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Orbital angular momentum of the vortex beams through a tilted lens

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ABSTRACT

Properties of orbital angular momentum (OAM) of a linearly polarized vortex beam traversing a tilted lens is investigated in detail, where the influences stemming from the tilted angle of the lens and the separation between the beam and the lens are mainly focused on. It was shown that such an optical system can invert the sign of the topological charge of the incident vortex beam. The possibility of reversing the sign of OAM was discussed and it was found that it only depends on the elements of the ray transfer matrix of this optical system. Finally, a simple experiment was carried out to verify the theoretical results.

1. Introduction

It has been well understood for quite some time that light carries both energy and momentum, and interaction of such light beams with matter will inevitably be accompanied by a transfer of momentum. The linear momentum is related to the light beam's wavelength ($\hbar k$) per photon and the angular momentum equals to an integer multiple of $\hbar$ per photon [1]. Angular momentum (AM) contains a spin component associated with the polarization of light and an orbital component in connection with the spatial distribution of light [2,3]. Orbital angular momentum (OAM) holds promise in various areas. For instances, it can exert forces and torques on both macroscopic and quantum objects [4–6]. Moreover, information can be encoded into higher dimensional OAM-alphabets for its application in free space communications systems [7], as well as being capable for optical imaging [8].

In less than three decades, investigations of OAM have immediately gained increasing attentions after it was recognized that Laguerre-Gaussian (LG) beams carrying a discrete OAM of $l\hbar$ per photon were readily realizable in the laboratory [9]. And the reader interested in the full and complete review of the historic development of the new area is referred to the articles including [10–13]. Following the discovery of OAM, the subsequent developments of this field have drawn ever closer to the work on optical vortices by Nye and Berry which began in the early 1970s [14], particularly for the LG beams. Initially, the concept of OAM was already long established in coherent beams, it did not take long for being a useful tool for descripting the partially coherent beams [15–17].

A significant activity in the early years of the development of OAM was directed at observing the transfer of OAM to matter [9,18]. And since the pioneering work that the transfer of OAM was first observed in an experiment of optical tweezers [4], much concern has been put to the subject. It was found that the transformation may be achieved by many astigmatic optical elements [19,20]. In this paper, we will discuss the behavior of OAM of a linearly polarized vortex beam during its propagation through a tilted lens. Specifically, we will concentrate on the effects of the tilted lens and the propagation distance on the transfer of OAM.

2. Theory

To study the properties of OAM along with propagation of light beams, let us begin by briefly reviewing the main results about the theory for following analysis. The AM density associated with the transverse electromagnetic field in vacuum is defined by [3]

$$\mathbf{M} = \varepsilon_0 \mathbf{r} \times (\mathbf{E} \times \mathbf{B}), \tag{1}$$

in terms of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$, where $\mathbf{r}$ is the position vector. As the primary intent of this paper is to explore the issue of the transfer of OAM, it is convenient to consider a linearly polarized beam as the incident light. On that basis, the intrinsic spin AM is zero and the total AM leaves only OAM. Within the paraxial approximation, it was readily shown that the time average of the real part of $\varepsilon_0 \mathbf{E} \times \mathbf{B}$, which is the linear momentum density of the beam, is given by [10]

$$\varepsilon_0 \langle \mathbf{E} \times \mathbf{B} \rangle = \frac{\varepsilon_0}{2}[(\mathbf{E}^* \times \mathbf{B}) + (\mathbf{E} \times \mathbf{B}^*)] = i\omega \frac{\varepsilon_0}{2}(u\nabla u^* - u^*\nabla u) + \omega k \varepsilon_0 |u|^2 \mathbf{z}, \tag{2}$$

for a beam of unit amplitude, where the asterisk denotes complex conjugation, $\mathbf{z}$ is the unit vector in the z direction.

Inserting Eq. (2) into (1), the time-averaged AM density of this mode has a component in the direction of propagation expressed as

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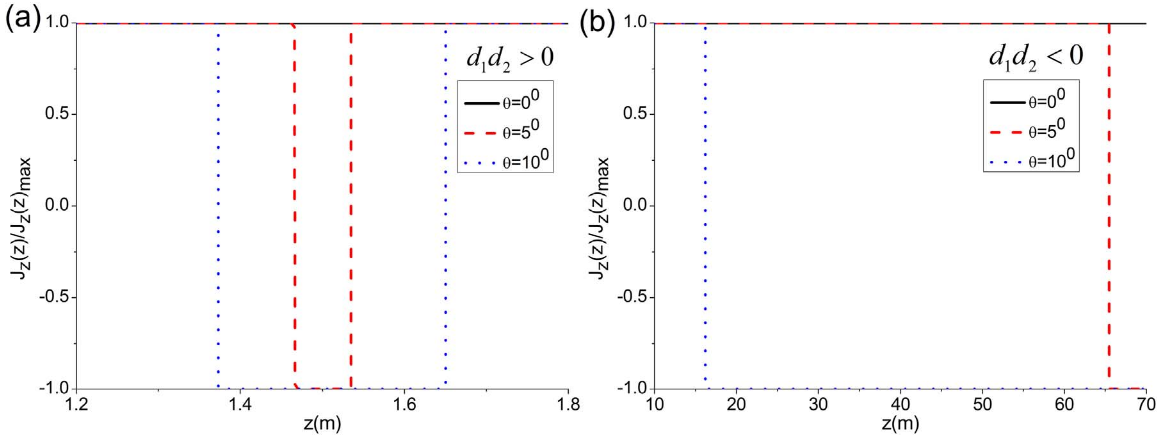

Fig. 1. Normalized OAM $\mathbf{J}_z(z)/\mathbf{J}_z(z)_{\max}$ of a linearly polarized vortex beam as a function of z for (a) $z_0 = 0.3$m, (b) $z_0 = 0.25$m.

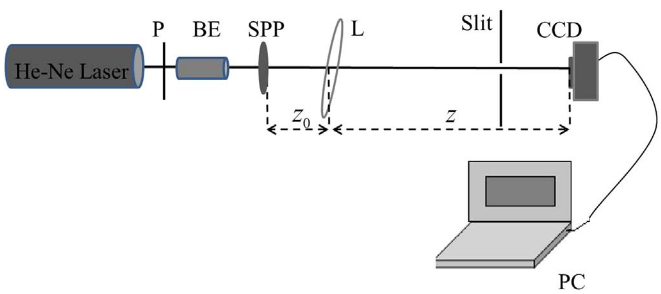

Fig. 2. Experimental setup for single-slit diffraction of a vortex beam through a tilted lens. P, polarizer; BE, beam expander; SPP, spiral phase plate; L, thin lens.

$$\mathbf{M}_z = i\omega\frac{\varepsilon_0}{2}\left[x\left(u\frac{\partial u^*}{\partial y} - u^*\frac{\partial u}{\partial y}\right) - y\left(u\frac{\partial u^*}{\partial x} - u^*\frac{\partial u}{\partial x}\right)\right]. \tag{3}$$

As a result, the OAM per unit length in the direction of propagation yields

$$\mathbf{J}_z = \int \mathbf{M}_z d\mathbf{r} = i\omega\frac{\varepsilon_0}{2}\iint\left[x\left(u\frac{\partial u^*}{\partial y} - u^*\frac{\partial u}{\partial y}\right) - y\left(u\frac{\partial u^*}{\partial x} - u^*\frac{\partial u}{\partial x}\right)\right]\mathrm{d}x\mathrm{d}y. \tag{4}$$

Consider a vortex embedded in a Gaussian beam at the waist plane, whose electric field has a form of

$$u^{(0)}(x', y') = (x' + iy')^m \exp\left[-\left(\frac{x'^2 + y'^2}{w_0^2}\right)\right], \tag{5}$$

where m and w_0 refer to the topological charge and characteristic beam size of the source, respectively.

Let such a vortex beam go through a tilted lens with a small angle θ about the y axis. The separations between the beam source and the thin lens, the lens and the out plane is z_0 and z, respectively. The overall ray transfer matrix of this optical system reads as [21,22]

$$T = T_z T_{lens} T_{z_0} = \begin{pmatrix} A & B \\ -C/f & D \end{pmatrix}. \tag{6}$$

where A, B, C and D are 2×2 diagonal matrices with diagonal elements given by a_j, b_j, c_j and d_j respectively. Explicitly, $c_1 = \sec\theta$, $c_2 = \cos\theta$, $a_j = 1 - zc_j/f$, $d_j = 1 - z_0c_j/f$, $b_j = z_0 + zd_j$, $j = 1, 2$.

Propagation of light beams through an optical system is commonly characterized by the extended Huygens-Fresnel integral. After taking the integrations over the whole beam source, the expression for the output field is derived

$$u(x, y) = \frac{(i/2)^{m+1}kw_1w_2}{(b_1b_2)^{1/2}}\gamma^m \exp[-(\beta_1x^2 + \beta_2y^2)]H_m[(\alpha_1x + i\alpha_2y)/\gamma], \tag{7}$$

during the calculations, the following notations were introduced

$$\frac{1}{w_j^2} = \frac{1}{w_0^2} + \frac{ika_j}{2b_j},\ \alpha_j = \frac{kw_j^2}{2b_j},\ \beta_j = \left(\frac{kw_j}{2b_j}\right)^2 + \frac{ikd_j}{2b_j},\ \gamma = (w_1^2 - w_2^2)^{1/2}. \tag{8}$$

The complexity of Hermite polynomial in Eq. (7) makes it more difficult to obtain a general expression. Thereby, for the sake of convenience and without loss of generality in our following calculations, we take the vortex beam with unit topological charge as example.

On substituting from Eq. (7) into (4), and performing the related integrations, we come to the final result for the analytical expression of the z component of OAM

$$\mathbf{J}_z = \frac{\varepsilon_0\omega\pi w_1w_1^*w_2w_2^*k^2[\alpha_1\alpha_2^*(\beta_1^* + \beta_2) + \alpha_1^*\alpha_2(\beta_1 + \beta_2^*)]}{8b_1b_2(\beta_1 + \beta_1^*)^{3/2}(\beta_2 + \beta_2^*)^{3/2}}. \tag{9}$$

Before proceeding further, we first consider the case when the lens is not tilted , i.e.$\theta = 0$. And hence, the contributions of j dependence of the parameters disappear, thus

$$c_1 = c_2,\ d_1 = d_2,\ b_1 = b_2 = b, \tag{10a}$$

$$w_1 = w_2 = w,\ \alpha_1 = \alpha_2 = \alpha,\ \beta_1 = \beta_2 = \beta,\ \gamma = 0. \tag{10b}$$

In such a manner, Eq. (9) reduces to a relatively simple form

$$\mathbf{J}_z(\theta = 0) = \frac{\varepsilon_0\omega\pi(ww^*)^2k^2\alpha\alpha^*}{4b^2(\beta + \beta^*)^2}. \tag{11}$$

As expected, OAM of light beams will hold a positive constant when traveling through a non-tilted lens, consistent with the fact of conservation of topological charge.

We now proceed with the analysis concerning our subject, Eq. (9) allows us to recognize the nature of the transfer of OAM of a linearly polarized vortex beam after propagating through a tilted lens. One may readily see from Eq. (9) that the factor b_1b_2 plays an important role in determining the sign of OAM. In the case of $b_1b_2 > 0$, $\mathbf{J}_z > 0$ satisfies, when $b_1b_2 < 0$, $\mathbf{J}_z < 0$. This comes to an interesting insight that whether the sign of OAM changes only depends on the one of the elements of the transfer matrix, irrelevant to the topological charge.

To have a more detail analysis on the probability of change of the sign of the topological charge, we start by discussing the factor b_1b_2, which is a parabolic function of propagation distance z written as

$$b_1b_2 = d_1d_2z^2 + (d_1 + d_2)z_0z + z_0^2, \tag{12}$$

two roots exist in the above equation $z_{1,2} = (-z_0/d_2, -z_0/d_1)$.

In view of Eq. (12), there are two ways to obtain the negative b_1b_2. For the first situation, if $d_1d_2 > 0$, the appropriate propagation distance is governed by $z_1 < z < z_2$. Alternatively, if $z_1 < z < z_2$, the inequality $z < z_1$ or $z > z_2$ need to be established.