Optics Communications 394 (2017) 135–138

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Energy density and energy flow of plasmonic waves in bilayer graphene

Afshin Moradi

Department of Engineering Physics, Kermanshah University of Technology, Kermanshah, Iran

ARTICLE INFO

Keywords:
Bilayer graphene
Energy flow
Surface wave
Hydrodynamic theory

ABSTRACT

The propagation of plasmonic waves in bilayer graphene is studied based on the classical electrodynamics. The interactions between conduction electrons confined to move on the surface of each layer are taken into account via the two-dimensional linearized hydrodynamic model. The energy theorem of electrodynamics is cast in a form which yields expressions for energy density and energy flow of p-polarized surface plasmon polariton waves in bilayer graphene. Numerical results show that the presence of two layers causes the appearance of two branches in the dispersion relation that introduce alterations in the physical behavior of the energy, power flow and the energy transport velocity, in comparison with the results of monolayer graphene.

1. Introduction

Following the discovery by Novoselov et al [1] of graphene, this truly two-dimensional (2D) electronic system has opened new frontiers in solid state physics due to its unique and superior electronic and optical properties [2–4]. Previous works demonstrated that monolayer graphene can support p-polarized or transverse magnetic (TM) surface plasmon polariton (SPP) waves [5–10]. Also, Mikhailov and Ziegler [11] predicted that monolayer graphene supports unusual transverse electric (TE) waves in a well defined and narrow frequency window. Then, Bludov et al. [12] discussed the possibility of nonlinear TE plasmonic wave oscillations on monolayer graphene.

Furthermore, bilayer graphene (BLG) has attracted much attention for fundamental physics as well as for possible technological applications. In this way, Wang and Chakraborty investigated the Coulomb screening properties and collective excitations in BLG [13] and BLG under a perpendicular electric bias [14]. Borghi et al. [15] studied the dynamical response functions and collective modes of BLG. Then, Sensarma et al. [16] analytically studied the dynamic screening properties of doped BLG systems within the random phase approximation and Jablan et al. [17] predicted the existence of TE plasmons in BLG in addition to the TM plasmonic waves [18,19]. Also, within the framework of 2D linearized quantum hydrodynamic model, Li et al. [20,21] investigated the interactions of a charged particle with two-layered 2D quantum electron gases as a simple model of BLG.

Nevertheless, no explicit calculation can be found for energy, power flow and the energy transport velocity associated with the plasmonic waves of BLG. We note the presence of two layers introduce new features which are the result of the combined effect of the geometry. Therefore, for BLG structure, we expect new physical behavior of the energy, power flow and energy transport velocity of plasmonic waves, in comparison with those obtained for monolayer graphene [22]. The aim of the present paper is to provide such an account, where we use the classical electrodynamics in conjunction with a 2D linearized hydrodynamic model [22] for electronic excitations on graphene surface, in order to study the plasmonic properties of BLG.

2. Formulation of the problem

Fig. 1 depicts schematic view of two laterally infinite graphene layers in a Cartesian coordinate system with coordinates (x, y, z). We define (x, y, z_j) to represent the coordinates of a point on the jth graphene surface $z = z_j$, (with j=1,2). In the present structure, a substrate with dielectric constant ϵ_1 is supposed to occupy the region $z < z_1$ underneath the BLG. Also, two graphene layers are separated by a dielectric material of thickness $z_2 - z_1 = d$ and dielectric constant ϵ_2, whereas the region $z > z_2$ is assumed to be a semi-infinite insulator with dielectric constant ϵ_3. Each graphene layer is modeled as a 2D Dirac massless electron fluid with a fixed equilibrium density of electrons having a typical value of $n_0 = 10^{13} cm^{-2}$ [23,24].

Now, we assume that $n_j(x, t)$ is the first-order perturbed density (per unit area) of the homogeneous electron fluid on the jth graphene surface, due to the propagating p-polarized SPP waves parallel to the jth layer along the x-direction. Based on the 2D linearized hydrodynamic theory [22,25], one may obtain the linearized continuity equation, for jth graphene surface, as

$$\partial_t n_j(x, t) + n_0 \partial_x v_{jx}(x, t) = 0, \tag{1}$$

and the linearized momentum-balance equation,

E-mail address: a.moradi@kut.ac.ir.

http://dx.doi.org/10.1016/j.optcom.2017.03.021
Received 13 January 2017; Received in revised form 27 February 2017; Accepted 7 March 2017
Available online 11 March 2017
0030-4018/

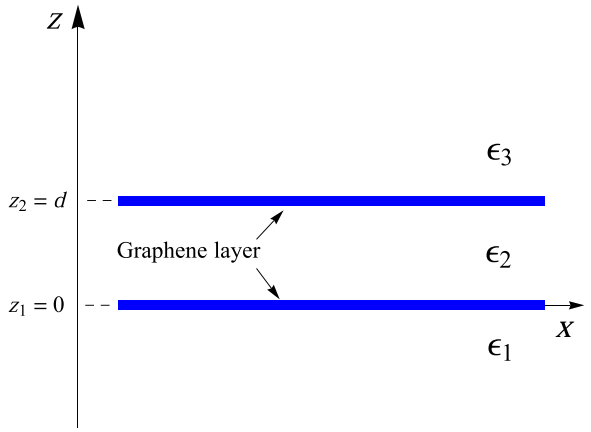

Fig. 1. The schematic view of BLG structure separated by a thin dielectric medium of thickness d.

$$\partial_t v_{jx}(x,\,t) = -\frac{e}{m_e}E_x|_{z=z_j} - \frac{\alpha}{n_0}\partial_x n_j(x,\,t), \tag{2}$$

where e is the element charge and $m_e = \hbar k_F/v_F$ is the effective mass of electron in graphene. We note that $k_F = \sqrt{\pi n_0}$, and $v_F \approx c/300$ [with c being the speed of light in free space] are Fermi wave number, and Fermi speed in graphene, respectively [25]. Furthermore, $v_{jx}(x,\,t)$ is the first-order perturbed values of velocity field of the electrons residing on the jth layer. In the right-hand side of Eq. (2), the first term is the force on electrons due to the tangential component of the electric field, evaluated at the graphene surface $z = z_j$, the second term is the force due to the 2D massless Dirac-fermion quantum statistical pressure in the electron fluid, with $\alpha = v_F^2/2$.

To go further, we assume that all physical quantities (electron density perturbation and electromagnetic fields) vary as $e^{i(kx-\omega t)}$ where k is the component of the wave vector along the x-axis. Therefore, from Eqs. (1) and (2) after the elimination of the velocity v_j, one obtain

$$N_j = -\frac{ien_0}{m_e}\frac{k}{\omega^2 - \alpha k^2}E_x(z)|_{z=z_j}, \tag{3}$$

and $n_j(x,\,t) = N_j e^{i(kx-\omega t)}$. By using Maxwell equations, electric component E_z of the TM waves can be written as

$$E_{1z} = Ae^{\kappa_1 z}e^{i(kx-\omega t)}, \tag{4}$$

in the region $z < z_1$,

$$E_{2z} = (Be^{\kappa_2 z} + Ce^{-\kappa_2 z})e^{i(kx-\omega t)}, \tag{5}$$

in the region $z_1 < z < z_2$, and as

$$E_{3z} = De^{-\kappa_3 z}e^{i(kx-\omega t)}, \tag{6}$$

in the region $z > z_2$, where $\kappa_\lambda^2 = k^2 - k_\lambda^2$, $k_\lambda = \sqrt{\epsilon_\lambda}\omega/c$, with $\lambda = 1,\,2$ and 3. The E_x and H_y components of the TM waves can also be calculated from Maxwell equations. The field components have to satisfy the usual boundary conditions at $z = z_j$ that is, continuity of E_z and discontinuity of E_x, due to the polarization of the electron gas on the BLG surfaces. Applying the boundary conditions at $z = z_1 = 0$ and $z = z_2 = d$ on the components E_x and E_z one can determine the coefficients A, C, and D as

$$A = (1 - \Gamma)\frac{\kappa_2}{\kappa_1}B, \tag{7}$$

$$C = \Gamma B, \tag{8}$$

$$D = \frac{\Gamma e^{-\kappa_2 d} - e^{\kappa_2 d}}{e^{-\kappa_3 d}}\frac{\kappa_2}{\kappa_3}B, \tag{9}$$

where

$$\Gamma = \left(\frac{\epsilon_1}{\kappa_1} - \frac{\epsilon_2}{\kappa_2} - \frac{n_0 e^2/m_e\epsilon_0}{\omega^2 - \alpha k^2}\right)\left(\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2} - \frac{n_0 e^2/m_e\epsilon_0}{\omega^2 - \alpha k^2}\right)^{-1}, \tag{10}$$

and ϵ_0 is the permittivity of free space. In the following, we obtain the dispersion relation which gives us information on the waves in order to obtain the energy flow, energy density and energy transport velocity of plasmonic waves of the system. Applying the mentioned boundary conditions the dispersion relation for the p-polarized SPP waves can be obtained as

$$\left(\frac{\epsilon_1}{\kappa_1} - \frac{\epsilon_2}{\kappa_2} - \frac{n_0 e^2/m_e\epsilon_0}{\omega^2 - \alpha k^2}\right)\left(\frac{\epsilon_3}{\kappa_3} - \frac{\epsilon_2}{\kappa_2} - \frac{n_0 e^2/m_e\epsilon_0}{\omega^2 - \alpha k^2}\right)$$

$$= \left(\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2} - \frac{n_0 e^2/m_e\epsilon_0}{\omega^2 - \alpha k^2}\right)\left(\frac{\epsilon_3}{\kappa_3} + \frac{\epsilon_2}{\kappa_2} - \frac{n_0 e^2/m_e\epsilon_0}{\omega^2 - \alpha k^2}\right)e^{2\kappa_2 d}. \tag{11}$$

From the above dispersion relation, we obtain two branches for ω defining the resonant frequencies of p-polarized SPP waves with one branch having higher frequency than the bare spectrum for monolayer graphene and other branch that is below the dispersion curve of monolayer graphene (see Section 3).

Also, the energy density u_j and the energy flow density S_{jx} on the jth graphene surface ($z = z_j$) can be written as [22]:

$$u_j = \frac{m_e n_0}{2}v_{jx}^2 + \frac{m_e}{2n_0}\alpha n_j^2, \tag{12}$$

$$S_{jx} = m_e \alpha n_j v_{jx}. \tag{13}$$

Eliminating v_{jx}, and n_j in Eqs. (12) and (13) by using Eqs. (1)–(3), the energy and energy flow densities on jth graphene surface may be expressed, in complex notation, as:

$$u_j^{layer}(z = z_j) = \frac{e^2 n_0}{4m_e}\frac{\omega^2 + \alpha k^2}{[\omega^2 - \alpha k^2]^2}E_{jx}E_{jx}^*, \tag{14}$$

$$S_{jx}^{layer}(z = z_j) = \frac{e^2 n_0}{2m_e}\frac{\alpha k\omega}{[\omega^2 - \alpha k^2]^2}E_{jx}E_{jx}^*. \tag{15}$$

In the insulators for $z \neq z_j$, we have:

$$u_\lambda(z) = \frac{1}{4}(\epsilon_0\epsilon_\lambda|\mathbf{E}_\lambda|^2 + \mu_0 H_{\lambda y}H_{\lambda y}^*), \tag{16}$$

$$S_{\lambda x}(z) = -\frac{1}{2}Re\,[E_{\lambda z}H_{\lambda y}^*], \tag{17}$$

where μ_0 is the permeability of free space and we put $\mu_\lambda = 1$. The total energy density and flow of energy associated with the surface waves are determined by an integration over z [22,26]. We have:

$$U = \int_{-\infty}^{z_1} u_1(z)dz + u_1^{layer}(z = z_1) + \int_{z_1}^{z_2} u_2(z)dz + u_2^{layer}(z = z_2) + \int_{z_2}^{+\infty} u_3(z)dz, \tag{18}$$

$$S_x = \int_{-\infty}^{z_1} S_{1x}(z)dz + S_{1x}^{layer}(z = z_1) + \int_{z_1}^{z_2} S_{2x}(z)dz + S_{2x}^{layer}(z = z_2) + \int_{z_2}^{+\infty} S_{3x}(z)dz. \tag{19}$$

After doing some algebra, the energy density per unit surface area and the energy flow per unit width are given by