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# Surface shape measurement by multi-illumination lensless Fourier transform digital holographic interferometry

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A B S T R A C T

This paper presents a multi-illumination lensless Fourier transform digital holographic interferometry method for surface shape measurement. In this method, the interference phases with different effective synthetic wavelengths are obtained by tilting the illumination angle several times, and all are wrapped. A Fourier-transform demodulation algorithm employing all these wrapped phases simultaneously is used to determine the object surface shape. No phase unwrapping procedure is required, and the shape information of each point is calculated independently, thereby offering great flexibility for measuring objects with discontinuities surface, such as holes, steps and gaps. Experimental results demonstrate the validity of the principle.



## 1. Introduction

Holography found a new life in the decades after the digital recording based an array detector and numerical reconstruction were achieved in 1994 [1]. Digital holography appears to have been a mature topic in the following decades, covering a wide range of areas, such as microscopy [2], three-dimensional (3D) imaging and display techniques [3], metrology and profilometry [4], and so on. Because digital holography can numerically reconstruct both the intensity and phase of the object light field, the special information contained in the phase allows us to achieve various measurements. For example, the object deformation and vibration, refractive index, material parameters and surface shape can all be determined by the phase distribution based on digital holography [5]. For profile measurement, the digital holographic interferometry method is frequently used, in which holograms corresponding to two different states are usually recorded, and this difference can result from the two-wavelength, two-source or two-refractive indices [6–8]. The phase distribution containing the surface shape information is then calculated by the phase-shifting or Fourier-transform method [9,10]. A holographic contour will appear by taking the subtraction between the phases of two different wave fields. However, this phase variation always has $2\pi$ ambiguity because of it being calculated from the arctangent function; hence, the phase-unwrapping procedure is necessary to obtain the continuous phase. However, obtaining a correct unwrapping phase still remains a challenge if the object has discontinuities, such as holes, steps and gaps, despite the many powerful phase-unwrapping methods proposed in recent years [11].

To avoid phase unwrapping, a multi-wavelength digital holographic method was proposed [12,13]. In this method, the light frequency is changed at a certain step equally while the hologram is recorded, and the phases of the object wave field are demodulated by the phase-shifting method. Unlike the two-wavelength method, no phase subtraction exists in this method. The surface shape is reconstructed based on a Fourier-transform demodulation algorithm, and the height information of every point is determined independently. Hence, no phase-unwrapping algorithm is required and high accuracy can be obtained with this method. However, due to the use of a tunable laser, the measurement system is usually costly. The chromatic aberration caused by the multi-wavelength source should be also considered and corrected [14]. In addition, a 4f image system and a phase-shifting setup are required in this method, which is not flexible and will complicate the measurement system.

In this paper, we proposed a multi-illumination lensless Fourier transform digital holographic interferometry method to measure the surface shape, that is also based on the Fourier-transform demodulation algorithm, but a 4f image system and a phase-shifting setup is not required. A lensless Fourier transform holographic configuration is adopted. In our method, the illumination angle is changed successively when the hologram is recorded. For each illumination angle, only one hologram is captured, and the phases of the object wave field are demodulated by the Fourier-transform method. Let the phase of the object wave field corresponding to the initial illumination be the

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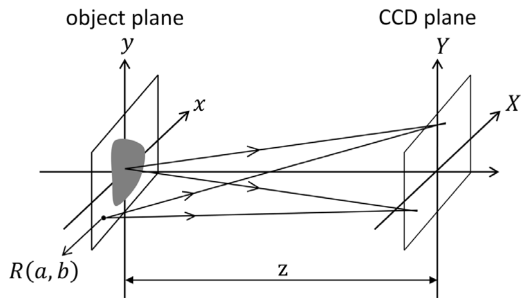


**Fig. 1.** Schematic of the holographic recording with a point source reference close to the object.

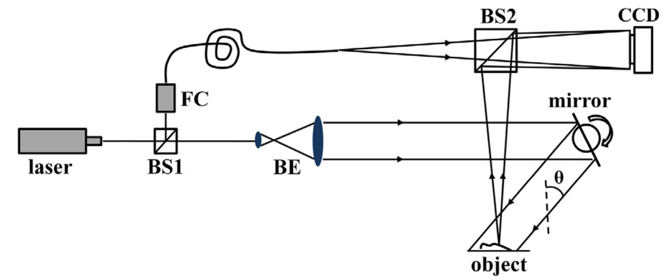


**Fig. 2.** Schematic of the basic set up of the lensless Fourier transform digital holography, including beam splitters (BS), abeam expander (BE), a fiber coupler (FC), a CCD sensor and a mirror mounted on a precision rotary stage.

reference phase. Consequently, the interference phase with different effective synthetic wavelength can be obtained, and all are wrapped. Based on the Fourier-transform demodulation algorithm, all wrapped phases are simultaneously used to demodulate the surface shape, which is calculated point by point independently. Hence, our method will be suitable for an object with a complicated and discontinuous surface. Because a laser with single wavelength is used, in addition to having a lower system cost, this method can be used for an object with wavelength-dependent reflectivity and is free of the chromatic aberration caused by different wavelengths.

## 2. Methodology

### 2.1. Lensless Fourier transform digital holographic interferometry by tilting illumination angle

The common characteristic of digital holography is the use of an image sensor such as a charge couple device (CCD) to record a hologram formed by the interference between the object beam and reference beam. Let the CCD plane be the $X$–$Y$ plane, and the object is placed in the $x$–$y$ plane. Assume that the object wave in the $x$–$y$ plane is $O(x, y)$. Under the Fresnel approximation, the object diffracted wave $U(X, Y)$ in the CCD plane will be defined as

$$U(X,Y) = \frac{\exp(jkz)}{j\lambda z}\exp\left[jk\frac{(X^2+Y^2)}{2z}\right]\int\int_{-\infty}^{+\infty} O(x,y) \times \exp\left[jk\frac{(x^2+y^2)}{2z}\right]\exp\left(-jk\frac{xX+yY}{z}\right)dxdy, \quad (1)$$

where $\lambda$ is the wavelength, $z$ is the distance between the object and detector, and $k$ denotes the wave number, $k = 2\pi/\lambda$. The recorded hologram $H(X, Y)$ can be expressed as

$$H(X,Y) = [U(X,Y)+R(X,Y)]\times[U(X,Y)+R(X,Y)]^*, \quad (2)$$

where $R(X, Y)$ is the reference wave, and $*$ indicates a complex conjugation. Note that the hologram includes four terms, and it will appear in the reconstruction wave field. By using phase-shifting or off-axis frequency filter method, the interest term $U(X, Y)R(X, Y)^*$ can be extracted. By multiplying a reconstruction reference wave, the object wave $O(x, y)$ can be finally reconstructed based on the Fresnel transform, convolution transform or angular spectrum method [4]. Particularly, in the case where the hologram is recorded by a point source close to the object as shown in Fig. 1, we have

$$R(X,Y) = A\exp\left\{\frac{jk}{2z}\left[(X-a)^2+(Y-b)^2\right]\right\}, \quad (3)$$

where $A$ is the amplitude of the reference wave, $(a, b)$ is the coordinate of the reference point source in the object plane.

The term $U(X, Y)R(X, Y)^*$ becomes

$$U(X,Y)R(X,Y)^* = \frac{\exp(jkz)}{j\lambda z}\int\int_{-\infty}^{+\infty} O(x,y) \times \exp\left[jk\frac{(x^2+y^2)}{2z}\right]\exp\left(-jk\frac{(x-a)X+(y-b)Y}{z}\right)dxdy. \quad (4)$$

It can be observed that the above equation is actually a quasi-Fourier transform. Besides a quadratic phase term, the object wave can be simply reconstructed by performing a Fourier transform of the recorded intensity, which will both simplify the holographic recording and reconstruction process.

The basic setup of our multi-illumination lensless Fourier transform digital holography method is illustrated in Fig. 2. The laser beam is split into two parts by the first cube beam splitter. One beam is collimated by a beam expander and is then reflected from the mirror and used to illuminate the object at a certain angle. The mirror is mounted on a precision rotary stage, which is used to tilt the illumination angle. The other beam is coupled into a single mode fiber through a fiber coupler, thereby forming a spherical reference beam. The reference point source and the object are located at the same plane equivalently based on a cube beam splitter. The light scattered from the object interferes at the CCD sensor plane with the spherical reference beam, thereby forming the lensless Fourier transform holography [15].

Without considering the constant term, from Eq. (4), the reconstruction object wave $O_r(x, y)$ can be expressed as

$$O_r(x,y) = O(x-a, y-b)\exp\left[jk\frac{(x-a)^2+(y-b)^2}{2z}\right]. \quad (5)$$

The phase distribution $\varphi$ of the $O_r(x, y)$ can be simply defined by

$$\varphi = arc\tan\left[\frac{\mathrm{Im}(O_r)}{\mathrm{Re}(O_r)}\right], \quad (6)$$

where the operators Im and Re denote the imaginary and real parts of a complex function, respectively. Since the phase does not affect the amplitude of the object wave, the object image can be directly reconstructed from Eq. (5). However, the phase information is important for us to achieve shape reconstruction, the additional phase distortion of the object wave need to be considered.

Assume that the initial illumination angle is $\theta_0$ and the reconstruction phase is $\varphi_0$. If we change the illumination angle at a step $\Delta\theta$ around $y$-axis, the phase $\varphi_1$ can be obtained. Let $\varphi_0$ be the reference phase; in this case, the phase difference $\Delta\varphi$ can be expressed as [7]

$$\Delta\varphi(x,y) = -kH\left[\cos\theta_0 - \cos\left(\theta_0+\Delta\theta\right)\right] - kx\left[\sin\theta_0 - \sin\left(\theta_0+\Delta\theta\right)\right], \quad (7)$$

where $H$ is the actual height of the object to be measured. It can be observed that the phase difference is not affected by the phase distortion in Eq. (5). Because of the stationary of the reference point source during the change of the illumination angle, the phase $\varphi_0$ and $\varphi_1$ have the same phase distortion, which is canceled out for the phase difference $\Delta\varphi$. Note that two terms are included in the phase difference. The first term contains the object height information while the second term as tilt component needs to be subtracted for shape measurement.

It is important to note that the illumination angle cannot be tilted at an arbitrary step $\Delta\theta$. Two main constrains exist. One is the limited diameter of the expanded laser beam collimated by a beam expander. If the $\Delta\theta$ is too large, the object cannot be covered by the illumination light. The other is the limited resolution of the CCD. From Eq. (7), the