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Improvement of image quality of holographic projection on tilted plane using iterative algorithm

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ABSTRACT

Holographic image projection on tilted plane has an important application prospect. In this paper, we propose a method to compute the phase-only hologram that can reconstruct a clear image on tilted plane. By adding a constant phase to the target image of the inclined plane, the corresponding light field distribution on the plane that is parallel to the hologram plane is derived through the titled diffraction calculation. Then the phase distribution of the hologram is obtained by the iterative algorithm with amplitude and phase constrain. Simulation and optical experiment are performed to show the effectiveness of the proposed method.

1. Introduction

Holographic image projection utilizing electronically addressed spatial light modulator (SLM) has become the current research hotspot because of its high-contrast images, low power consumption, compact optical setup, and potentials for 3D display [1–4]. Different from the traditional projection technology, which directly loads the target image on a micro display or Liquid-Crystal panel, the key task in holographic projection is to calculate the hologram. There exist a lot of algorithms to design the phase-only holograms, whereas the image quality of the reconstructed image is still too poor to make this technology commercialization. Thereafter many attempts have been proposed to improve the image quality, such as pixel separation method [5,6], random phase-free method [7,8], double-constraint iterative algorithm [9,10], over sampling iterative algorithm [11,12], and the error diffusion method [13–15].

All of these image quality enhancement methods assume that the image plane is parallel to the hologram plane. However, some special projection systems require inclined image plane. To this end, Chang et al. reported a holographic projection to tilted plane using Gerchberg–Saxton (GS) algorithm [16]. Since the phase distribution of the image on the tilted plane is not constrained during the iteration, speckle noise caused by the destructive interference between adjacent sampling points is generated, which deteriorate the image quality seriously and make the details blurred and undiscerned. Subsequently, Tomoyoshi et al. proposed a multi-random phase method that prepares multiple holograms with different random phase and then reduce the speckle noise by fast switching the hologram [17]. However, increasing the number of the superimposing holograms require more calculation time and high-refresh rate SLM.

Recently, an iterative algorithm to control both the phase and amplitude of an optical beam on the output plane is proposed [18–20]. Based on this algorithm and the tilted diffraction calculation [21], a new method to design the phase-only hologram to reconstruct a clear image on the tilted plane is proposed in this paper. The effectiveness of the method is verified with the simulations and the optical experiments.

2. Proposed method

The schematic diagram of a typical holographic image projection on a tilted plane is illustrated in Fig. 1. Three coordinate systems correspond to three planes are defined. Here, we assume that the tilted plane is slanted at θ degree around the y_p axis and shares the same origin with the parallel plane. The basic problem in the calculation of a phase-only hologram can be described as follows: a pattern is initially given on the tilted plane and then optimize the phase distribution of the hologram so that its diffraction pattern infinitely approximates the target intensity distribution. The design method proposed in this paper can be divided into two steps. The first step is the diffraction calculation from the tilted plane to the parallel plane. The second step is the iterative optimization between the hologram plane and the parallel plane.

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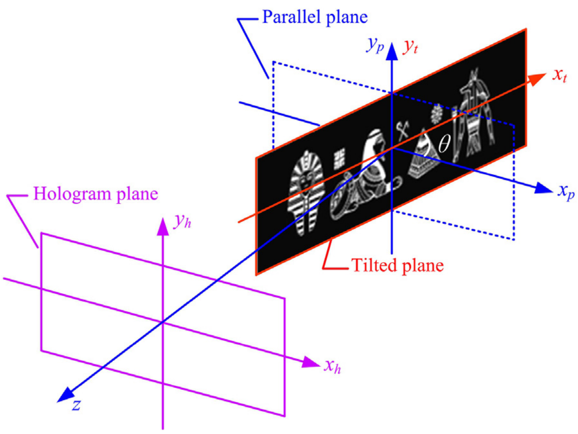

Fig. 1. Schematic of holographic projection on tilted plane.

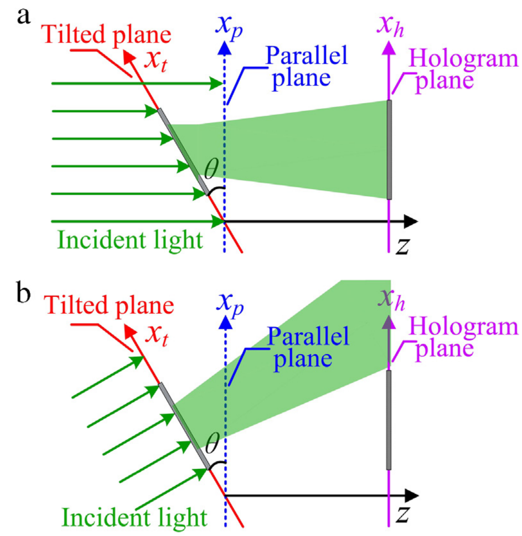

Fig. 2. Influence of incident light on the emitted light field (a) tilted illumination (b) perpendicular illumination.

2.1. Diffraction calculation

Target image $I(x_t, y_t)$ is initially given on the tilted plane. For holographic image projection, we only focus on the intensity distribution. So the phase as a design freedom can be arbitrarily specified. In the traditional hologram design, target image is usually superimposed with a random phase. However, the random phase will cause serious speckle noise in the reconstructed image. Here, we add a uniform constant phase to the target image to eliminate the speckle noise came from the destructive interference between the sampling points with random and erratic phase. Consequently, the light field distribution in the tilted plane can be expressed as:

$$t\left(x_t, y_t\right) = \sqrt{I\left(x_t, y_t\right)}. \tag{1}$$

To calculate the light field on the parallel plane, we first need to calculate the spectrum of the light field emitted from the titled plane. In many tilted diffraction calculations, the tilted plane is directly regarded as the light-emitting surface source, and the two-dimensional Fourier transform of Eq. (1) is considered as the spectrum of the outgoing light field. This is equivalent to the case of illuminating the tilted plane with plane wave perpendicularly, as shown in Fig. 2(b). however, in this case, especially when the tilted angle θ is large, many of the low-frequency components of the light field cannot reach the hologram plane. And the image usually contains a lot of low-frequency components. On the contrary, when the tilted plane is illuminated by the plane wave along the optical axis, most of the low-frequency components can reach the hologram, as shown in Fig. 2(a). Then the spectrum of the light field emitted from the tilted plane can be written as:

$$T\left(fx_t, fy_t\right) = \mathcal{F}\left\{t\left(x_t, y_t\right) \times \exp\left[-ikx_t \sin(\theta)\right]\right\}, \tag{2}$$

where F denotes the two-dimensional Fourier transform operation, $k = 2\pi/\lambda$ is the wave number in the free space, fx_t and fy_t are the spatial frequency coordinates in the x_t and y_t direction, respectively.

After obtaining the Fourier spectrum, the amplitude and phase of each spatial frequency component can be known. After that, these frequency components propagate to the parallel plane and participate in imaging. Due to the rotation of the coordinate system, spatial frequency in the tilted and parallel plane can be mutually transformed by the rotation matrix M as follows:

$$\begin{bmatrix} fx_t \\ fy_t \\ fz_t \end{bmatrix} = M \begin{bmatrix} fx_p \\ fy_p \\ fz_p \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} fx_p \\ fy_p \\ fz_p \end{bmatrix}. \tag{3}$$

By simply solving Eq. (3), we can get the relation:

$$\begin{aligned} fx_t &= \alpha\left(fx_p, fy_p\right) = fx_p \cos(\theta) - fz_p \sin(\theta) \\ fy_t &= \beta\left(fx_p, fy_p\right) = fy_p. \end{aligned} \tag{4}$$

Then the Fourier spectrum in the parallel plane can be written as:

$$P\left(fx_p, fy_p\right) = T\left(\alpha\left(fx_p, fy_p\right), \beta\left(fx_p, fy_p\right)\right) J\left|\left(fx_p, fy_p\right)\right|, \tag{5}$$

where J is the Jacobian determinant and is defined as follows:

$$J\left(fx_p, fy_p\right) = \frac{\partial\alpha}{\partial fx_p}\frac{\partial\beta}{\partial fy_p} - \frac{\partial\alpha}{\partial fy_p}\frac{\partial\beta}{\partial fx_p} = \frac{fx_p}{fz_p}\sin(\theta) + \cos(\theta). \tag{6}$$

Finally, complex amplitude distribution on the parallel plane can be obtained by an inverse two-dimensional Fourier transformation as follow:

$$U_p\left(x_p, y_p\right) = \mathcal{F}^{-1}\left\{P\left(fx_p, fy_p\right)\right\}. \tag{7}$$

2.2. Iterative optimization

After calculating the light field on the parallel plane, the iterative optimization algorithm is adopted to design the phase distribution of the hologram. The flow chart of the algorithm is shown in Fig. 3. Firstly, random phase φ, which acts as the initial distribution of the hologram, is generated. Subsequently, this light field is propagated to the parallel plane with Fresnel diffraction. If the deviation between the obtained complex amplitude $P(x_p, y_p)$ and the desired one is small enough in the signal area, output φ as the phase distribution of the hologram. Otherwise, take the complex amplitude constraint on the parallel plane as follows:

$$P'\left(x_p, y_p\right) = \begin{cases} cU_p, & x_1, y_1 \in S \\ P\left(x_p, y_p\right), & x_1, y_1 \in N \end{cases} \tag{8}$$

where S denotes the signal area, N denotes the noise area. The definition of signal window and noise window can be seen in Fig. 3, which represents a set of sampling points in the plane. Scale factor c is a constant and utilized to balance the diffraction efficiency and root mean square error. Secondly, propagate this modified optical field to the hologram plane with inverse Fresnel diffraction. Then the amplitude distribution on hologram plane is replaced by unity and the phase is kept.

After performing of a number of quantities of iterations, obtained phase distribution on the hologram plane is the synthesized phase-only hologram. To evaluate the convergence of the iteration, we adopt root mean square error (RMSE) as the criteria, which are defined as follows:

$$RMSE = \sqrt{\sum\left(u - u_d\right)^2 / \sum\left(u_d\right)^2}, \tag{9}$$