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Fast diffraction calculation of cylindrical computer generated hologram based on outside-in propagation model

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ABSTRACT

Cylindrical computer-generated hologram is a promising approach to realize a display with 360°field of view. However, conventional cylindrical hologram employs an inside-out propagation model and suffers from two main drawbacks: limited object size and lack of effective reconstructed method. Previously, we proposed to fix these problems using an outside-in propagation model with reversed propagation direction of the inside-out model. We also derived corresponding diffraction calculation formula for the outside-in propagation model. In this work, we investigate a non-constant obliquity factor in the outside-in propagation model, and show that it is the projection of the unit complex amplitude in the propagation direction onto the outer normal of the observation point. We then propose to apply fast Fourier transform to accelerate the convolution operation needed for diffraction calculation. We conducted experiments on inverse diffraction and reconstruction of the cylindrical objects. Very encouraging results demonstrate the validity of this proposed approach.

1. Introduction

Holography is a promising three-dimensional (3D) display technique because it can record and reproduce wholistic 3D information such as a motion parallax, a convergence, an occlusion, an accommodation, and so on. Usually holograms are made on flat surfaces which limit the viewing angles. This limitation can be alleviated if holograms are made on cylindrical surfaces [1–3]. Beside a 3D display, a cylindrical geometry is also preferred for 3D volumetric imaging, such as computed tomography, magnetic resonance images, and integral-floating display [4]. A cylindrical computer generated hologram (CGH) can provide 360° horizontal field of view (FoV) as opposed to the limited FoV offered by the conventional planar CGH. Furthermore, the amount of data needed in a curved hologram is smaller than that in a planar hologram for the same range of FoV [5]. With the developments of curved display and recording devices, a cylindrical CGH with a "look around property" has attracted much attention recently.

Sakamoto et al. [6] proposed a fast calculation method for the cylindrical hologram of a planar object using the angular spectrum of plane wave and the fast Fourier transform (FFT) algorithm which exploits the rotational shift invariance property between a planar surface and a cylindrical surface. Continue on this work, Kashiwagi and Sakamoto generated a cylindrical hologram of 3D object by slicing it into planar segments [7]. Sando et al. [8] demonstrated the propagation from cylindrical surface as cylindrical waves by defining wave propagation in cylindrical coordinates. By choosing the object also to be a concentric cylindrical surface with the hologram, the shift invariance was preserved, and the FFT algorithm is applicable. Jackin et al. [9] define wave propagation in the spectral domain in cylindrical coordinates, hence one fewer FFT operation is needed. By applying this approach, they reported successful reconstruction of a simple 3D object [10]. For a complex 3D object, an algorithm of wavefront recording surface for fast calculation of cylindrical CGH was proposed by Kim et al. [11]. Different from the previous approaches, Yamaguchi et al. [12] generated the cylindrical holograms by segmenting the cylindrical surface into elemental plane surfaces. They also developed a computer generated cylindrical rainbow hologram using the same method [13]. Above mentioned methods consider wave propagation from the inner object surface to the outer cylindrical observation surface using an inside-out propagation (IOP) model. Due to the limited observation or hologram surface area, the size of object may be limited. Moreover, the IOP model also needs to calculate the diffraction distribution, which is propagated from outside to inside cylindrical surface, during the reconstruction process.

Previously, by placing the object surface on the outer cylinder and observation surface on the inner cylinder, we proposed an outside-in propagation (OIP) model [14] with opposite propagation direction

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of IOP model to fix the above problems. We derived corresponding diffraction calculation formula for the OIP model, yet the direct calculation is extremely time-consuming. In this paper, a fast method based on the convolution theorem using FFT algorithm, which is hereinafter referred as *convolution method*, is proposed to accelerate the diffraction calculation of OIP model. Compared against an approximation constant obliquity factor for IOP model, a non-constant obliquity factor is derived for the calculation of OIP model, and its physical significance is analyzed. In addition, the conditions of Nyquist-sampling theorem for the proposed convolution method are discussed. The experimental results have verified the physical significance of the obliquity factor as well as the correctness and accuracy of the proposed calculation for OIP model. The experiments of inverse diffraction as well as generating a cylindrical hologram and reconstructing the cylindrical objects have been carried out successfully. Further, the realization of the cylindrical CGH is discussed.

2. Theory of cylindrical CGH

Since the observation surface is a cylinder in cylindrical CGH, we can construct an abstract model of object and observation in cylindrical coordinate. To simplify the calculation, we make a hypothesis that the shape of the object surface is also cylindrical, which is the same as the observation surface, and the pair of the surfaces are concentric. The schematic of the geometrical relation between them is shown in Fig. 1, where R and r denote the radii of the outer and inner cylinders, respectively. The point O is origin of rectangular and cylindrical coordinates, where P and Q denote any points on the outer and inner cylinders, respectively. The d_{QP} and d_{PQ} represent the distances between any two points P and Q, respectively. The θ_R and θ_r denote the angle variables of the points P and Q, respectively. The point O' is the projection of points P and Q onto the Z axis shown in Figs. 1(a) and (b), respectively. The P' is the projection of the point P onto the outer normal of the point Q. Obviously, the abstract models have inside-out and outside-in two directions of propagation, which is so-called (IOP) and (OIP) models, as shown in Figs. 1(a) and (b), respectively.

2.1. Conventional calculation model — IOP Model

The distributions of diffraction (Hereafter referred to as distribution) on the object and the observation surfaces are represented by $u_r(\theta_r, z_r)$ and $u_R(\theta_R, z_R)$, respectively, in the case that observation surface is outside in IOP model. Based on the assumption that the object is the aggregation of many point light sources, the distribution $u_R(\theta_R, z_R)$ is the summation of the spherical wavefront that emerge from the point light sources according to the Huygens–Fresnel principle [8], and it is given by

$$u_R\left(\theta_R, z_R\right) = C \iint_{\Sigma} u_r\left(\theta_r, z_r\right) \frac{\exp\left(jkd_{QP}\right)}{d_{QP}}\, d\theta_r dz_r, \tag{1}$$

$$d_{QP} = \left[R^2 + r^2 - 2Rr\cos\left(\theta_R - \theta_r\right) + \left(z_R - z_r\right)^2\right]^{1/2}, \tag{2}$$

where k, Σ, and C denote the wavenumber of the incident light, the object surface, and a constant, respectively.

2.2. Previously proposed calculation model — OIP Model

To solve the issues of limited object size and lack of effective reconstructed method in the conventional cylindrical CGH, we consider the OIP model in our previous research [14]. For the case that observation surface is inside in OIP model, on which we concern, the distributions on the object and observation surfaces are represented by $u_R(\theta_R, z_R) f_R(\theta, \eta)$ and $g_r(\varphi, z)\, u_r(\theta_r, z_r)$, respectively, as shown in Fig. 1(b). Applying to the Huygens–Fresnel principle, the distribution $u_r(\theta_r, z_r) g_r(\varphi, z)$ can be given by

$$u_r\left(\theta_r, z_r\right) = C \iint_{\Sigma} u_R\left(\theta_R, z_R\right) \frac{\exp\left(jkd_{PQ}\right)}{d_{PQ}} \cos\alpha\ d\theta_R dz_R, \tag{3}$$

$$d_{PQ} = \left[R^2 + r^2 - 2Rr\cos\left(\theta_r - \theta_R\right) + \left(z_r - z_R\right)^2\right]^{1/2} = d_{QP} \tag{4}$$

where Σ denotes the object surface. Here, the *obliquity angle* is defined as the angle between the outer normal of the point Q on the diffraction surface and the vector of the point P on the object surface point to the point Q on the diffraction surface, which is the angle α between vectors **O'Q** and **PQ** in Fig. 1(b). The cos a means the *obliquity factor* of the first solution of Rayleigh Sommerfeld under the condition that the distance between two points, d_{PQ}, is far greater than the wavelength. And it is proportional to the projection vector **P'Q** that the vector **PQ** projected onto the outer normal of the point Q on the diffraction surface. Therefore, it is easy to achieve the expression of

$$\cos\alpha = \cos\left(\mathbf{O'Q}, \mathbf{PQ}\right) = \mathbf{P'Q}/d_{PQ}. \tag{5}$$

3. The proposal

3.1. Derivation and analysis of the obliquity factor

In this session, the physical significance of the obliquity factor and the details of the derivation process will be discussed. To determine the obliquity factor, cos α, the 2D top view is considered first for simplify as shown in Fig. 2 for OIP model. Figs. 2(a) and (b) are the situations that the angles, $|\theta_R - \theta_r|$, are acute and obtuse angles, respectively.

When the observation surface is inside, the obliquity factor is proportional to the projection vector **P'Q** as shown in red bold line in Fig. 2. In the case of acute angle shown in Fig. 2(a), the value of the vector **P'Q** is negative since the direction of the vector **P'Q** is opposite to that of the outer normal of the point Q, **OQ**. While in the case of obtuse angle shown in Fig. 2(b), the value of the vector **P'Q** is positive since the direction of the vector **P'Q** is the same with that of the outer normal of the point Q, **OQ**. According to the geometric relationship in Fig. 2, it is easy to obtain the expression of the obliquity factor that,

$$\cos\alpha = \cos\left(\angle PQP'\right) = \mathbf{P'Q}/PQ,$$
$$\cos\alpha = \begin{cases} -\left(OP' - OQ\right)/PQ \\ \quad = -\left[R\cos\left(\theta_r - \theta_R\right) - r\right]/d_{PQ} \quad \text{............acute angle} \\ \quad = \left[r - R\cos\left(\theta_r - \theta_R\right)\right]/d_{PQ} \\ \left(OP' + OQ\right)/PQ \\ \quad = \left\{R\cos\left[\pi - \left(\theta_r - \theta_R\right)\right] + r\right\}/d_{PQ} \quad \text{....obtuse angle} \\ \quad = \left[r - R\cos\left(\theta_r - \theta_R\right)\right]/d_{PQ} \end{cases}. \tag{6}$$

When it comes to the 3D cylindrical model as shown in Fig. 1(b), it is easy to derive out that the expression of the obliquity factor is the same with that of 2D top view. While the d_{PQ} of 2D top view is replaced by the form of 3D cylindrical model. Therefore, the distribution $u_r(\theta_r, z_r) g_r(\varphi, z)$ can be rewritten by

$$u_r\left(\theta_r, z_r\right) = C \iint_{\Sigma} u_R\left(\theta_R, z_R\right) \frac{\left[r - R\cos\left(\theta_r - \theta_R\right)\right]}{\left[R^2 + r^2 - 2Rr\cos\left(\theta_r - \theta_R\right) + \left(z_r - z_R\right)^2\right]} \times \exp\left(jk\left[R^2 + r^2 - 2Rr\cos\left(\theta_r - \theta_R\right) + \left(z_r - z_R\right)^2\right]^{1/2}\right) d\theta_R dz_R. \tag{7}$$

From the above analysis, it shows:

(1) The diffraction distribution of any point on the observation plane is the projection of the complex amplitude of the observation point propagated from the object point onto the outer normal of the observation point.
(2) The physical meaning of the obliquity factor is the projection of the unit complex amplitude in the propagation direction onto the outer normal of the observation point.