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Phase extraction for dual-wavelength phase-shift Fizeau interferometry in the presence of multi-beam interference

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ABSTRACT

Dual-wavelength interferometry could extend the measured range of single-wavelength interferometry by combining the two single wavelength phases, particularly for the measurement of step height. When testing the high-reflectivity surfaces with the single wavelength Fizeau interferometer, we have presented the $\pi/4$ phase-shift carrier squeezing interferometry (QCSI) method for phase extraction with multi-beam interference (*Appl. Opt.* 55, 1920–1928, 2016). In this paper, we propose an integer and decimal portions synthetization (IDS) method for the multi-beam interference in the dual-wavelength Fizeau interferometer. One of the single wavelength wrapped phases is demodulated by the multi-beam interference QCSI algorithm, while the second of the single wavelength wrapped phases is extracted by the conventional two-beam interference phase-shift algorithm, so the equivalent wavelength unwrapped phase is derived from the two single wavelength wrapped phase. The decimal portion of synthesized phase is then obtained directly from the first single wavelength wrapped phase, and the integer portion of synthesized phase is obtained from the fringe order of the first single wavelength wrapped phase determined by the equivalent wavelength unwrapped phase. The proposed non-iterative IDS method avoids the common error magnification effect in the two-wavelength techniques, and only requires no more than 8 frame phase-shift interferograms for each single wavelength. Its robust performance is validated by both simulations and experiments in the presence of multi-beam interference as well as phase-shift error for measuring objects with height discontinuities.

1. Introduction

The interferometry is a powerful, typical and direct choice to measure the three-dimensional surface profile [1]. However, if the surface has a height discontinuity larger than a quarter of the wavelength of the illumination laser, the profile cannot be measured correctly using single-wavelength interferometry, due to the 2π phase ambiguity problem in the phase unwrapping procedure. Dual-wavelength interferometry provides a solution to solve this problem [2,3]. Subtracting the unwrapped phase at the first wavelength from the other unwrapped phase at the second wavelength, the phase for the long equivalent wavelength is produced, enabling measurement of height discontinuities larger than $\lambda/4$ at either single wavelength. A variety of remarkable researches to demodulate the phase for the equivalent wavelength have been reported, which are based on the two-beam interference [4–6].

However, when testing surface with high reflectivity, the multi-beam interference exists and the interference intensity is not strictly cosine distributed [7,8]. The ripple error presents obviously in the extracted phase for the equivalent wavelength from the combination of the two phases at each single wavelength when using the routine phase-shift algorithms suitable only for two-beam interference. Placing an attenuator in interferometric cavity is a usual approach to suppress multi-beam interference. However, for the test surfaces with different reflectivity or apertures, attenuators should be fabricated with different transmissivities or apertures at both of the two wavelengths in dual-wavelength Fizeau interferometry.

The other way to handle this problem is to separately demodulate the phase at each single wavelength using the multi-beam interference algorithms. These multi-beam interference algorithms have been made aiming at reducing the effect of harmonics, which could be divided into

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the non-iterative and iterative methods according to the demodulation process. The multi-beam interference error could be suppressed by the $\pi/4$ phase-shift averaging method [9], principal component analysis (PCA) [10–12], 4N-3 algorithm [13,14] and so on, all of which are non-iterative algorithms. However, their precision is limited by the accuracy of phase shift. Using the iterative algorithm, phase distribution can be extracted from the interferograms with random phase shifts [15,16]. Actually, the phase shifts in the well-calibrated Fizeau interferometer are not arbitrary but with minor difference from the demanded nominal value, so the time-consuming iterative algorithm will not be the best choice. To suppress the ripple error in the retrieved phase induced by the multi-beam interference and the phase-shift errors simultaneously, a $\pi/4$ phase-shift carrier squeezing interferometry (QCSI) algorithm is proposed [17], which is based on carrier squeezing interferometry (CSI) by converting the temporal phase shift into spatial carrier and establishing the relationship of the temporal domain and spatial domain [18–20]. However, as mentioned above, the equivalent wavelength phase is obtained by subtracting the phase at one wavelength from the phase at the other wavelength. Therefore, both phases should be demodulated by the multi-beam interference algorithms mentioned above respectively, which would be a complex and time-consuming process. Moreover, for the error magnification effect in the two wavelength techniques [2], all the retrieve errors of the two single-wavelength phases would be superposed and magnified in the equivalent wavelength phase.

This paper is organized mainly as follows. In Section 2, for single wavelength phase in the presence of multi-beam interference as well as phase-shift error, the calculated residual phase error by the conventional phase-shift algorithm such as the de Groot 7-frame algorithm [21] is analyzed. Based on the analysis of the demodulated phase error and the two-wavelength techniques, we present an integer and decimal portions synthetization (IDS) method for multi-beam interference in the dual-wavelength Fizeau interferometer in Section 3. By the calculation of the integer-portion phase, the residual phase error for the conventional phase-shift algorithm could be suppressed. And then one of the single-wavelength wrapped phases could be extracted by the multi-beam interference algorithm, while the second could be demodulated by the conventional two-beam interference phase-shift algorithm. So the process of phase demodulation is simplified by the proposed IDS method. Finally, in Sections 4 and 5, numerical simulations and experiments are executed to demonstrate the performance of the proposed IDS method in multi-beam interference, compared with the 7-frame algorithm, $\pi/4$ phase averaging algorithm and the iterative algorithm.

2. Analysis for the demodulated error

In the presence of multiple-beam interference as well as phase-shift error for Fizeau interferometer, the calculated residual phase error for the conventional phase-shift algorithm could be expressed as:

$$\Delta\phi = o\left(a_i\right) + o\left(\varepsilon_j\right) \tag{1}$$

where $o\left(a_i\right)$ and $o\left(\varepsilon_i\right)$ are the residual calculated phase errors resulted by the harmonics in multi-beam interferometry and the phase-shift error separately, and a_i is the coefficients of harmonic in multi-beam interferometry, while ε_i is the error coefficient of phase shift. To estimate the influences of the retrieve errors for the single-wavelength phase in dual-wavelength interferometry, the multi-beam interferometry error $o\left(a_i\right)$ and phase-shift error $o\left(\varepsilon_i\right)$ would be analyzed separately in following section.

2.1. demodulated error in presence of multi-beam interference

The intensity distribution of multi-beam interference in Fizeau interferometer could be developed in Fourier series from the expression of the Airy formula [22]. And the intensity distribution of multi-beam interference could be expressed approximately as the sum of harmonics [23]:

$$I_n = I_0\left\{\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\left[k\cdot\left(\phi+\delta_n\right)\right]\right\} \tag{2}$$

where I_0 and ϕ are the local mean intensity and measured phase separately, and δ_n is the temporal phase shift, and n is the frame number, and the pixel coordinate (x, y) in Eq. (2) is omitted for simplicity. Besides, the coefficients in Eq. (2) are depended on the reflection coefficients of reference flat r_1 and test surface r_2, which are defined as follows:

$$a_0 = \frac{2\left(r_1^2 + r_2^2 - 2r_1^2r_2^2\right)}{1 - r_1^2r_2^2} \tag{3}$$

$$a_k = \frac{2\left(1-r_1^2\right)\left(1-r_2^2\right)}{r_1^2r_2^2 - 1}\left(r_1r_2\right)^k, \quad k = 1, 2, \ldots. \tag{4}$$

For the conventional phase-shift algorithm such as the 7-frame phase-shift algorithm proposed by de Groot [21], the calculated phase error $o\left(a_i\right)$ in Eq. (1) from the multi-beam interferograms with $\pi/2$ phase shift could be expressed as:

$$\begin{aligned} o\left(a_i\right) = \sum_{m=1}^{\infty}\Bigg[& \frac{\left(a_{(4m-1)} - a_{(4m+1)}\right)}{a_1}\sin(4m\phi) \\ & - \frac{1}{2}\frac{\left(a_{(4m-1)} - a_{(4m+1)}\right)}{a_1}\frac{\left(a_{(4m-1)} + a_{(4m+1)}\right)}{a_1}\sin(8m\phi) + \cdots\Bigg]. \end{aligned} \tag{5}$$

The term of $\sin(8m\phi)$ and the higher order ones in Eq. (5) could be omitted due to their small coefficients. The calculated error is then approximated as the sinusoidal function with the period 4 times the modulation frequency of fringes. And substituting the coefficients of harmonics in Eqs. (3) and (4) into Eq. (5), the calculated phase error $o\left(a_i\right)$ could also be rewritten as:

$$o\left(a_i\right) = r_1^2r_2^2\left(1 - r_1^2r_2^2\right)\cos(4\phi). \tag{6}$$

Since the reflection coefficients $r_1, r_2 < 1$, the coefficient of sine term in Eq. (6) satisfies the condition:

$$r_1^2r_2^2\left(1-r_1^2r_2^2\right) < \frac{1}{2}\left[r_1^2r_2^2 + \left(1-r_1^2r_2^2\right)\right]^2 = \frac{1}{2}. \tag{7}$$

Therefore the calculated phase error of 7-frame phase-shift algorithm for multi-beam interferometry satisfies the condition $o\left(a_i\right) < 1/2$ rad. To provide a more intuitive explanation, Fig. 1 illustrates the values of the coefficient for sine term in the calculated phase error $o\left(a_i\right)$ in Eq. (6) with the test surface reflection coefficient r_2 varying from 0.2 to 0.99, while the reflection coefficient of reference flat r_1 is set from 0.2 to 0.9. Inferred from the relationship curve presented in Fig. 1, the value of the coefficient in Eq. (6) is less than 0.3. And then the values of calculated phase error satisfies the condition $o\left(a_i\right) < 0.3$ rad.

Actually in dual-wavelength Fizeau interferometer developed by ourselves with the working wavelength 632.8 nm and 532 nm, the reference flat is made by silica with the reflection coefficient about 0.2. Therefore, the values of calculated phase error satisfies the condition $o\left(a_i\right) < 0.04$ rad from Fig. 1.

2.2. demodulated error in presence of phase-shift error

When the phase shifter is not calibrated well and is non-linear for the existence of the phase-shift error, the phase shift δ_n could be expressed as a function of the unperturbed phase-shift values δ_{0n}, which is given by the superposition of different-order polynomials:

$$\delta_n = \delta_{0n}\left[1 + \varepsilon_1 + \varepsilon_2\frac{\delta_{0n}}{\pi} + \varepsilon_3\left(\frac{\delta_{0n}}{\pi}\right)^2 + \cdots + \varepsilon_p\left(\frac{\delta_{0n}}{\pi}\right)^{p-1}\right], \quad n = 1, \ldots, N \tag{8}$$