Optics Communications 405 (2017) 334–342

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Phase-step retrieval for tunable phase-shifting algorithms

Gastón A. Ayubi [a,*], Ignacio Duarte [a], César D. Perciante [b], Jorge L. Flores [c], José A. Ferrari [a]

[a] Instituto de Física, Facultad de Ingeniería, UdelaR, J. Herrera y Reissig 565, Montevideo, Uruguay

[b] Departamento de Ciencias Naturales, Facultad de Ingeniería y Tecnologías, Universidad Católica del Uruguay, Av. 8 de Octubre 2738, Montevideo, Uruguay

[c] Electronic Engineering Department, University of Guadalajara, Av. Revolución #1500, CP. 44430 Guadalajara, Jalisco, Mexico

ARTICLE INFO

Keywords:
Phase retrieval
Fringe analysis
Phase measurements
Interferometry

ABSTRACT

Phase-shifting (PS) is a well-known technique for phase retrieval in interferometry, with applications in deflectometry and 3D-profiling, which requires a series of intensity measurements with certain phase-steps. Usually the phase-steps are evenly spaced, and its knowledge is crucial for the phase retrieval. In this work we present a method to extract the phase-step between consecutive interferograms. We test the proposed technique with images corrupted by additive noise. The results were compared with other known methods. We also present experimental results showing the performance of the method when spatial filters are applied to the interferograms and the effect that they have on their relative phase-steps.

1. Introduction

Phase-shifting (PS) is a powerful technique for phase retrieval in interferometry (see e.g. [1–4] and the references therein), which presents a high accuracy and has been widely used in wavefront reconstruction, optical element testing, and refractive index measurement. Also, it has been used in deflectometry and 3D-profiling by fringe projection [5]. In general, it requires a series of intensity measurements $I_k(i,j)$ (were $k = 1, 2, \dots, K$) with known phase-shifts δ_k,

$$I_k(i,j) = a(i,j) + b(i,j)\cos\left(\phi(i,j) + \delta_k\right), \tag{1}$$

where (i,j) are spatial coordinates (pixels), and $\phi(i,j)$ is the phase introduced by the test object. The functions $b(i,j)$ and $a(i,j)$ are the modulation and the mean intensity of the interferograms, respectively. Usually the phase-shifts are evenly spaced, and thus they can be written as $\delta_k = k\alpha$, with α being a constant, so

$$I_k(i,j) = a(i,j) + b(i,j)\cos\left(\phi(i,j) + k\alpha\right). \tag{2}$$

It is possible to classify the PS algorithms into two groups, the fixed coefficient PS algorithms and the tunable PS algorithms. The first category includes algorithms with a given (constant) phase-step such as the four step Bruning algorithm which requires $\alpha = \pi/2$ [1,6]. On the other hand tunable PS algorithms are valid for a continuous range of phase-step values. Hariharan five step algorithm [1,7], Carré algorithm [8] and the Stoilov and Dragostinov [9] algorithms are examples of this kind of algorithms.

In order to apply a tunable PS algorithm it is important to know the phase-step (α), but in practice it is known only approximately, e.g. due to miscalibration of the phase shifter. [For example, PZT is the most popular phase stepper; unfortunately, its response presents hysteresis and it is sensitive to temperature and aging. Moreover, the phase-step miscalibration can be due to the electronic signal driving the phase stepper device.] Thus, the actual phase-step values could be deviated from the theoretical ones, which leads to phase reconstruction errors, as discussed in [10–12] and the references therein. Therefore, a precise knowledge of the phase-step is essential for a reliable phase retrieval.

Additionally, the intensity measurements are usually corrupted by noise, so that instead of Eq. (2) one has

$$I_k(i,j) = a(i,j) + b(i,j)\cos\left(\phi(i,j) + k\alpha\right) + n_k(i,j), \tag{3}$$

where for simplicity we are assuming that the noise ($n_k(i,j)$) is additive. Thus, the problem of retrieving the phase-step value from the intensity measurements is far away to be from a trivial problem, and this continues to be an area of active research.

In interferometry is common to assume that $n_k(i,j)$ is additive white Gaussian noise with a mean of zero and variance σ^2. This under the assumption that we are dealing with smooth-surface interferometric metrology, where the main source of noise is additive electronic noise. According with Servin et al. [13], in interferometry one deal with two main kind of noise: the first one is the additive noise, where the acquired irradiance is corrupted by additive uncorrelated noise and it arises in

* Corresponding author.
E-mail address: ayubi@fing.edu.uy (G.A. Ayubi).

http://dx.doi.org/10.1016/j.optcom.2017.08.045
Received 14 June 2017; Received in revised form 11 August 2017; Accepted 20 August 2017
0030-4018/

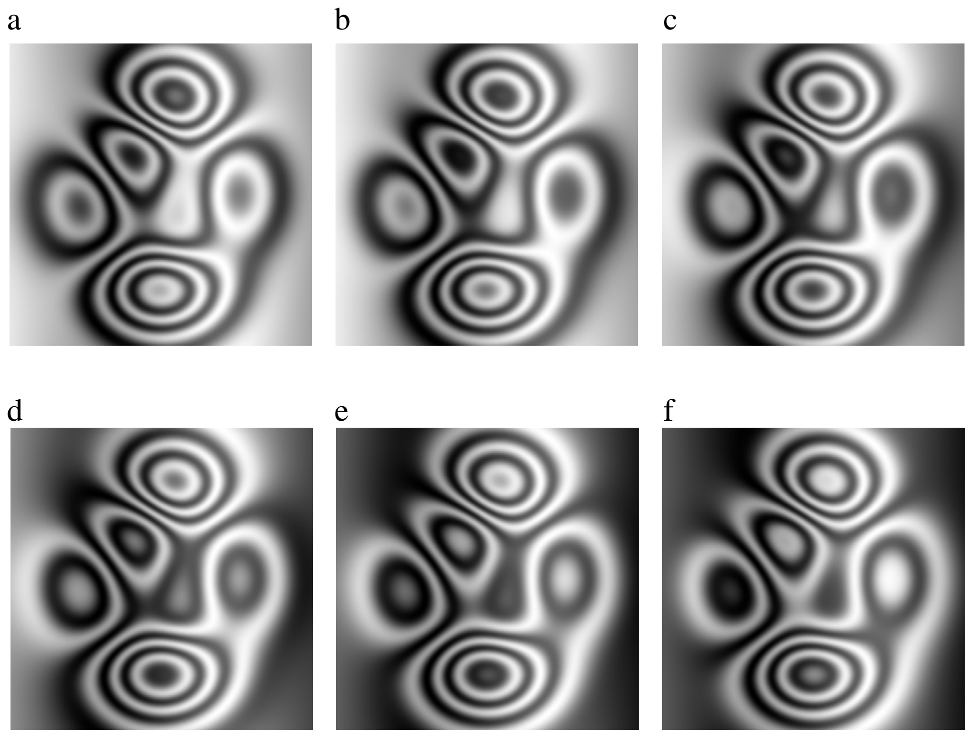

Fig. 1. Noiseless interferograms $I_k(i,j)$ with $k = 1, 2, \ldots, 6$ respectively.

optical metrology of smooth surfaces such as mirrors where most of the noise comes from ambient and the electronic equipment used. The second one is the phase (speckle) noise, which normally arises in TV speckle metrology or ESPI. Both kind of noise are present in the acquired interferograms containing a mixture of these two extreme cases. Also, according with Servin et al. [13], when dealing with smooth-surface interferometric metrology the main source of noise is additive electronic noise.

Over the last decades, several methods for phase-step retrieval in presence of noise have been proposed. In general, the phase-step retrieving methods reported in the literature can be classified into two categories: iterative and noniterative. The iteratives (see e.g. [14–16] and the references therein) are greatly time-consuming since a procedure has to be repeated many times to achieve acceptable accuracy. Also, the number of iterations depends on the guessed initial phase-steps.

Several noniterative approaches have been proposed in the literature (see e.g. [17–19]). For example, Farrell and Player [17] suggested a Lissajous elliptic fitting algorithm. Its main drawback is that the calculation of the phase-steps is easily affected by noise, modulation of the background intensity, etc.

Recently, other noniterative algorithms less sensitive to noise have been proposed. For example, Xu et al. [18] proposed a Euclidean matrix norm (EMN) algorithm to extract the unknown phase-steps from three interferograms. The method assumes that the interferograms contain more than one fringe and that certain inequality holds (see Eq. (4) of [18]). Under similar assumptions, Guo and Zhang [19] suggested a simple algorithm that estimates phase-steps from variances of fringe pattern differences. However, the accuracy in measurement of phase-steps obtained with this algorithm (even under noise free condition) depends on the number of fringes in the interferograms.

Additionally, there are many other algorithms proposed in the literature for phase-step retrieval that assume certain homogeneity (or certain previous knowledge) of the background intensity or/and modulation of the interferograms (see e.g. [20,21]).

In this work we present a new noniterative method for estimating the phase-step (α) under the assumption that the phase-shifts between interferograms are evenly spaced. Unlike most algorithms published in the literature, the proposed procedure does not assume any constraints over the mean intensity or modulation of the interferograms. The method is easy to implement and it works well even with very small signal-to-noise ratios (SNR), e.g. $SNR \approx 0.1$ or even smaller. The details of the method will be discussed in Sections 2–4. Simulations and comparison with other methods are presented in Sections 5 and 6. Section 7 shows experimental results.

2. Phase-step extraction algorithm

Defining $t_k(i,j) = \phi(i,j) + k\alpha$, from Eq. (2) it results

$$I_k(i,j) - a(i,j) = b(i,j)\cos\left(t_k(i,j)\right), \tag{4}$$

and then,

$$I_{k+1}(i,j) - a(i,j) = b(i,j)\cos\left(t_k(i,j) + \alpha\right). \tag{5}$$

Expressions (4) and (5) resemble the parametric equations of an ellipse centered at the point of coordinates $(a(i,j), a(i,j))$ with semi-axes oriented at 45°, and whose semi-axes ratio is equal to $\tan^2(\alpha/2)$,

$$x(t) - a(i,j) = b(i,j)\cos(t) \tag{6}$$

and

$$y(t) - a(i,j) = b(i,j)\cos(t + \alpha), \tag{7}$$

where $(x(t), y(t))$ are Cartesian coordinates and t is a real parameter.

From (4) and (5) it is clear that in absence of noise, for each pixel of the interferogram, each particular value $k = 1, 2, \ldots, K-1$ generates a point $(I_k(i,j), I_{k+1}(i,j))$ over the ellipse described by Eqs. (6) and (7), as shown in Fig. 2.

By performing a least-squares fitting, one can find the best ellipse fitting the experimental data (with $K > 3$), and thus, the semi-axes ratio of this ellipse provides an estimation for the phase-step (α). A discussion of least-squares techniques for fitting ellipses is presented by Fitzgibbon et al. [22].