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Integration method to calculate the stress field in the optical fiber

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A B S T R A C T

An integration method based on superposition theorem to calculate the stress field in the optical fiber with arbitrary shape stress elements is derived. The identity between the theoretical analysis result and the integration method in the optical fiber with sector shape bow-tie stress elements is proved. The integration method calculation is compared with the Comsol Multiphysics software simulation and they are agreed well with each other.

1. Introduction

The stress type optical fibers [1–5], such as the Bow-Tie fiber, Panda fiber and Elliptical Cladding fiber, are widely used in coherent optical communications, fiber-optic sensing systems [6–9], and polarization controlling devices. It is well known that the stress induced birefringence is highly related to the stress distribution in the stress induced polarization maintaining optical fiber. But it is hard to acquire an exact analytical stress expression except in the case with circular stress inclusion, elliptical stress inclusion, or sector shape Bow-Tie inclusion [10–12]. Though some numerical approaches based on finite element method [13–18] have been developed to calculate the stress distribution of the optical fiber with arbitrary stress doped region or non-circular core, unfortunately, there is no explicit expression or formula for the relationship between the stress components and the stress region shape or other related parameters. This will result in disadvantage that the stress distribution characteristics and the transmission property of the optical fiber cannot be analyzed more generally and theoretically. In this paper, the calculation method of the stress distribution in the stress type optical fiber is studied comprehensively. An integration method to precisely calculate the stress distribution with arbitrary shape stress elements is introduced and derived. The effectiveness of this method is proved theoretically based on superposition theorem and demonstrated numerically by COMSOL Multi-physics software simulation.

2. Basic theory of the stress analysis

As is illustrated by Fig. 1, the stress field in elasticity material can be expressed as a stress tensor:

$$\mathrm{T} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \tag{1}$$

where σ_x, σ_y, σ_z are the normal stresses, τ_{xy}, τ_{yx}, τ_{xz}, τ_{zx}, τ_{yz}, τ_{zy} are the shear stresses, and $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$ according to elastic theory.

Because of the infinite elongation to the both ends and the bilateral symmetry of the optical fiber, the displacement related to the strain is only occurred in the cross section of the optical fiber. The stress analysis in the optical fiber is a plane strain problem. The force balance equation is expressed as [19]

$$\begin{cases} \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{yx}}{\partial y} = 0 \\ \dfrac{\partial \sigma_y}{\partial y} + \dfrac{\partial \tau_{xy}}{\partial x} = 0 \end{cases} \tag{2}$$

where the shear stress τ_{xz} and τ_{yz} disappear in the equation because $\tau_{xz} = \tau_{yz} = 0$, and $\sigma_z = \upsilon\left(\sigma_x + \sigma_y\right) - E\alpha\Delta T$.

The compatibility equation in terms of stress is expressed as

$$\nabla^2\left(\sigma_x + \sigma_y\right) = 0. \tag{3}$$

Eqs. (2) and (3) are the primary equations to solve the stress field in the stress type optical fiber when they are combined with the free

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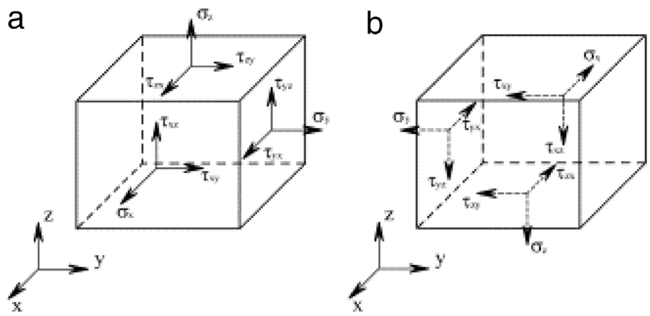

Fig. 1. Illustration of the stress tensor components.

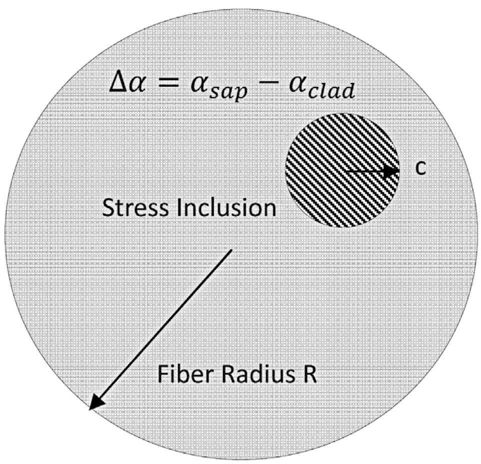

Fig. 2. Cross section of the optical fiber with circular stress inclusion.

boundary condition and the connection condition between different elastic material parts.

To acquire an exact solution in the stress type optical fiber the potential function method based on complex variable is employed [12]. The stress components can be expressed by two complex potential functions $\varphi(z)$ and $\psi(z)$ as follows:

$$\sigma_x + \sigma_y = 4Re\left[\varphi'(z)\right] \tag{4a}$$

$$\sigma_x - \sigma_y = -2Re\left[\bar{z}\varphi''(z) + \psi'(z)\right] \tag{4b}$$

$$\tau_{xy} = Im\left[\bar{z}\varphi''(z) + \psi'(z)\right]. \tag{4c}$$

When the potential functions $\varphi(z)$ and $\psi(z)$ are given, the stress components σ_x , σ_y and τ_{xy} or the stress tensor $\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$ in the cross section of the optical fiber will be determined.

3. Integration method and the identity with the theoretical analysis

Firstly, let us consider the stress field with a circular stress inclusion as is demonstrated in Fig. 2. As it is shown in reference [12], the complex potential functions $\varphi(z)$ and $\psi(z)$ can be written as follows when the stress region is a circular inclusion.

$$\begin{cases} \varphi^{In}(z) = \sigma^0\left[\dfrac{z}{2} - \dfrac{c^2}{2} z \dfrac{1 + z\overline{z_c}}{1 - z\overline{z_c}}\right] \\ \psi^{In}(z) = \sigma^0 c^2 \overline{z_c} \dfrac{2 - z\overline{z_c}}{\left(1 - z\overline{z_c}\right)^2} \end{cases} \tag{5a}$$

$$\begin{cases} \varphi^{Out}(z) = -\sigma^0 \dfrac{c^2}{2} z \dfrac{1 + z\overline{z_c}}{1 - z\overline{z_c}} \\ \psi^{Out}(z) = \sigma^0 c^2\left[\dfrac{1}{z - z_c} + \overline{z_c}\dfrac{2 - z\overline{z_c}}{\left(1 - z\overline{z_c}\right)^2}\right] \end{cases} \tag{5b}$$

where $\varphi^{In}(z)$, $\psi^{In}(z)$ and $\varphi^{Out}(z)$, $\psi^{Out}(z)$ represent the corresponding complex potential functions in the stress doped region and out of the stress doped region (the region except the stress doped parts on the cross section of the fiber). It should be noted that all length quantities, such as the complex variable z, the circular stress inclusion radius c and its center location variable z_c, are normalized by the optical fiber radius R. $\sigma^0 = -\frac{E\Delta\alpha\Delta T}{2(1-\upsilon)}$ is the normalized unit of the stress. E and υ are the Young's modulus and Poisson ratio related to the elasticity. $\Delta\alpha = \alpha_{sap} - \alpha_{clad}$ is the expansion coefficient difference between the stress doped region and the substrate (e.g. the cladding of the optical fiber in this case). $\Delta T = T_{cool} - T_{hot}$ is the temperature difference between the cooled fiber room temperature and the melted fiber hot temperature in the optical fiber drawing process.

Because Eqs. (2) and (3) and the boundary condition connecting different elastic parts are all linear equations, the solution of the stress in the elasticity should satisfy the superposition theorem. The contribution from all stress doped parts as a whole can be acquired by adding together the individual contribution from the stress doped parts.

Since the contribution from a single circular stress element with radius c is as that shown by Eq. (5b), the contribution from a stress element with small enough area ds may be expressed as

$$\begin{cases} \varphi_{ds}^{out}(z) = -\sigma^0 \dfrac{ds}{2\pi} z \dfrac{1 + z\overline{z_c}}{1 - z\overline{z_c}} \\ \psi_{ds}^{out}(z) = \sigma^0 \dfrac{ds}{\pi}\left[\dfrac{1}{z - z_c} + \overline{z_c}\dfrac{2 - z\overline{z_c}}{\left(1 - z\overline{z_c}\right)^2}\right]. \end{cases} \tag{6}$$

The contribution from all stress elements can be acquired by integration on the stress doped region and may be expressed as follows:

$$\begin{cases} \varphi^{Out}(z) = -\dfrac{\sigma^0}{2\pi} \displaystyle\iint z \dfrac{1 + z\overline{z_c}}{1 - z\overline{z_c}} ds \\ \psi^{Out}(z) = \dfrac{\sigma^0}{\pi} \displaystyle\iint \left[\dfrac{1}{z - z_c} + \overline{z_c}\dfrac{2 - z\overline{z_c}}{\left(1 - z\overline{z_c}\right)^2}\right] ds. \end{cases} \tag{7}$$

But here there is a problem that the stress doped region cannot be fully filled by the circular stress elements because there is always some interspace between them, as is shown by Fig. 3a. The uncovered ratio in the stress doped region is more than $\frac{\sqrt{3}-\pi/2}{\sqrt{3}} \geq 9.31\%$. Eq. (7) cannot be used directly before it is confirmed.

Fortunately, as is shown by Fig. 3b, any shape stress doped region can be divided into some small enough sector shape Bow-Tie stress elements and there is no gap between them. The stress doped region can be fully filled by the sector shape stress elements, just as the case the stress doped region is divided into some rectangular or triangular shape stress elements. Then the whole contribution can be expressed explicitly if a general expression of the complex potential functions $\varphi(z)$ and $\psi(z)$ about a sector shape Bow-Tie stress element is acquired. In the following paragraphs, it will be proved that the theoretical analysis result about a sector shape Bow-Tie stress element can be derived identically from the integration Eq. (7), which is based on the superposition theorem and Eq. (5b).

To calculate the integration in Eq. (7) more simply and generally, a sector shape Bow-Tie stress element is symmetrically put on the x-axis as is shown by Fig. 4 and the stress normalized unit σ^0 is ignored temporarily. The complex potential function $\varphi^{Out}(z)$ is derived by the