side of a system of an arbitrary number N of resonators. In other words, we want to find the matrix $T^{(N)}$ that satisfies the following condition:

$$\begin{pmatrix} S'_{-N} \\ S'_{+N} \end{pmatrix} = T^{(N)} \begin{pmatrix} S_{+1} \\ S_{-1} \end{pmatrix}. \tag{9}$$

We divide the problem of finding $T^{(N)}$ into two smaller ones: (i) obtaining the single-ring transfer matrix T_i and (ii) determining the relation of the signal between two adjacent resonators.

(i) *Transfer matrix* T_i:

The transfer matrix relates the left port of the ith resonator to its right port. It is defined as

$$\begin{pmatrix} S'_{-i} \\ S'_{+i} \end{pmatrix} = T_i \begin{pmatrix} S_{+i} \\ S_{-i} \end{pmatrix}. \tag{10}$$

By using $a_i(\omega)$ and $b_i(\omega)$ in Eqs. (7) and (8) together with the following relations between the cavity modes and the signal

$$\begin{cases} S'_{-i} = e^{-i\Phi_L}\left(S_{+i} - i\kappa^*_{a_i} a_i\right) \\ S_{-i} = e^{-i\Phi_L}\left(S'_{+i} - i\kappa^*_{b_i} b_i\right), \end{cases} \tag{11}$$

the transfer matrix for the ith ring is given by

$$T_i = \begin{pmatrix} e^{-i\Phi_L}\left(t_{B_i}(\omega) + \frac{u_i^2|\kappa_{a_i}|^2|\kappa_{b_i}|^2}{D_i(\omega)^2 t_{A_i}(\omega)}\right) & i\frac{u_i\kappa^*_{a_i}\kappa_{b_i}}{D_i(\omega)t_{A_i}(\omega)} \\ -i\frac{u_i\kappa^*_{b_i}\kappa_{a_i}}{D_i(\omega)t_{A_i}(\omega)} & e^{i\Phi_L}\frac{1}{t_{A_i}(\omega)} \end{pmatrix}. \tag{12}$$

We note that in the above formula, Φ_L is just the phase factor that the light picks up when traveling across the ith microtoroidal resonator.