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# An accelerated hologram calculation using the wavefront recording plane method and wavelet transform

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ABSTRACT

Fast hologram calculation methods are critical in real-time holography applications such as three-dimensional (3D) displays. We recently proposed a wavelet transform-based hologram calculation called WASABI. Even though WASABI can decrease the calculation time of a hologram from a point cloud, it increases the calculation time with increasing propagation distance. We also proposed a wavefront recoding plane (WRP) method. This is a two-step fast hologram calculation in which the first step calculates the superposition of light waves emitted from a point cloud in a virtual plane, and the second step performs a diffraction calculation from the virtual plane to the hologram plane. A drawback of the WRP method is in the first step when the point cloud has a large number of object points and/or a long distribution in the depth direction. In this paper, we propose a method combining WASABI and the WRP method in which the drawbacks of each can be complementarily solved. Using a consumer CPU, the proposed method succeeded in performing a hologram calculation with $2048 \times 2048$ pixels from a 3D object with one million points in approximately 0.4 s.

## 1. Introduction

Due to the heavy calculation amounts of holograms, fast hologram calculation methods are essential in real-time holography applications such as three-dimensional (3D) displays [1]. Multiple algorithms have been proposed: point cloud methods [2–8], polygon methods [9–11], holographic stereograms [12–15], and RGB-D [16,17]. The details of these methods can be found in Refs. [18,19].

In point cloud methods, a 3D object is expressed as the aggregation of object points, and the light waves emitted from these object points are superposed in the hologram plane. The superposition is the most time-consuming part of the hologram calculation. To accelerate the superposition, we recently proposed a wavelet transform-based hologram calculation, which is referred to as the WAvelet ShrinkAge-Based superpositIon (WASABI) [20]. WASABI accelerates a hologram calculation by representing the object light with a few wavelet coefficients. Even though the WASABI can considerably decrease the calculation time of a hologram, it increases the calculation time with increasing propagation distance.

We also proposed a wavefront recoding plane (WRP) method [21]. Subsequently, various improved methods for the WRP method have been proposed [22–35]. The WRP method is a two-step fast hologram calculation in which the first step calculates the superposition of light waves emitted from object points in a virtual plane, and the second step performs diffraction calculation from the virtual plane to the hologram plane. A drawback of the WRP method is in the first step when the point cloud has a large number of object points and/or a long distribution in the depth direction.

Several methods to alleviate this problem have been proposed. For example, multiple WRP methods introduce multiple virtual planes to split the long distribution [25,26,28,30], and the placement of the virtual planes can be optimized [35]. The tilted WRP method [27] decreases the calculation cost of the first step by using a tilted diffraction calculation [9], and the stretched WRP method [31] decreases the calculation cost using a non-uniform sampled diffraction calculation [36]. More recently, a WRP method using the sparse fast Fourier transform (FFT) has been proposed to accelerate the second step in the WRP method [33].

In this paper, we propose a method combining WASABI and the WRP method in which the drawbacks of each can be complementarily solved. That is, the calculation cost of the first step in the WRP method is accelerated using WASABI, resulting in a decrease in the total calculation time. Using a consumer CPU, the proposed method succeeded in completing a hologram calculation with $2048 \times 2048$

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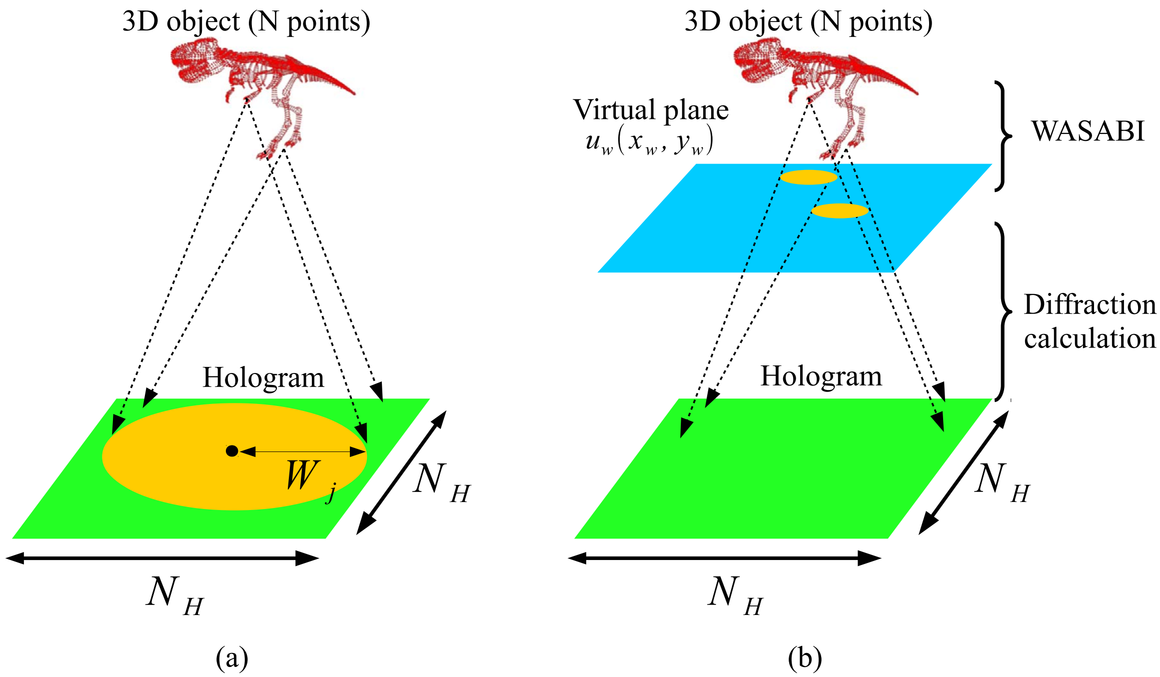


**Fig. 1.** Hologram calculation for (a) a conventional hologram calculation and (b) the proposed method.

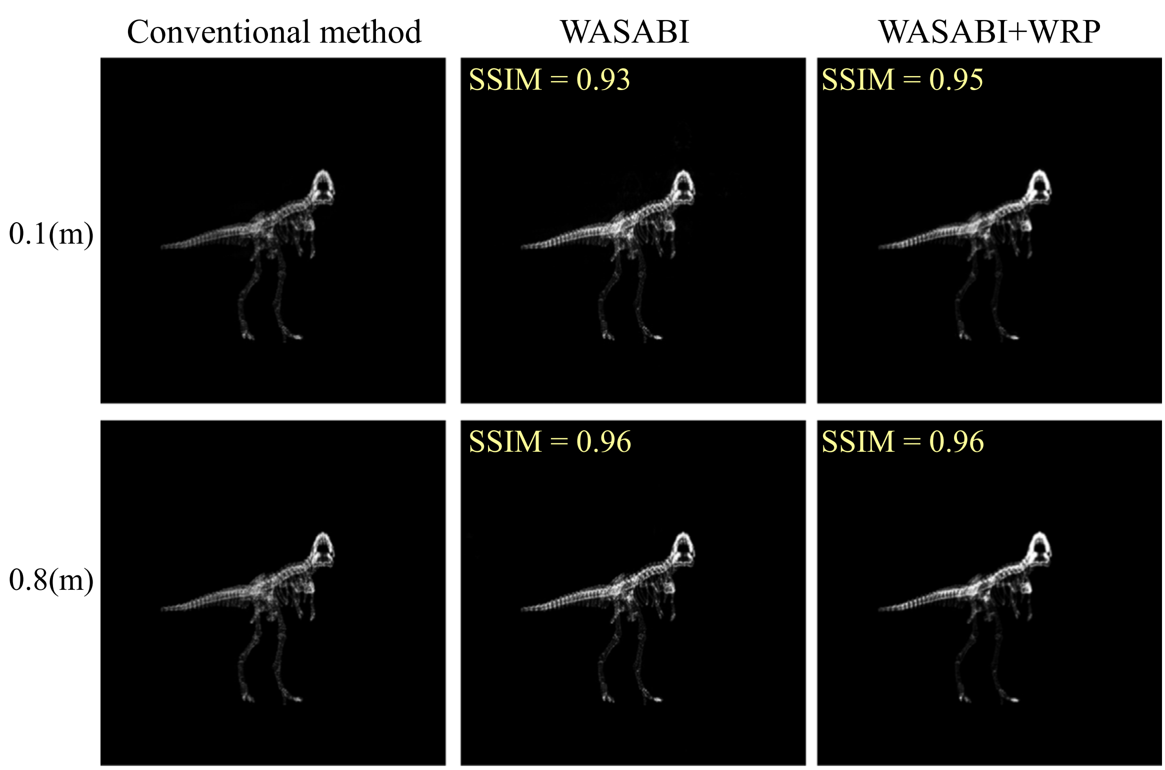


**Fig. 2.** Reconstructed images of a dinosaur composed of 11,646 object points at the two different distances of 0.1 m and 0.8 m.

pixels from a 3D object with one million points in approximately 0.4 s Section 2 describes the proposed method, and Section 3 shows the effectiveness of the proposed method. Section 4 concludes this study.

## 2. Proposed method

The proposed method combines WASABI and the WRP method to further accelerate the hologram calculation at long propagation distances.

Fig. 1(a) shows a conventional hologram calculation from the 3D object points. The light waves emitted from these object points are superimposed in the hologram plane using

$$u(x_h, y_h) = \sum_j^N a_j \exp\left(i\frac{2\pi}{\lambda}r_{hj}\right) = \sum_j^N a_j u_{z_j}(x_h - x_j, y_h - y_j), \tag{1}$$

where $i$ is the imaginary unit number, $N$ is the number of object points, $(x_h, y_h)$ are the coordinates of the hologram plane, $(x_j, y_j, z_j)$ and $a_j$ are the coordinates and amplitude of the $j$-th object point, respectively, $\lambda$ is the wavelength, and $u_{z_j}$ is the point spread function (PSF) at a distance $z_j$.