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Intensity and coherence modulation of scattering of multi-Gaussian Schell-model beams from a quasi-homogeneous medium with adjustable boundary

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ABSTRACT

We investigate the intensity and coherence modulation of the far-zone scattered field generated by scattering of multi-Gaussian Schell-model beams from a quasi-homogeneous medium with adjustable boundary. The influences of the scatterer parameters and the beam parameters on the scattered field have been illustrated in detail by numerical simulations. The results indicate that the correlation length of the medium is the key to determine whether the beam or the scatterer plays a leading role in the modulation of the scattered field. When the value of the correlation length of the medium is on the order of the wavelength, the modulation is dominated by the scatterer, while in the case of the value being much larger than the wavelength, the scatterer-led modulation is transformed into the beam-led modulation. Besides, during the transformation process there is a critical value of the correlation length of the medium for the beam to dominate the modulation, and the correlation width of the beam has an impact on the critical value.

1. Introduction

The light scattering is of great importance in physics, astronomy, meteorology, biology, and in other fields, because the information about the unknown object can be obtained from the knowledge of the scattered field. There are many potential applications of the light scattering in areas like remote sensing, target detection, medical diagnosis, and so on. In the past few decades, continuous efforts have been devoted to the studies of the scattering from different kinds of media, including continuous media and particle collections [1–8]. It has been shown that the characteristics of the scattered field are closely related to the properties of the media, which may provide a potential method to reconstruct the structural features of the media from the measurements of the scattered field. For example, the correlation-induced spectral change and the cross-spectral density function of the far-zone scattered field have been used to determine the correlation function of the scattering potential of random media, respectively [9,10]. Besides, the inverse scattering of a collection of particles has also been addressed in [11]. Since the ultimate aim of investigating the characteristics of the scattered field is to obtain the information about the object, it is of greater significance to explore the light scattering from more realistic media. In the previous studies, the most commonly used media are the soft-edge (Gaussian) media [12] and the hard-edge media [13], the refractive index distributions of which are modeled by the Gaussian function and the sign function, respectively. Although being mathematically convenient, the two models are only the idealizations. Recently, Sahin et al. have introduced a more realistic medium with variable rates of change in the index of refraction at the edge, and the two idealized media mentioned above are the two limiting cases of this medium [14]. Subsequently, the media with adjustable boundaries have been extended to more specific ones that have ellipsoid-, cylinder-, and parallelepiped-like shapes [15]. It will have more practical value to study the scattering of light waves from the more realistic medium with adjustable boundary.

On the other hand, since the development of the laser in the 1960s, lots of scattering experiments have been and are being performed with various laser beams rather than plane waves. Therefore, compared with the scattering of plane waves, the scattering of various laser beams is more valuable for the practice, and it has attracted a lot of attention in recent years. For instance, Dijk et al. discussed the effects of spatial coherence of the incident beam on the intensity of the far-zone scattered field [13]. Zhang and Zhao have carried out a series of research on scattering of ordinary Gaussian Schell-model beams, multi-Gaussian Schell-model beams and rectangular Gaussian Schell-model beams [16–18]. In addition, a new method has been proposed for the determination of the correlation function of the scattering potential of random media by using the Gaussian vortex beams on scattering [19]. These studies about the scattering of laser beams and the inverse problems have all revealed that besides the medium, the incident beam also has an important impact on the scattered field.

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Apparently, when a laser beam is incident on a medium, the modulation of the scattered field consists of two components: one from the beam, and the other from the medium. However, there are some important questions that have not been addressed before: which factors could influence the proportion of the beam role and that of the scatterer role in the modulation; under what circumstance would the modulation be dominated by the beam or the scatterer; how does the same parameter affect the scattered field under different circumstances, etc. The solution of these problems will help us to understand the deeper nature of the scattering of laser beams from a medium, and thus promote the development of the inverse problem of the scattering.

In this paper, we consider a relatively realistic case in which the multi-Gaussian Schell-model (MGSM) beam is scattered by a quasi-homogeneous (QH) medium with adjustable boundary and aim to solve the problems raised above. First, we derive the expressions for the spectral density and the spectral degree of coherence of the far-zone scattered field. Then, the intensity and coherence modulation of the scattered field are illustrated by numerical simulations and the effects of the scatterer parameters and the beam parameters on the scattered field are discussed in detail. Finally, we conclude our results which may have applications on the inverse scattering.

2. Theory

Let us consider a scalar wave propagating in a direction specified by a unit vector $\mathbf{u_0}$, incident on a scatterer occupying a finite domain D. Assume that the scalar wave is not a plane wave but is of a more general form. Such a field may be represented as an angular spectrum of plane waves propagating into the $z \geq 0$ half-space as [20]

$$U^{(i)}(\mathbf{r}', \omega) = \int_{|\mathbf{u_{0\perp}}|^2 \leq 1} a(\mathbf{u_{0\perp}}, \omega)\exp(ik\mathbf{u_0}\cdot\mathbf{r}')d^2u_{0\perp}, \tag{1}$$

where $\mathbf{u_{0\perp}} = (u_{0x}, u_{0y})$ is a two-dimensional vector, $\mathbf{r}'$ denotes the position vector in the incident field, ω denotes the angular frequency, and $k = 2\pi/\lambda$ is the wave number with λ being the wavelength.

Within the accuracy of the first-order Born approximation, the scattered field can be expressed as [12]

$$U^{(s)}(\mathbf{r}, \omega) = \int_D F(\mathbf{r}', \omega)U^{(i)}(\mathbf{r}', \omega)G(|\mathbf{r} - \mathbf{r}'|, \omega)d^3r', \tag{2}$$

where $\mathbf{r}$ is the position vector in the scattered field, $F(\mathbf{r}', \omega)$ denotes the scattering potential function of the scatterer, and $G(|\mathbf{r} - \mathbf{r}'|, \omega)$ is the free-space Green function. Generally, the actual measurements are taken far away from the scatterer, so we concentrate on the far-zone properties of the scattered field. In this case, the Green function can make the approximation as [21]

$$G(|\mathbf{r} - \mathbf{r}'|, \omega) \cong \frac{\exp(ikr)}{r}\exp(-ik\mathbf{u}\cdot\mathbf{r}'), \tag{3}$$

where $\mathbf{u}$ is a unit vector representing the scattering direction and $\mathbf{r} = r\mathbf{u}$.

On substituting from Eq. (1) into Eq. (2) and using Eq. (3), we obtain the expression for the far-zone scattered field as

$$U^{(s)}(\mathbf{r}, \omega) = \frac{\exp(ikr)}{r}\int_{|\mathbf{u_{0\perp}}|^2 \leq 1} a(\mathbf{u_{0\perp}}, \omega)f(\mathbf{u}, \mathbf{u_0}, \omega)d^2u_{0\perp}, \tag{4}$$

where

$$f(\mathbf{u}, \mathbf{u_0}, \omega) = \int_D F(\mathbf{r}', \omega)\exp[-ik(\mathbf{u} - \mathbf{u_0})\cdot\mathbf{r}']d^3r' \tag{5}$$

is the scattering amplitude.

We consider the case where the incident light wave is partially coherent and the scatterer is a random medium, then the statistical properties of the scattered field can be characterized by the cross-spectral density function (CSDF), which is defined by [12]

$$W^{(s)}(r\mathbf{u_1}, r\mathbf{u_2}, \omega) = \langle U^{(s)*}(r\mathbf{u_1}, \omega)U^{(s)}(r\mathbf{u_2}, \omega)\rangle, \tag{6}$$

where the angle brackets denote the statistical ensemble average in the space-frequency domain and the asterisk denotes the complex conjugate. On substituting from Eq. (4) into Eq. (6), the CSDF of the far-zone scattered field can be expressed as

$$W^{(s)}(r\mathbf{u_1}, r\mathbf{u_2}, \omega) = \frac{1}{r^2}\int_{|\mathbf{u_{01\perp}}|^2 \leq 1}\int_{|\mathbf{u_{02\perp}}|^2 \leq 1}\langle a^*(\mathbf{u_{01\perp}}, \omega)a(\mathbf{u_{02\perp}}, \omega)\rangle \times \langle f^*(\mathbf{u_1}, \mathbf{u}_{01}, \omega)f(\mathbf{u_2}, \mathbf{u}_{02}, \omega)\rangle d^2u_{01\perp}d^2u_{02\perp}. \tag{7}$$

From Eq. (7) we can find that the kernels in the integral consist of two parts: one is related to the incident light wave and the other is determined by the scatterer. The former is the so-called angular correlation function of the stochastic incident field, and it can be given by the formula as a four-dimensional Fourier transform of the CSDF in the light source plane as [20]

$$\begin{aligned}A(\mathbf{u_{01\perp}}, \mathbf{u_{02\perp}}, \omega) &= \langle a^*(\mathbf{u_{01\perp}}, \omega)a(\mathbf{u_{02\perp}}, \omega)\rangle\\ &= \left(\frac{k}{2\pi}\right)^4\iint_{-\infty}^{+\infty} W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)\exp[-ik(\mathbf{u_{02\perp}}\cdot\boldsymbol{\rho}_2 - \mathbf{u_{01\perp}}\cdot\boldsymbol{\rho}_1)]d^2\rho_1 d^2\rho_2,\end{aligned} \tag{8}$$

where $W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)$ is the CSDF in the light source plane, $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are two-dimensional position vectors in the plane.

Besides, the other part associated with the scatterer in Eq. (7) can be expressed as a six-dimensional Fourier transform of $C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$ with $C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = \langle F^*(\mathbf{r}'_1, \omega)F(\mathbf{r}'_2, \omega)\rangle$ being the correlation function of the scattering potential of the random medium, and it has a form of

$$\begin{aligned}\widetilde{C}_F(-\mathbf{K_1}, \mathbf{K_2}, \omega) &= \langle f^*(\mathbf{u_1}, \mathbf{u}_{01}, \omega)f(\mathbf{u_2}, \mathbf{u}_{02}, \omega)\rangle\\ &= \int_D\int_D C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega)\exp[-i(\mathbf{K_2}\cdot\mathbf{r}'_2 - \mathbf{K_1}\cdot\mathbf{r}'_1)]d^3r'_1 d^3\mathbf{r}'_2,\end{aligned} \tag{9}$$

where $\mathbf{K_1} = k(\mathbf{u_1} - \mathbf{u}_{01})$ and $\mathbf{K_2} = k(\mathbf{u_2} - \mathbf{u}_{02})$.

We assume that the medium is illuminated by the MGSM beams, which are an important class of partially coherent laser beams. For such beams the CSDF in the source plane is given by [22]

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \frac{1}{C_0}\exp\left(-\frac{\boldsymbol{\rho}_1^2 + \boldsymbol{\rho}_2^2}{4\sigma^2}\right)\sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}\exp\left[-\frac{|\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1|^2}{2m\delta^2}\right], \tag{10}$$

where $C_0 = \sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}$ is the normalization factor, $\binom{M}{m}$ denote binomial coefficients, σ is the transverse beam width, and δ is the correlation width of the source. On substituting from Eq. (10) into Eq. (8), we obtain the angular correlation function of the MGSM beam expressed as

$$A(\mathbf{u_{01\perp}}, \mathbf{u_{02\perp}}, \omega) = \frac{1}{C_0}\left(\frac{k^2\sigma}{2\pi}\right)^2\exp\left[-\frac{k^2\sigma^2}{2}(\mathbf{u_{01\perp}} - \mathbf{u_{02\perp}})^2\right] \times \sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}\sigma_{eff}^2\exp\left[-\frac{k^2\sigma_{eff}^2}{8}(\mathbf{u_{01\perp}} + \mathbf{u_{02\perp}})^2\right], \tag{11}$$

where

$$\frac{1}{\sigma_{eff}^2} = \frac{1}{4\sigma^2} + \frac{1}{m\delta^2}. \tag{12}$$

In the light scattering theory, the soft-edge (Gaussian) model and the hard-edge model are the most commonly used models for scattering media. Take a deterministic medium, for example. The soft-edge deterministic medium has a Gaussian distribution of its scattering potential, which is given by

$$F^{(G)}(\mathbf{r}', \omega) = \exp\left(-\frac{\mathbf{r}'^2}{2\sigma_I^2}\right), \tag{13}$$