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Electromagnetic fields of an ultra-short tightly-focused radially-polarized laser pulse

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ABSTRACT

Fully analytic expressions, for the electric and magnetic fields of an ultrashort and tightly focused laser pulse of the radially polarized category, are presented to lowest order of approximation. The fields are derived from scalar and vector potentials, along the lines of our earlier work for a similar pulse of the linearly polarized variety. A systematic program is also described from which the fields may be obtained to any desired accuracy, analytically or numerically.

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1. Introduction

Present-day high-power laser systems, and ones that are currently envisaged for the near future [1–3], are mostly of the pulsed type. For many applications [4–11], including laser acceleration of electrons, ions and bare nuclei, and laser interactions with bulk matter, the need for super-intense pulses which contain merely a few laser cycles, continues to grow. Hence, a proper theoretical representation for the electric and magnetic fields of a pulse of this type is very much in demand and continues to motivate work, and stimulate efforts, in the field [12–37].

A radially-polarized laser beam or pulse has two electric field components, one radial, which helps to confine a beam of particles to a region close to the propagation axis, and a second that is axial, which comes in handy for particle acceleration [38–41]. This work is also motivated by recent advances in the technology of radially polarized laser systems [42,43]. For modeling ultra-short ultra-strong laser pulses, the paraxial solutions of Maxwell's equations are no longer adequate [12–14]. The need for more accurate solutions to be employed [15,30] seems timely now. This paper focuses on a pulse propagating in vacuum, while some of the cited references treat cases involving media, such as an under-dense plasma [15] or a dielectric interface [27].

This paper contributes to those continuing efforts and builds upon former investigations [35–37]. Analytic expressions for the most dominant terms in the description of the electric and magnetic fields of an ultra-short and tightly-focused laser pulse of the radially-polarized category, will be derived. For our purposes in this work, ultra-short will mean of axial length, L, small compared to a Rayleigh length z_r, and by tightly-focused will be meant of waist radius at focus, $w_0 \leq \lambda_0$, where λ_0 is a central wavelength.

This paper is organized as follows. First, key points of the background material are briefly reviewed in Section 2. Based on that, analytic expressions for the zeroth-order fields, in some truncated series, assumed to be the most dominant ones, will be derived in Section 3, following the work of Esarey et al. [15] and our own earlier investigations [35,37]. Higher-order corrections will only be used in numerical simulations, as they turn out to be quite cumbersome. To distinguish the present paper from [35], which employs a uniform distribution of wavenumbers, two different initial pulse frequency-distributions will be employed here, one Gaussian and the other Poissonian. Two sets of zero-order fields, obtained from the two distributions, will be derived and compared analytically, as well as numerically. A summary and our main conclusions will be given in Section 4.

2. Background

The fields will be derived from a vector potential $\boldsymbol{A}$ which satisfies the wave equation

$$\nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = 0, \tag{1}$$

and a similar equation for the scalar potential Φ in vacuum, where c is the speed of light. The vector and scalar potentials will be assumed to be linked via a Lorentz condition. The Following is a brief outline of the steps leading from Eq. (1) to an expression for $\boldsymbol{A}$, and the associated

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scalar potential, from which the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields of a radially polarized laser pulse will ultimately be obtained [35]. The starting point is a change of variables, to the set (ρ, η, ζ), where $\rho = r/w_0$, $r = \sqrt{x^2+y^2}$, w_0 is the initial waist radius at focus, $\zeta = z - ct$, and $\eta = (z+ct)/2$. In terms of the new variables, and for propagation along the z-axis, the ansatz

$$A = A_0 a(\rho, \eta, \zeta) e^{ik_0\zeta}, \tag{2}$$

for the single-component vector potential is then introduced, in which A_0 is a constant complex amplitude, and $k_0 = 2\pi/\lambda_0$ is a central wavenumber corresponding to the central wavelength λ_0. The coordinate transformations turn (1) into an equation for the amplitude $a(\rho, \eta, \zeta)$, which may then be synthesized from Fourier components a_k, according to

$$a(\rho, \eta, \zeta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} a_k(\rho, \eta, k) e^{ik\zeta} dk. \tag{3}$$

It can be easily shown that each Fourier component satisfies an equation of the form

$$\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial}{\partial \rho} + 4iz_{rk}\frac{\partial}{\partial \eta}\right] a_k = 0; \quad z_{rk} = (k+k_0)\frac{w_0^2}{2}. \tag{4}$$

Eq. (4) admits an exact analytical solution, which may be written as

$$a_k(\rho, \eta, k) = f_k \psi_k, \tag{5}$$

where

$$\psi_k = \beta_k e^{-\beta_k \rho^2}; \quad \beta_k = \frac{1}{1+i\alpha_k}; \quad \alpha_k = \frac{\eta}{z_{rk}}. \tag{6}$$

As has been pointed out elsewhere [15,35,36], f_k is a function that must be adopted to appropriately represent the wavenumber distribution in k-space of the initial pulse, and whose Fourier transform will be related below to the spatio-temporal pulse envelope. Unfortunately, the Fourier transform of a_k according to Eq. (3) cannot be evaluated analytically, in general. Resort to approximation is, therefore, inevitable. Viewing ψ_k as a function of $k' \equiv k + k_0$, the following series expansion may be the natural approach to follow, namely

$$\begin{aligned}\psi_k &= \sum_{m=0}^{\infty} \frac{(k'-k_0)^m}{m!} \left.\frac{\partial^m \psi_k}{\partial k'^m}\right|_{k'=k_0}, \\ &= \sum_{m=0}^{\infty} \frac{k^m}{m!}\psi_0^{(m)}; \quad \psi_0^{(m)} \equiv \left.\frac{\partial^m \psi_k}{\partial k^m}\right|_{k=0}.\end{aligned} \tag{7}$$

With the help of this expansion, the vector potential amplitude may be written as

$$a(\rho, \eta, \zeta) = \sum_{m=0}^{\infty} \frac{\psi_0^{(m)}}{m!} F_m(\zeta), \tag{8}$$

where

$$F_m(\zeta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(f_k k^m\right) e^{ik\zeta} dk, \tag{9}$$

is Fourier transform of the product $f_k \psi_k$. In anticipation of the conclusion, to be arrived at shortly, that terms in the sum (8) beyond the first will contribute negligibly in real applications, Eq. (8) will be replaced by the truncated series

$$a^{(n)}(\rho, \eta, \zeta) \approx \sum_{m=0}^{n} \frac{\psi_0^{(m)}}{m!} F_m(\zeta). \tag{10}$$

At this stage, the complex amplitude will be written as $A_0 = a_0 e^{i\varphi_0}$, in which φ_0 is an initial phase and a_0 is a real amplitude for the exact vector potential. The status of a_0 will be slightly modified in the approximate solutions to be presented below, by introducing model-dependent normalization factors, defined appropriately at each level of truncation. With this, the truncated vector potential (to order of truncation n) takes the form

$$A^{(n)}(\rho, \eta, \zeta) \approx a_0 e^{i\varphi_0 + ik_0\zeta} \sum_{m=0}^{n} \frac{\psi_0^{(m)}}{m!} F_m(\zeta). \tag{11}$$

Eq. (11) can be employed to obtain the electric and magnetic fields, to any desired order of truncation. However, on account of the conclusion to be made shortly for some initial frequency spectra, the zeroth-order terms may be the most dominant ones and only the lowest-order corrections may be necessary. For book-keeping purposes, explicit expressions for $\psi_0^{(m)}$, with $m = 0-3$, are [35,36]

$$\psi_0^{(0)} = \beta e^{-\beta\rho^2}, \tag{12}$$

$$\psi_0^{(1)} = \frac{i\alpha}{k_0}(1-\beta\rho^2)\beta^2 e^{-\beta\rho^2}, \tag{13}$$

$$\psi_0^{(2)} = \frac{i\alpha}{k_0^2}\left[-2 + (4\beta - 2)\rho^2 + i\alpha\beta^2\rho^4\right]\beta^3 e^{-\beta\rho^2}, \tag{14}$$

$$\psi_0^{(3)} = \frac{i\alpha}{k_0^3}\left[6 + 6(2-3\beta)\rho^2 + 3i\alpha\beta(1-3\beta)\rho^4 + \alpha^2\beta^3\rho^6\right]\beta^4 e^{-\beta\rho^2}. \tag{15}$$

In Eqs. (12)–(15)

$$\beta = \frac{1}{1+i\alpha}; \quad \alpha = \frac{\eta}{z_r}; \quad z_r \equiv z_{r0} = \frac{1}{2}k_0 w_0^2, \tag{16}$$

where z_r is the Rayleigh length. The zeroth-order electric and magnetic fields of an ultrashort and tightly focused laser pulse will be derived from the above equations, fully analytically. The first-order, and possibly higher-order corrections, can in principle be used in numerical calculations.

3. The fields

The radially polarized $\boldsymbol{E}$ and $\boldsymbol{B}$ fields will be obtained below from the one-component axially polarized vector potential

$$\boldsymbol{A} = \hat{z}A; \quad A = a_0 a e^{i\varphi_0 + ik_0\zeta}, \tag{17}$$

where $\hat{z}$ is a unit vector in the direction of propagation of the pulse, taken along the z-axis of a cylindrical coordinate system, together with the associated scalar potential [35,36,44,45]. Employing SI units, the radial and axial electric field components may be obtained, respectively, from [35]

$$E_r = -\frac{c^2}{R}\frac{\partial}{\partial r}\left(\frac{\partial A}{\partial z}\right) - \frac{c^2}{R^2}\left(\frac{\partial A}{\partial z}\right)\frac{\partial}{\partial r}\left(\frac{1}{a}\frac{\partial a}{\partial t}\right), \tag{18}$$

$$E_z = -\frac{\partial A}{\partial t} - \frac{c^2}{R}\frac{\partial^2 A}{\partial z^2} - \frac{c^2}{R^2}\left(\frac{\partial A}{\partial z}\right)\frac{\partial}{\partial z}\left(\frac{1}{a}\frac{\partial a}{\partial t}\right), \tag{19}$$

in which

$$R = ick_0 - \frac{1}{a}\frac{\partial a}{\partial t}. \tag{20}$$

Furthermore, the only (azimuthal) magnetic field component will follow from

$$B_\theta = -\frac{\partial A}{\partial r}. \tag{21}$$

Derivation of analytic expressions for the electric and magnetic fields from the appropriate vector potential will be done below. The starting point along this path is a choice that must be made for an appropriately defined initial pulse spectrum. In the next two subsections, two such choices will be made.

3.1. An initial Gaussian spectrum

An initial Gaussian spectrum is considered first, following Esarey et al. [15] and our own earlier work [37]. Employing the same notation as in [15,37] the initial wavenumber distribution will be given by

$$f_k = \frac{\sigma}{k_0}\left(1 + \frac{k}{k_0}\right)\exp\left(-\frac{k^2\sigma^2}{2k_0^2}\right), \tag{22}$$

in which σ is the pulse's initial full-width-at-half-maximum, given in terms of its spatial extension in the forward direction, $L \sim c\tau_0$, where τ_0 is the temporal pulse duration, by $\sigma = k_0 L/(2\sqrt{2\ln 2})$.

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