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Continuous variables quantum computation over the vibrational modes of a single trapped ion

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ABSTRACT

We consider the quantum processor based on a chain of trapped ions to propose an architecture wherein the motional degrees of freedom of trapped ions (position and momentum) could be exploited as the computational Hilbert space. We adopt a continuous-variables approach to develop a toolbox of quantum operations to manipulate one or two vibrational modes at a time. Together with the intrinsic non-linearity of the qubit degree of freedom, employed to mediate the interaction between modes, arbitrary manipulation and readout of the ionic wave function could be achieved.

1. Introduction

The current paradigm of implementations of quantum computing consists in the coherent manipulation of discrete two-level systems, the qubits, by sequences of quantum gates [1]. Ion traps stand as one of the most successful experimental implementations of a quantum processor, a system for which all basic elements required for computation universality have been demonstrated in proof-of-principle experiments [2–4] and scalability is not unlikely [5,6].

A chain of ions trapped in a harmonic potential functions as a quantum register wherein each ion encodes one qubit in energy eigenstates of its electronic configuration [7]. The external confinement and the electric repulsion among ions give rise to collective modes of vibration which are employed to mediate the interaction between any chosen pair of qubits. Quantum operations are accomplished by resonant or near-resonant laser pulses with the qubit transitions. Other internal energy levels of the ions are employed to initialize and measure the qubits. Even though the specifics of this manipulation scheme has evolved enormously since its inception [8,9,3,10,11], it would not be inappropriate to name it as the 'Cirac & Zoller (CZ) paradigm' of ion trap quantum computing. In short, the CZ paradigm has each ion storing a single qubit in internal electronic energy levels and different qubits interacting via the quantum information 'bus' provided by one or more motional modes.

An alternative route to quantum computation considers physical observables with continuous spectra – continuous variables (CV) – to realize the physical encoding and manipulation of quantum information [12,13]. In the continuous variables quantum computing (CVQC) paradigm, Gaussian states and operations are usually considered as the building blocks of quantum logic [14,15], as well as a single non-Gaussian operation needed to achieve universality [16,17]. The basic physical object of quantum computing is embedded in this case in an infinite-dimensional Hilbert space. And although it may be regarded as continuous in the eigenbasis of certain observables, it can many times also be understood as a discrete configuration space in the eigenbasis of other observables. More concretely, as considered in this paper, a vibrational mode of the ion chain [18–22] can be either described in the continuous phase space of position and momentum observables, e.g. by the Wigner function, or in terms of superpositions of the quantized energy eigenstates of the harmonic oscillator, the number or Fock states [23]. The generation, control [24–26] and measurement [27–29] of vibrational modes have been approached in the recent literature in various ways: by entangling them with optical resonator modes [30], using them as equivalent models for the study of vibrational states of optomechanical systems [31], by generating exotic quantum states [32,33], or by implementing quantum simulations of solid state systems [34–36].

In this paper, we investigate the idea of exploiting the vibrational modes of trapped ions as the physical platform of quantum computing, i.e. for the implementation of quantum gates in the motional modes of vibration [37,38]. We consider the feasibility and particularities of inverting the CZ paradigm to employ the qubit degree of freedom as the

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mediator of interaction among a set of motional modes of vibration. Even though our proposal can be extended to a system of different singly trapped ions [39], we focus here on the simplest case of a single trapped ion and its corresponding set of three vibrational modes as a starting point. By following this approach, we try to establish the potential capabilities brought by this minimalistic quantum system and the likely limitations on the size of the configuration space made available by this simple change of perspective in the use of the ion trap. We develop a CV quantum computation toolbox to manipulate each of the single modes and to make them interact in pairs, in particular to show that conditional dynamics (entangling gates) would be available. The proposed quantum gates are realizable by bichromatic laser fields with tunable frequencies. Readout of the quantum state can be performed using number-dependent Rabi flops on the qubit [40–42].

This paper is organized as follows. In Section 2, we present the quantum processor based on the ion trap and recall the basic manipulation of a single trapped ion by an external laser source. Section 3 presents the CV quantum gates that can be realized in the motional modes with bichromatic laser fields. We detail our CVQC proposal and develop the necessary toolbox of quantum gates in Section 4. Our concluding remarks follow in Section 5.

2. Basic implementation

2.1. Physical system

In this proposal, the physical objects to be manipulated are the different oscillation modes of a quantum harmonic oscillator. There are three available modes in the simplest case of a single trapped ion oscillator. The size of the Hilbert space associated with each vibrational mode and available to manipulation in actual experimental conditions is better quantified in the eigenbasis of the number operators. The basis for each mode is assumed to be truncated at a maximum phonon number N, and is hence composed of the eigenstates

$$\{|0\rangle, |1\rangle, |2\rangle, \ldots, |N\rangle\}. \tag{1}$$

The representation of the quantum state in terms of phonon number eigenstates refers to a 'particle-like' description of the quantum system. The quantum state of the harmonic oscillator also admits a CV representation in the position and momentum phase space, a 'wave-like' description employing the Wigner function. In phase space, quantum gates are transformations of the Wigner function. In the ion trap processor, they can be performed by coupling vibrational modes to the qubit (internal) degree of freedom by means of bichromatic laser light [8,9,43,44]. The qubit is by construction a highly non-linear physical system – one that saturates with a single quantum –, a property here employed to generate non-Gaussian operations on the vibrational modes. Since in our proposed ion trap CVQC architecture the qubit is only an auxiliary source of non-linearity and coupling among motional modes, the desired quantum operations must start and end with quantum states $\hat{\rho}$ which are separable in the qubit $\hat{\rho}_q$ and motional modes $\hat{\rho}_m$, i.e. we impose that $\hat{\rho} = \hat{\rho}_q\hat{\rho}_m$ before and after the application of quantum gates.

The CV toolbox of quantum operations to be developed below can be separated in Gaussian and non-Gaussian operations. The class of Gaussian operations maintains as Gaussian an initially Gaussian Wigner function. There are single- and two-mode Gaussian operations. Single-mode displacements and squeezers respectively displace the origin of phase space or the scaling of the position and momentum axis. Both of them have already been experimentally demonstrated in the ion trap processor [45,46]. Two-mode operations comprise the beam splitter and the two-mode squeezer. The beam splitter is a passive transformation that linearly combines two field modes. The two-mode squeezer, an active transformation, can be understood as two single-mode squeezers simultaneously acting on orthogonal combinations of two modes. One can also include two-mode conditional gates as generalizations of such operations.

2.2. Hamiltonian of the ion trap

Our CVQC toolbox is built upon the simplest implementation of an ion trap processor: a single ion furnishes the qubit and three independent modes of vibration. We consider in this section the basic coherent manipulation of a single trapped ion by an external source of coherent light [47,23].

To establish notation, we recall below the elementary dynamics of one qubit and two motional modes coupled to it by a monochromatic external laser. The generalization of the interaction to three oscillator modes and bichromatic lasers capable of producing the desired quantum gates follows next.

The ion trap Hamiltonian reads in this case as

$$\hat{H} = \hat{H}_0 + \hat{H}_I, \tag{2}$$

where $\hat{H}_I$ is the interaction Hamiltonian discussed below and $\hat{H}_0$ provides the free dynamics of qubit and motional modes,

$$\hat{H}_0 = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \hbar\omega_a\hat{a}^\dagger\hat{a} + \hbar\omega_b\hat{b}^\dagger\hat{b}. \tag{3}$$

The qubit transition frequency is ω_0 and its two-dimensional Hilbert space is described in terms of the excited $|e\rangle$ and ground $|g\rangle$ internal states of the ion, with which we write $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$. The two independent vibrational modes under consideration are described in terms of the annihilation operators $\hat{a}$ and $\hat{b}$ and the respective creation operators satisfying the commutation relations $[\hat{a}, \hat{a}^\dagger] = [\hat{b}, \hat{b}^\dagger] = 1$. Their oscillation frequencies are ω_s, where $s \in \{a, b\}$ denotes the mode.

The simplest model of interaction Hamiltonian $\hat{H}_I = -\vec{d}\cdot\vec{E}$ comprises a dipolar coupling between the ion and an external coherent light source. The atomic dipole operator is $\vec{d} = \vec{\mu}(\hat{\sigma}_+ + \hat{\sigma}_-)$, with dipole moment $\vec{\mu}$ and operators $\hat{\sigma}_+ = |e\rangle\langle g|$ and $\hat{\sigma}_- = |g\rangle\langle e|$. The light source drives the ion by means of the electric field $\vec{E} = \vec{E}_0\exp(i\vec{k}\cdot\vec{r} - i\omega_L t)$ with wavevector $\vec{k}$ and frequency ω_L. The interaction Hamiltonian can be made to account for the free evolution associated with $\hat{H}_0$ (interaction picture), yielding

$$\begin{aligned}\hat{H}_I = \ & \tfrac{1}{2}\hbar\Omega\hat{\sigma}_+e^{-i\delta t}\exp[i\eta_a(\hat{a}e^{-i\omega_a t} + \hat{a}^\dagger e^{i\omega_a t}) \\ & + i\eta_b(\hat{b}e^{-i\omega_b t} + \hat{b}^\dagger e^{i\omega_b t})] + \text{h. c.},\end{aligned} \tag{4}$$

where $\delta = \omega_L - \omega_0$ is the radiation-atom detuning, $\Omega = |\vec{\mu}\cdot\vec{d}|/\hbar$ is the Rabi frequency, and $\eta_s = kx_s\cos\theta$ are the Lamb-Dicke parameters, defined in terms of the typical scale of the ground state oscillator wavefunction $x_s = \sqrt{\hbar/(2m\omega_s)}$ and the direction of propagation θ of the laser with respect to the direction of vibration of mode s. Typical experimental conditions in optical qubits imply $\eta_s \ll 1$, values for which the Lamb-Dicke regime can be evoked to expand the interaction Hamiltonian in powers of η_s.

The CV quantum gates we consider in the next section are obtained by expanding the interaction Hamiltonian up to second order in η_s [48,49], as

$$\begin{aligned}\hat{H}_I = \ & \hat{H}^{(0)} + \eta_a\hat{H}_a^{(1)} + \eta_b\hat{H}_b^{(1)} - \eta_a^2\hat{H}_a^{(2)} \\ & - \eta_b^2\hat{H}_b^{(2)} - 2\eta_a\eta_b\hat{H}_{ab}^{(2)} + O(\eta_s^3)\end{aligned} \tag{5}$$

The effect of each Hamiltonian is easily understood in the Fock basis of the motional states. The single-quantum saturation associated with the qubit degree of freedom plays the fundamental role of allowing the coherent manipulation of single quanta in the motional modes.

The zeroth-order term is the carrier transition Hamiltonian,

$$\hat{H}^{(0)} = \frac{1}{2}\hbar\Omega'(e^{-i\delta t}\hat{\sigma}_+ + e^{i\delta t}\hat{\sigma}_-), \tag{6}$$

resonant for $\delta = 0$. The Rabi frequency is modified due to the motional coupling as $\Omega' = (1 - \eta_a^2 - \eta_b^2)\Omega$. This Hamiltonian induces qubit transitions without affecting the motional state of the ion. It may be