Optics Communications 400 (2017) 34–37

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optics

Invited paper

Effect of nonlinearity on the dynamics of Bragg-induced optical Rabi oscillations in a one-dimensional periodic photonic structure

P.A. Brandão *, S.B. Cavalcanti

Instituto de Física, UFAL, Cidade Universitária, Maceió-AL, 57072-970, Brazil

ARTICLE INFO

Keywords:
Photorefractive lattice
Rabi oscillation
Nonlinear optics
Self phase modulation

ABSTRACT

Propagation of wide optical beams in transverse periodic lattices have been reported to induce power oscillations between Fourier modes related by the Bragg resonance condition, resulting from the coupling between the beam and the periodic structure. These oscillations have been referred to as Rabi optical oscillations due to the analogy with matter Rabi oscillations. In this work, we investigate the behavior of Bragg-induced Rabi-type oscillations of a multimode Gaussian beam in the presence of optical nonlinearity. We find a combination of oscillation and spectrum broadening under both self-focusing and self-defocusing nonlinearities, in the sense that the oscillations are maintained while the spectrum is broadened and therefore partially transferred to the twin frequency. For intense self-focusing nonlinearities a complete leak of the initial mode profile to other modes is rapidly attained so that no oscillation is observed. In contrast, for intense self-defocusing nonlinearities the redistribution rate is so dramatic that oscillations cease and power only fades away.

1. Introduction

The effect of spatially periodic potentials on the propagation of quantum particles have been remarkably illustrated in recent years in optical periodic systems. These optical systems have provided a myriad of experiments that exhibit phenomena well known in its electronic counterpart. Photonic lattices with a transverse refractive index gradient have served as an experimental tool that permits the observation of coherent phenomenon such as Bloch oscillations and Zener tunneling [1–7]. Bloch oscillations result from the coherent scattering from the periodic structure in a Bragg fashion [8]. Recently, Bragg-resonance-induced Rabi oscillations of a beam propagating through a photonic lattice have been reported in the literature [9], presented as an analog of Rabi oscillations in a two level system driven by an external optical field. In the optical analog, the photonic lattice plays the role of the field which couples the pair of resonant modes of the beam, which plays the role of the two-level system, inducing Rabi-type power oscillations. Rabi oscillations have also been reported in waveguide arrays where power transfer occur between two modes under phase-matching conditions [10]. Nonlinear light propagation in photonic lattices, where there is an interplay between diffraction and nonlinearity, may be experimentally carried out to show dramatic changes in spectrum [11]. Work on a nonlinear periodic photonic lattice, under the assumption of a constant field amplitude in a Bragg fashion configuration, has reported a mode trapping phenomena (whereby Rabi oscillations cease under the influence of self-focusing nonlinearity) and a nonlinear modulation of the Rabi oscillation period [12].

As the power spectra and phase structures of a spatial beam play completely different roles under linear or nonlinear propagation regimes, in this work we wish to study the influence of nonlinearity on the optical Rabi oscillations induced upon a multimode beam, centered around the Bragg resonance mode, propagating through a photonic lattice with a period much smaller than the beam width. Numerical results based on the nonlinear Gross–Pitaevskii equation show that for a Gaussian input, a weak and positive (self-focusing) nonlinearity promotes the redistribution of the energy among neighboring modes while keeping the oscillations, so that the distorted spectrum is partially transferred to its twin frequency. In contrast, in case of a strong and positive nonlinearity we have found that power redistribution occurs in a faster rate so that the energy transferred has become a tiny fraction of the input. For a negative (self-defocusing) weak nonlinearity, the results are practically identical to the positive weak nonlinear case. In both cases damping is always present due to the redistribution of the initial mode power among its neighbors so that the power transfer to the twin mode is only partial. However for a strong negative nonlinearity, the

* Corresponding author.
E-mail address: paulocabf@gmail.com (P.A. Brandão).

http://dx.doi.org/10.1016/j.optcom.2017.05.004
Received 21 December 2016; Received in revised form 24 April 2017; Accepted 3 May 2017
0030-4018/

redistribution rate is so dramatic that oscillations cease and power only fades away.

2. Theory and results

To model the beam dynamics in a physically realizable scheme, we follow the theory developed in [4] by assuming a one-dimensional photorefractive optically induced photonic lattice. The equations governing the evolution of the lattice waves W, responsible for the creation of the periodic potential, and the probe U, that will perform Bragg-induced Rabi oscillations, may be described by the following pair of coupled nonlinear partial differential equations:

$$i\frac{\partial U}{\partial z}+\frac{1}{2k_1}\frac{\partial^2 U}{\partial x^2}-\Delta n_e(I)U=0, \quad (1)$$

$$i\frac{\partial W}{\partial z}+\frac{1}{2k_2}\frac{\partial^2 W}{\partial x^2}-\Delta n_o(I)W=0, \quad (2)$$

where $k_1 = 2\pi n_e/\lambda$, $k_2 = 2\pi n_o/\lambda$, n_e and n_o are the refractive index for the extraordinary and ordinary directions, respectively, Δn is the nonlinear index change, $I = |U|^2 + |W|^2$ the total intensity of the two orthogonally polarized fields and λ the free space wavelength [4]. By considering only the dominant screening nonlinearity, the terms Δn_e and Δn_o are given by

$$\Delta n_e=\frac{k_0 n_e^3 r_{33}}{2}\frac{E_0}{1+\bar{I}(x)}\approx\frac{k_0 n_e^3 r_{33}E_0}{2}[1-\bar{I}(x)], \quad (3)$$

$$\Delta n_o=\frac{k_0 n_o^3 r_{13}}{2}\frac{E_0}{1+\bar{I}(x)}\approx\frac{k_0 n_o^3 r_{13}E_0}{2}[1-\bar{I}(x)], \quad (4)$$

where $k_0 = 2\pi/\lambda$, r_{ij} is the electro-optic tensor coefficient, E_0 is the applied bias field and $\bar{I}(x) = I(x)/I_d$ with I_d the dark irradiance of the crystal. By externally illuminating the photorefractive crystal, I_d may attain large values and here, we assume that this fact justifies the binomial expansion used on the right side of Eqs. (3) and (4) [13,14]. As Δn_o is typically much smaller than Δn_e, the propagation of the lattice waves V is linear to a good approximation and $|W|^2$ plays the role of a potential in the evolution of U. Quantitatively this means that one may substitute Eq. (3) by Eq. (1):

$$i\frac{\partial U}{\partial z}+\frac{1}{2}\frac{\partial^2 U}{\partial x^2}-\frac{n_e^2 r_{33}E_0}{2}\left[1-\frac{|W|^2}{I_d}\right]U+\frac{n_e^2 r_{33}E_0}{2I_d}|U|^2U=0, \quad (5)$$

where we have scaled $k_1 z \to z$ and $k_1 x \to x$. The bias electric field E_0 may assume positive or negative values, $E_0 = \pm|E_0|$, and therefore two cases may be distinguished and treated separately. If one chooses $E_0 > 0$, then it is useful to define $\kappa = |\kappa| = n_e^2 r_{33}|E_0|/2I_d$ which plays the role of a positive nonlinear constant, or, a self-focusing nonlinearity. For this case, the potential term in Eq. (5) is $V(x) = |\kappa|[I_d - |W|^2]$ and by writing $|W|^2 = I_d - |w_0|^2\cos x$ (with $|w_0|^2 \le I_d$) then $V(x) = |\kappa||w_0|^2\cos x = V_0\cos x$ in which V_0 may be controlled through $|\kappa|$. The Gross–Pitaevskii equation in this case is written as:

$$i\frac{\partial U}{\partial z}+\frac{1}{2}\frac{\partial^2 U}{\partial x^2}-V(x)U+|\kappa||U|^2U=0. \quad (6)$$

Likewise, if $E_0 = -|E_0|$ then, following the same steps as above, one may write $V(x) = |\kappa|[|W|^2 - I_d]$, and if $|W|^2 = I_d + |w_0|^2\cos x$ (no restrictions on $|w_0|^2$ and I_d in this case) then $V(x) = |\kappa||w_0|^2\cos x = V_0\cos x$ as well. The Gross–Pitaevskii equation in this situation has the same form as in Eq. (6) except for a minus sign before the nonlinear constant, which now represents a self-defocusing nonlinearity. Typical values for the system material parameters are $n_e = 2.299$, $n_o = 2.312$, $r_{33} = 1340$ pm/V, $r_{13} = 67$ pm/V and $E_0 \approx 102$ V/mm. By using these values, the nonlinear coefficient is of the order $\kappa = 3.612\times10^{-4}/I_d$ which can be easily tuned through the dark irradiance. With the formalism just developed, our next objective is to solve Eq. (6) using a multimode beam centered at the Bragg resonant frequency, as initial condition to observe the effect of the nonlinearity (positive and negative) on the Rabi oscillation dynamics exhibited by the spectrum.

To this end, let us begin by assuming the initial condition for the Gross–Pitaevskii Eq. (6), a multimode Gaussian beam centered at the Bragg resonant frequency k_B, that is,

$$U(x,0)=\exp\left[-\frac{1}{2}\left(\frac{x}{X_0}\right)^2\right]\exp(ik_Bx), \quad (7)$$

where X_0 is the normalized beam initial width and k_B the normalized wavevector component in the x direction at the particular value $k_B = 1/2$ (edge of the Brillouin zone) and we initially assume that $X_0 = 100P$, where P is the lattice period, in what follows. Later we study the influence of the beam width and discuss it in Fig. 5 where X_0 varies. As the beam width is many times the lattice period, we are in the opposite regime of discrete diffraction [15–19]. The numerical method used to solve Eq. (6) was a pseudospectral Fourier method which is well described in [20]. The number of points used in the x direction was 2048, the computational window had a transverse length of $5X_0$ in order to make border effects negligible and the propagation step was 0.01 ranging from zero to 60. Let us treat the linear case ($\kappa = 0$) first, which could be achieved if $|w_0|^2 \gg |U|^2$ so we can work with the usual Schrödinger equation. The upper part of Fig. 1 shows the solution of Eq. (6) in Fourier space with potential strength $V_0 = 0.1$. This numerical value of V_0 was assumed in all subsequent simulations in order to guarantee that the initial mode localized at $k_x = k_B$ remains strongly coupled only to modes inside the first Brillouin zone. The Fourier transform is taken only in the x direction, $U_k(k_x, z) = \mathfrak{F}_x[U(x,z)]$, where $\mathfrak{F}_x[\cdot]$ represents the Fourier transform operator in the x direction. For small z, all spatial frequencies are situated around $k_B = 1/2$ and this simply represents the Fourier transform of Eq. (7), where a Gaussian profile centered at $k_B = 1/2$ with a width $\Delta k_x \sim 0.0012$ is initially expected. As the beam propagates inside the medium, Bragg-induced effects act on the spatial mode profile and it is useful to define the population difference as $|U_k(1/2,z)|^2 - |U_k(-1/2,z)|^2$. With this definition, it can be seen that for $z \approx 30$, all Fourier components are shifted to $-k_B$, as the bottom part of Fig. 1 indicates. At $z \approx 60$ all Fourier mode is traced back to the initial one around k_B and therefore this value represents the approximate oscillation period between the two modes. In this linear regime, there is no bandwidth distortion and the cycle continues oscillating forever between $\pm k_B$. Since there is no addition of spatial frequencies in the linear approximation, we expect the spatial dependence of $U(x,z)$ to maintain its Gaussian shape so that it is possible to define a beam center through $\bar{x} = \int x|U|^2dx/\int|U|^2dx$ and to show that it exhibits spatial oscillations [9]. These spatial oscillations are a consequence of the Bragg-induced mode transfer between $\pm k_B$. It should be noted that the lack of spatial frequency broadening (even in a linear propagation regime with no lattice present) is due to the spatial range of z considered in the analysis. In a linear approximation we expect the beam to diffract for $z_d \sim X_0^2$ and, as we chose as maximum value $z = 65$ one may safely neglect diffraction effects for which the second term in Eq. (6) is responsible. Therefore, all bandwidth modifications are due to the lattice or the nonlinear coupling. This does not mean that one may discard the diffraction term in Eq. (6) for our beam propagates with a transverse wavevector k_B in the x direction such that the effect of $\partial^2/\partial x^2$ is essential.

Let us now solve Eq. (6) with $\kappa > 0$, representing a self-focusing nonlinearity. The results for the evolution of Fourier modes, using the same parameters as in Fig. 1, except for the nonlinear coefficient which is now taken as $\kappa = 0.1$, are shown in Fig. 2. This figure illustrates how the initial Gaussian distribution in Fourier space changes dramatically as the beam propagates inside the nonlinear periodic medium. At $z \approx 60$ the Fourier peak broadens and the spatial frequency spectrum is approximately zero at the Fourier mode k_B (see the bottom part of Fig. 2) while a periodic pattern starts to develop inside the initial beam bandwidth. This effect is the spatial analog of the self-phase modulation (SPM) effect that occurs with optical pulses propagating in dispersive nonlinear media [21]. The main effect of SPM is to create an oscillatory pattern structure inside the beam's initial frequency bandwidth. But, in our