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Surface plasmon dispersion relation at an interface between thin metal film and dielectric using a quantum hydrodynamic model

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A R T I C L E   I N F O

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A B S T R A C T

The plasmon excited at the surface of a thin metal film covering on a semi-infinite dielectric substrate is investigated in detailed. The surface plasmon dispersion relation is derived and presented by using the quantum hydrodynamic theory with taking into the quantum statistical and quantum diffraction effects. The effects of the thin metal film character r_s and the dielectric constant ε_1 on the dispersion relations are shown and discussed, i.e., without and with quantum effects. The results show that increasing r_s weakens the quantum effects while increasing ε_1 can enhance the quantum effects. In addition, the plasmon dispersion relation with quantum effects is also compared to a classical model. This simple structure proposed in the paper with quantum hydrodynamic theory can be used for yielding meaningful results for studying more complex systems related to surface plasmons.

1. Introduction

A plasmon is a quantized collective oscillation of electron in metals or semiconductors. In recent years, renewed interest in plasmon has attracted a great deal attention in investigation of two-dimensional (2D) materials [1–4], due to its ability to localize the electromagnetic radiation to subwavelength scales and enhance the local electric fields by means of its strong interaction with light and free electrons [5–7]. These novel properties make plasmonic structures valuable candidates in various worth seeing applications, such as surface enhance Raman spectroscopy [8,9], molecule sensors [10], enhancing nonlinear optical phenomena [11], quantum computing and data storage [12,13], plasmon enhanced fluorescence [14], solar cell [15], and studying fundamental phenomena [16,17].

In many plasmonic systems, the case of thin metal film with certain thickness on a dielectric substrate is more important because the electric fields of both surface interact and make the electron oscillations different from the normal surface plasmon. It is to be noted that plasmonics restrict to a coupling between electromagnetic radiation and metal film for the generation of plasmons. The assembly of thin metal film structures into small size, provides suitability to control and tune their optical properties by coupling it to incident light. It has been shown that there is a strong correlation between the film thickness and film excitation features [18,19]. The behavior of plasmon is quite unique at nanoscale. In that case the quantum effects cannot be neglected which may yield some new interesting phenomena [20].

In this paper, we investigate the properties of surface plasmon of a thin metal film covering on the semi-infinite dielectric substrate. Although it was first introduced to solve the nonlinear Schrördinger–Poisson or Winger–Poisson kinetic models by Haas et al. [21–23], the quantum hydrodynamic (QHD) model has become popular for its extension of the usual fluid model to one incorporating the quantum effects. Here, we employ QHD method to examine quantum effects on the surface plasmon dispersion. Two quantum effects are considered in the paper. One is quantum statistical effect which is the force due to the internal interactions in the electron species, the other is quantum diffraction effect which is coming from the quantum pressure due to the Bohm force. We examine the plasmon dispersion relations in four kinds of situations and discuss the influence of different electron density and dielectric constant on the dispersion relations. Further, the dispersion relations with the quantum effects are compared to a classical model.

This paper is organized as follows. The QHD model and corresponding analytical formulas are introduced in Section 2. Numerical results are presented and discussed in Section 3. Finally, conclusions are given in Section 4. Gauss units will be adopted throughout this paper, except in the case of specific definitions.

2. QHD model and formalism

We consider a thin metal film covering on a semi-infinite dielectric substrate with dielectric constant ε_1, as the schematic diagram shown

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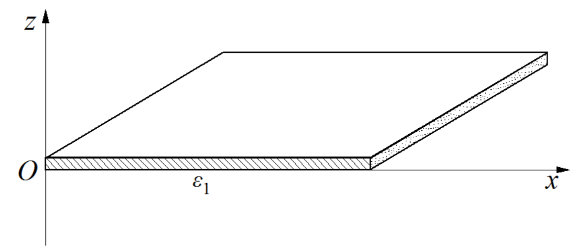

Fig. 1. Schematic view of a thin metal film covering on a semi-infinite dielectric substrate. The metal is located at $z = 0$, the substrate with dielectric function ε_1 is in the region $z < 0$, while the vacuum is in the region $z > 0$.

in Fig. 1. The thin metal film is considered as two-dimensional electron gas (2DEG) which is located in the plane $z = 0$, and the free space in the region $z > 0$ is vacuum.

In the $z > 0$ regime, using Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{1}$$

and

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}. \tag{2}$$

we can get

$$\nabla^2 \boldsymbol{B} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} = 0, \tag{3}$$

where c is the speed of light. In our proposed case, we consider the magnetic field propagation as a plane wave along y axis, i.e., B $= \{0, B_y, 0\}$, and $B_y(x,z,t) = b(z)e^{i(kx-\omega t)}$. Under these assumption, from Eq. (3), we can obtain,

$$\frac{d^2 b(z)}{dz^2} - (k^2 - k_0^2)b(z) = 0, \tag{4}$$

where $k_0 = \omega/c$ and k is the propagation constant. Similar with the literature [24], the solution of Eq. (4) for $z > 0$ region is

$$b(z) = A_1 \exp(-\kappa_0 z), \tag{5}$$

where $\kappa_0 = \sqrt{k^2 - k_0^2}$ which is a real number. By using Eq. (2) we can get

$$E_x = i\frac{c\kappa_0}{\omega}B_y, \tag{6}$$

$$E_z = -\frac{ck}{\omega}B_y. \tag{7}$$

Similarly, in the $z < 0$ regime, we also get

$$b(z) = A_2 \exp(\kappa_1 z), \tag{8}$$

$$E_x = -i\frac{c\kappa_1}{\varepsilon_1 \omega}B_y, \tag{9}$$

$$E_z = -\frac{ck}{\varepsilon_1 \omega}B_y \tag{10}$$

where $\kappa_1 = \sqrt{k^2 - k_1^2}$ with $k_1 = \varepsilon_1 k_0$.

Imposing the continuity condition of E_x while jump condition of B_y at the surface $z = 0$, we can obtain:

$$E_x|_{z=0^+} = E_x|_{z=0^-}, \tag{11}$$

$$B_y|_{z=0^+} - B_y|_{z=0^-} = -\frac{4\pi}{c}\alpha_x \tag{12}$$

where α_x is the x-component of the area current density $\boldsymbol{\alpha}$, which is defined as $\boldsymbol{\alpha} = -n_e e \boldsymbol{u}_e$, here n_e is the electron density, e is the elementary charge, and $\boldsymbol{u}_e$ is the velocity of electron in the thin metal film.

According to Eqs. (6), (9) and (11), we get

$$\kappa_0 B_y|_{z=0^+} = -\frac{\kappa_1}{\varepsilon_1}B_y|_{z=0^-}. \tag{13}$$

Combining Eqs. (12) and (13), we can obtain the plasmon dispersion as

$$\left(1 + \frac{\kappa_1}{\varepsilon_1 \kappa_0}\right) B_y|_{z=0^-} = \frac{4\pi}{c}\alpha_x. \tag{14}$$

It is easy to see that, if $\alpha_x = 0$, i.e., there is no thin metal film, the dispersion relation is $1 + \frac{\kappa_1}{\varepsilon_1 \kappa_0} = 0$, which is the dispersion relation of surface plasmons at a surface of a semi-infinite solid [5].

The area current density can be determined by the QHD equations for the 2DEG. Here, we treat the valence electrons in the thin metal film as free electron gas immersed in a uniform background of positive charges with the density per unit area n_0. The homogeneous electron gas in the thin metal film will be perturbed with a light propagation, and can be regarded as a charge fluid with 2D scalar density field $n_e(\boldsymbol{r},t) = n_0 + n_{e1}(\boldsymbol{r},t)$ and a vector velocity field $\boldsymbol{u}_e(\boldsymbol{r},t) = \boldsymbol{u}_{e1}(\boldsymbol{r},t)$, where n_{e1} and $\boldsymbol{u}_{e1}$ are the disturbed values of the electron gas density and velocity, and can be written as $n_{e1}(\boldsymbol{r},t) = n_{e1}e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$ and $\boldsymbol{u}_{e1}(\boldsymbol{r},t) = \boldsymbol{u}_{e1}e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$, respectively. Therefore, we can linearize the QHD equations of the electron excitations on the thin metal film as below

$$\frac{\partial n_{e1}}{\partial t} + \nabla_{\parallel} \cdot (n_0 \boldsymbol{u}_{e1}) = 0, \tag{15}$$

$$\frac{\partial \boldsymbol{u}_{e1}}{\partial t} = -\frac{e}{m_e}\boldsymbol{E} - \frac{\pi\hbar^2}{m_e^2}\nabla_{\parallel} n_{e1} + \frac{\hbar^2}{4m_e^2 n_0}\nabla_{\parallel}(\nabla_{\parallel}^2 n_{e1}) \tag{16}$$

where m_e is the mass of electron and $\hbar$ is the reduced Planck constant which is defined as $\hbar = h/2\pi$. The operator $\nabla_{\parallel}$ in Eqs. (15) and (16) is defined as $\nabla_{\parallel} = \frac{\partial}{\partial x}\hat{\boldsymbol{e}}_x + \frac{\partial}{\partial y}\hat{\boldsymbol{e}}_y$ and it is restricted only to the quantum electron gas surface. The first term in the right-hand side of Eq. (16) is the force on electron in the thin metal film due to the tangential component of the electric field, the second term is the quantum statistical (QS) effect and the last term is the quantum diffraction (QD) effect. Note that here we only consider the in-plane (x, y) components of the electric field, as only the in-plane motion of a 2DEG (surface plasmon) is studied, similar with our previous paper [25]. In particular, a 2D ideal gas Fermi pressure [26], P_f is considered in above Eq. (16),

$$P_f = \frac{\pi\hbar^2 n_e^2}{2m_e}. \tag{17}$$

Based on the linearized QHD equations (15) and (16) with the assumptions of n_{e1} and $\boldsymbol{u}_{e1}$, we can get

$$\boldsymbol{u}_{e1} = -i\frac{e}{m_e \omega}\frac{\boldsymbol{E}}{D(k,\omega)}. \tag{18}$$

The expression of $D(k,\omega)$ in Eq. (18) is divided into four kinds of situations: (1) without quantum effect; (2) with only QS effect; (3) with only QD effect; and (4) with both QS and QD effects. It can be expressed as below

$$D(k,\omega) = \begin{cases} 1, \text{ without quantum effect} \\ 1 - \frac{1}{2}\left(\frac{ku_F}{\omega}\right)^2, \text{ with only QS} \\ 1 - \frac{1}{4}\left(\frac{ku_F}{\omega}\right)^2\left(\frac{k}{k_F}\right)^2, \text{ with only QD} \\ 1 - \frac{1}{2}\left(\frac{ku_F}{\omega}\right)^2 - \frac{1}{4}\left(\frac{ku_F}{\omega}\right)^2\left(\frac{k}{k_F}\right)^2, \text{ with QS and QD} \end{cases} \tag{19}$$

where $k_F = (2\pi n_0)^{1/2}$ is the Fermi wave number of 2D electron gas, and $u_F = \hbar k_F/m_e$ is the Fermi velocity.

Thus, the area current density is

$$\boldsymbol{\alpha} = i\frac{e^2 n_0}{m_e \omega}\left[\boldsymbol{E} + \frac{1 - D(k,\omega)}{D(k,\omega)}\frac{\boldsymbol{k}\cdot\boldsymbol{E}}{k^2}\boldsymbol{k}\right]. \tag{20}$$

And then the x-component of the area current density is

$$\alpha_x = i\frac{e^2 n_0}{m_e \omega}\left[1 + \frac{1 - D(k,\omega)}{D(k,\omega)}\right]E_x = \frac{e^2 n_0 c\kappa_1}{\varepsilon_1 m_e \omega^2}\left[1 + \frac{1 - D(k,\omega)}{D(k,\omega)}\right]B_y|_{z=0^-}. \tag{21}$$