Optics Communications 405 (2017) 29–34

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Quantum behaviour of pumped and damped triangular Bose–Hubbard systems

C.V. Chianca, M.K. Olsen *

School of Mathematics and Physics, University of Queensland, Brisbane, QLD 4072, Australia

ARTICLE INFO

Keywords:
Bose–Hubbard model
Pumped and damped
Squeezing
Entanglement

ABSTRACT

We propose and analyse analogs of optical cavities for atoms using three-well Bose–Hubbard models with pumping and losses. We consider triangular configurations. With one well pumped and one damped, we find that both the mean-field dynamics and the quantum statistics show a quantitative dependence on the choice of damped well. The systems we analyse remain far from equilibrium, preserving good coherence between the wells in the steady-state. We find quadrature squeezing and mode entanglement for some parameter regimes and demonstrate that the trimer with pumping and damping at the same well is the stronger option for producing non-classical states. Due to recent experimental advances, it should be possible to demonstrate the effects we investigate and predict.

1. Introduction

In this article we analyse pumped and damped three-well triangular Bose–Hubbard models [1–6] in terms of their mean field behaviour and their quantum statistical properties. Our proposal in this work is inspired by recent advances in the techniques of configuring optical potentials [7,8], which allow for the fabrication of a variety of geometric configurations. Together with the ability to cause controlled loss from particular lattice sites, utilising either electron beams [9], or optical methods [10], this allows for the manufacture and study of innovative lattice configurations. In some ways the pumped and damped systems we investigate here are similar to coupled nonlinear optical cavities [11,12], but there is one very important difference. While not every optical cavity in a cluster need be pumped, it is impossible to construct an optical resonator without losses. As we will demonstrate in what follows, the ability to choose which well is to be damped leads to some very interesting behaviour in both the mean fields and the quantum correlations.

Driven dissipative dimers with damping at both wells have been analysed by Casteels and Wouters [13], in terms of both entanglement and bistability. Casteels and Ciuti [14] have examined phase transitions in the same system. Triangular dimers and inline chains with dissipation at one well have also been analysed [15,16], finding some interesting physical effects. Pižorn has analysed Bose–Hubbard models with pumping and dissipation [17], using density matrix techniques, which are useful for moderate numbers of atoms and wells. In this work we examine Bose–Hubbard trimers in the triangular configuration, with pumping and dissipation at one well each. A triangular lattice with damping at the middle well, but without pumping, has been analysed by Shchesnovich and Mogilevstev, showing that the mean-field analysis is not accurate [18]. Since we do not wish to limit ourselves to the smallish number of atoms used in other treatments, we use the truncated Wigner representation [19,20], as in previous works examining different configurations [21,22]. Unlike some of the other methods, the truncated Wigner is easily extensible to higher well and atom numbers, with a system of 11 wells and 2000 atoms having been analysed recently [23].

2. Hamiltonian and equations of motion

In a three-well triangle with pumping and damping at one well each, there are two possible geometric configurations. Labelling the pumped well as number one, we can choose to damp either well one or well two. By symmetry, damping at well three is the same configuration as that with damping at well two. We begin with the three-well triangular Bose–Hubbard unitary Hamiltonian,

$$\mathcal{H} = \hbar\chi \sum_{i=1}^{3} \hat{a}_i^{\dagger\,2}\hat{a}_i^2 - \hbar J\left[\hat{a}_1^\dagger(\hat{a}_2+\hat{a}_3) + \hat{a}_2^\dagger\hat{a}_3 + h.c.\right], \quad (1)$$

where $\hat{a}_i$ is the bosonic annihilation operator for the ith well, χ represents the collisional nonlinearity and J is the tunnelling strength.

* Corresponding author.
E-mail address: mko@physics.uq.edu.au (M.K. Olsen).

http://dx.doi.org/10.1016/j.optcom.2017.07.075
Received 5 July 2017; Received in revised form 26 July 2017; Accepted 27 July 2017
Available online 12 August 2017
0030-4018/

We will always consider that the pumping from the larger condensate is at well 1 and can be represented by the Hamiltonian

$$\mathcal{H}_{pump} = i\hbar\left(\epsilon\hat{a}_1^\dagger - \epsilon^*\hat{a}_1\right), \tag{2}$$

which is of the form commonly used for the investigation of optical cavities. The basic assumption here is that the first well receives atoms from a coherent condensate, represented by the complex amplitude ϵ, which is much larger than any of the modes in the wells we are investigating, so that it will not become depleted over the time scales of interest. The damping term for well i acts on the system density matrix as the Lindblad superoperator

$$\mathcal{L}\rho = \gamma\left(2\hat{a}_i\rho\hat{a}_i^\dagger - \hat{a}_i^\dagger\hat{a}_i\rho - \rho\hat{a}_i^\dagger\hat{a}_i\right), \tag{3}$$

where γ is the coupling between the damped well and the atomic bath, which we assume to be unpopulated. If the lost atoms fall under gravity, we are justified in using the Markov and Born approximations [24].

Following the usual procedures [25,26], we may map the master equation for the density operator onto a generalised Fokker–Planck equation in the Wigner representation. This is not a true Fokker–Planck equation because it has third-order derivatives and, although it can be mapped onto stochastic difference equations [27], these are not easy to integrate. By dropping the third-order terms, usually under the assumption that they are small, we may map the problem onto Itô stochastic equations [28] in the truncated Wigner representation. As a representative example, the equations for pumping at well 1 and loss at well 2 are

$$\begin{aligned}\frac{d\alpha_1}{dt} &= \epsilon - 2i\chi|\alpha_1|^2\alpha_1 + iJ(\alpha_2+\alpha_3),\\ \frac{d\alpha_2}{dt} &= -\gamma\alpha_2 - 2i\chi|\alpha_2|^2\alpha_2 + iJ(\alpha_1+\alpha_3) + \sqrt{\gamma}\eta,\\ \frac{d\alpha_3}{dt} &= -2i\chi|\alpha_3|^2\alpha_3 + iJ(\alpha_1+\alpha_2),\end{aligned} \tag{4}$$

where ϵ represents the rate at which atoms enter well 1, γ is the loss rate from the selected well, and η is a complex Gaussian noise with the moments $\overline{\eta(t)} = 0$ and $\overline{\eta^*(t)\eta(t')} = \delta(t-t')$. The variables α_i correspond to the operators $\hat{a}_i$ in the sense that averages of products of the Wigner variables over many stochastic trajectories become equivalent to symmetrically ordered operator expectation values, for example $\overline{|\alpha_i|^2} = \frac{1}{2}\langle\hat{a}_i^\dagger\hat{a}_i + \hat{a}_i\hat{a}_i^\dagger\rangle$. The initial states in all wells will be vacuum. We note here that we will use $\epsilon = 10$ and $\gamma = J = 1$ in all our numerical investigations, while using two values of χ, 10^{-3} and 10^{-2}. The equations for configurations with damping at a different well are found by the simple transfer of the terms involving γ.

The parameters used here are consistent with known experimental values. Fixing the tunnelling rate at $J = 1$ sets the scale for all the other parameters. Physically, the pumping rate and the loss rate can be varied by adjusting well geometries and the strength of the method used for outcoupling. J itself can be changed by changes in the well depths and separation. The most difficult parameter to change experimentally would be χ, which is possible using Feshbach resonance techniques [29]. Using the published results of Albiez et al. [30] and setting their tunnelling equal to one, we find that their $\chi \approx 10^{-4}$ in our units. While this is smaller than what we have used, deeper wells would lower J and give a ratio χ/J consistent with our two values, or χ could be changed using Feshbach techniques. By reference to the same article, we can also say that our system is in the regime where the three-mode approximation is valid.

We note here that the truncated Wigner has been chosen as an alternative to the exact positive-P representation [31] because this was found to be unstable for the trimer configuration with loss at only one well. For a similar configuration with loss at the two unpumped wells [32], the positive-P representation was used, and gave indistinguishable results to those found with the truncated Wigner for the same system. For a dimer system with loss at one well, the truncated Wigner reproduced all the positive-P results for first and second order moments [33], while being qualitatively correct for higher moments. Since we are not interested in the investigation of any moments higher than second order in this work, we feel justified in using this method.

3. Quantities of interest

In this work we are interested in the number of atoms in each mode and the correlation functions which are used to detect squeezing in each mode and entanglement between the modes. The populations in each well are calculated as $N_i = \overline{|\alpha_i|^2} - \frac{1}{2}$ while the correlations we use to detect quantum statistical properties are constructed from expectation values of moments of the mode operators. In order to proceed, we define the atomic quadratures as

$$\hat{X}_j(\theta) = \hat{a}_j e^{-i\theta} + \hat{a}_j^\dagger e^{i\theta}, \tag{5}$$

so that the $\hat{Y}_j(\theta) = \hat{X}_j(\theta+\pi/2)$. Single mode squeezing exists whenever a particular quadrature variance is found to be less than 1, for any angle. As is well known, one of the effects of a $\chi^{(3)}$ nonlinearity can be to cause maximum squeezing to be found at a non-zero quadrature angle [11], so that it becomes important to investigate all angles. This is not the case for resonant $\chi^{(2)}$ systems such as second harmonic generation, where the best squeezing is found for $\theta = 0$.

In order to detect bipartite entanglement and inseparability, we will use the Duan–Simon inequality [34,35] which states that, for any two separable states,

$$V(\hat{X}_j + \hat{X}_k) + V(\hat{Y}_j - \hat{Y}_k) \geq 4, \tag{6}$$

where the variance of any quantity is defined as $V(A) = \langle A^2\rangle - \langle A\rangle^2$. We define the correlation function

$$DS_{jk} = V(\hat{X}_j + \hat{X}_k) + V(\hat{Y}_j - \hat{Y}_k), \tag{7}$$

for which a value of less than 4 means that modes j and k are inseparable. Note that the angular dependence has been suppressed here for clarity of notation.

The next quantum statistical effect is the Einstein–Podolsky–Rosen (EPR) paradox [36], also known as EPR-steering [37,38]. For a continuous variable pumped and damped system, the usual method for demonstrating the presence of steering is with the Reid criterion [39]. This is based on the fact that the Heisenberg Uncertainty Principle requires that

$$V(\hat{X}_i)V(\hat{Y}_i) \geq 1.. \tag{8}$$

Reid defines the inferred quadrature variances of two modes labelled i and j, with an observer of mode j inferring values of mode i, as

$$\begin{aligned}V_{inf}(\hat{X}_{ij}) &= V(\hat{X}_i) - \frac{\left[V(\hat{X}_i,\hat{X}_j)\right]^2}{V(\hat{X}_j)},\\ V_{inf}(\hat{Y}_{ij}) &= V(\hat{Y}_i) - \frac{\left[V(\hat{Y}_i,\hat{Y}_j)\right]^2}{V(\hat{Y}_j)},\end{aligned} \tag{9}$$

where the θ dependence is again suppressed, and $V(AB) = \langle AB\rangle - \langle A\rangle\langle B\rangle$. When the product of these two inferred variances is less than one, this means that mode i can be steered by mode j. As shown by Reid, a violation of the inequality signifies a two-mode state which demonstrates the EPR paradox. It can be seen that this criterion is directional, with the ability to swap i and j in Eq. (9). In what follows, we will denote the value of the product of the inferred variances as EPR_{ij} when the quadrature variances of mode i are inferred by measurements at mode j. When one of the pair (EPR_{ij}, EPR_{ji}) is less than one and the other is more than one, we have a phenomenon known as asymmetric Gaussian steering. This property has been predicted in optical [40–42] and atomic systems [43], and measured in the laboratory [44]. It is now established that it is a general property, and may also exist for non-Gaussian measurements [45]. Since states which are steerable