Optics Communications 393 (2017) 1–4

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

# Impact of linear coupling on nonlinear phase noise in two-core fibers

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ARTICLE INFO

*Keywords:*
Nonlinear phase noise
Optical pulses
Two-core fiber
Linear coupling
Self-phase modulation
Kerr nonlinearity

ABSTRACT

We study the effect of linear coupling on nonlinear phase noise in two-core fibers (TCFs). Considering the elementary TCF switching process, we demonstrate reduction of nonlinear phase noise in the propagation of optically amplified pulses through TCFs. Our results show that the reduction occurs just when the first maximum transfer of optical power is carried out between the TCF cores.

## 1. Introduction

While single-core fiber (SCF) networks are gradually approaching their theoretical capacity limits [1], new types of fibers such as multi-core fibers (MCFs) have been the focus of worldwide research to overcome critical transmission capacity barriers and boost the capability of modern optical fiber communication (OFC) systems [2]. This is so because their signal-carrying capacity is many times greater than that of traditional SCFs. In MCFs, a number of cores is introduced at different positions, in a preselected array, in the fiber cross-section and within a single cladding. In the most typical case, each core accommodates a single guided mode, depending on the size of the MCF cores and some other design parameters, but there may be a number of guided modes [2,3]. MCFs have attracted attention for enhancing the capacity of OFC systems through space-division multiplexing (SDM) [3], and several experiments have demonstrated high-speed data transmission over MCFs at rates that approach a petabit per second [4–6]. However, when we consider a MCF as a medium for SDM transmission, the linear coupling between cores is a key feature to be considered and analyzed [7,8]. Therefore, before MCFs become a practical solution, the linear coupling effect in MCFs needs to be studied theoretically and experimentally because co-propagating modes in such fibers can interact both linearly and nonlinearly [8].

Essential to SCF-OFC systems is the use of optical amplifiers to compensate losses due to material absorption and scattering [9]. However, the process of optical amplification introduces amplified spontaneous emission (ASE) noise in the transmitted optical signal fields [9]. The field amplitude fluctuations caused by the ASE noise are translated into phase fluctuations, nonlinear phase noise, because of the SCF nonlinearity [10]. Specifically, the SCF nonlinearity refers to the phenomenon of self-phase modulation (SPM), which originates from the variation of the refractive index of the guided medium dependent of the launched power. SPM causes the signal field to change its own phase through a nonlinear phase shift [11]. The nonlinear phase noise is detrimental in SCF-OFC systems based on modulation schemes in which the information is encoded in the optical phase, leading to bit errors in transmission and limiting the regeneration transmission distance [10]. Therefore, it is expected that this limitation is extended to MCFs described as a collection of coupled SCFs.

In this work, we describe how the linear coupling in a two-core fiber (TCF) affects the nonlinear phase noise of initial amplified optical pulses. For this purpose, the analysis of unchirped Gaussian input pulses and the elementary TCF switching process were sufficient and useful to obtain considerable and comprehensive results. We focus on a TCF, which is the simplest, but most important, setup of MCFs to obtain physical insight of the effect of linear coupling on nonlinear phase noise. We consider the TCF system because it is the workhorse of the field of MCFs for exploring those aspects that more complex systems as MCFs would not be able to see. We neglect the effect of the initial chirp because the performance of the TCF changes minimally when chirped Gaussian input pulses are considered.

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http://dx.doi.org/10.1016/j.optcom.2017.02.020
Received 18 November 2016; Received in revised form 20 January 2017; Accepted 7 February 2017
Available online 10 February 2017

## 2. Nonlinear coupled-mode equations and their analytic solution

Let us begin by considering a symmetric TCF with identical fiber cores of radius $a$, separated by a distance $d$ between their centers. The TCF consists of a launch fiber core coupled with an unlaunch fiber core. The launch fiber core is a fiber core that is initially pumped with an optical pulse and the unlaunch fiber core is not initially pumped. We are considering the simplest situation in which a single-input pulse is launched into one TCF core such that it excites a single transverse electric polarization mode of that TCF core. To neglect the dispersive effects and focus in the analysis of the effect of linear coupling on nonlinear phase noise, we consider the propagation of picosecond pulses through the TCF. Introducing the dispersion length in the usual way as $L_D = T_0^2/|\beta_2|$ [11], where $T_0$ is the pulse width and $\beta_2$ is the group-velocity dispersion parameter, the nonlinear coupled-mode equations for optical pulses, wide enough that $L_D$ is much larger than the TCF length $L$, propagating in a lossless TCF are given by [12].

$$\frac{dU_1}{dz} = i\kappa U_2 + i\frac{1}{L_{NL}}(|U_1|^2 + b|U_2|^2)U_1, \tag{1a}$$

$$\frac{dU_2}{dz} = i\kappa U_1 + i\frac{1}{L_{NL}}(|U_2|^2 + b|U_1|^2)U_2, \tag{1b}$$

where $U_1(z, T)$ and $U_2(z, T)$ are the normalized slowly varying amplitudes, $L_{NL} = (\gamma P_0)^{-1}$ is the nonlinear length, $P_0$ is the peak power of the input optical pulse, and $\gamma$ is the nonlinear parameter of each TCF core. The factor $i = \sqrt{-1}$ represents the imaginary unit. Here we are considering that the modes in the TCF cores are perfectly matched. The time $T = t - z/v_g$ is measured in a frame of reference moving with the optical pulse at the group velocity $v_g$, and $z$ is the standard notation for the propagation distance. The coupling coefficient $\kappa$ and cross-phase modulation (XPM) parameter $b$, which represent the linear and nonlinear coupling coefficients, respectively, depend on the distance $d$ between the TCF cores, and are given by [13].

$$\kappa = \sqrt{\frac{\Delta\pi a}{Wd}}\frac{U^2}{V^3}\frac{\exp(-Wd/a)}{aK_1^2(W)}, \tag{2}$$

$$b = 2\exp(-d^2/w^2), \tag{3}$$

where $\Delta$ is the relative core-cladding index difference, $K_1(W)$ is the modified Bessel function of the second kind of order 1, $V$ is the waveguide parameter, $U$ and $W$ are the normalized transverse wave-numbers, and

$$w = a(0.65 + 1.619V^{-3/2} + 2.879V^{-6}). \tag{4}$$

is the width parameter [11]. To obtain Eq. (3), we have used the definition of the XPM parameter [12] and the Gaussian approximation for the spatial distribution of the fundamental mode of each TCF core. Although the value of $b$ is quite weak and could be zero, we consider $b$ in our calculations for more general situations where $b$ can not be neglected.

The complex slowly varying amplitude $U_j(z, T)$ of the $j$th TCF core (with $j$=1,2) has an instantaneous optical power of $P_j(z, T)$. Both optical powers, $P_1(z, T)$ and $P_2(z, T)$, can vary along $L$ because of the overlap of the two modes. Using amplitude and phase terms, we can represent to the slowly varying amplitudes as [14]

$$U_j(z, T) = i^{(j-1)}\frac{U_1(0, T)}{\sqrt{2}}\sqrt{1-(-1)^j u(z, T)} \times \exp\{i[\phi_{NLj}(z, T)]\}, \tag{5}$$

where $u(z, T)$ is a real function and $\phi_{NLj}(z, T)$ is the nonlinear phase shift of a picosecond pulse propagating inside the $j$th TCF core. Here we are considering the specific case in which all the input power is initially launched into one TCF core (i.e., $U_2(0, T) = 0$ at any time). Therefore, initial conditions are such that $u(0, T) = 1$ and $\phi_{NLj}(0, T) = 0$ for each time element. Using the above relation [Eq. (5)] and elliptic integrals, we can analytically solve Eqs. (1a) and (1b). Two kind of solutions of Eqs. (1a) and (1b), that satisfy the initial conditions, are available [14].

Solution 1:

$$u(z, T) = \mathrm{cn}(2\kappa z|m_1), \tag{6a}$$

$$\phi_{NL1}(z, T) = \frac{z}{4L_{NL}}|U_1(0, T)|^2(b+3) - \frac{(-1)^{k_1}}{2}\arccos\{\mathrm{dn}(2\kappa z|m_1)\}, \tag{6b}$$

$$\phi_{NL2}(z, T) = \frac{z}{4L_{NL}}|U_1(0, T)|^2(b+3) + \frac{(-1)^{k_1}}{2}\arccos\{\mathrm{dn}(2\kappa z|m_1)\}, \tag{6c}$$

where $k_1$ is an integer that depends on the argument and period of the Jacobian elliptic function $\mathrm{dn}(2\kappa z|m_1)$. For this case, the modulus $m_1(T) = [(b-1)|U_1(0, T)|^2/4\kappa L_{NL}]^2$ is a time dependent parameter.

Solution 2:

$$u(z, T) = \mathrm{dn}\left(\frac{2\kappa z}{\sqrt{m_2}}|m_2\right), \tag{7a}$$

$$\phi_{NL1}(z, T) = \frac{z}{4L_{NL}}|U_1(0, T)|^2(b+3) - \frac{(-1)^{k_2}}{2}\arcsin\left\{\mathrm{sn}\left[\frac{z}{2L_{NL}}|U_1(0, T)|^2(b-1)|m_2\right]\right\} - \frac{\pi k_2}{2}, \tag{7b}$$

$$\phi_{NL2}(z, T) = \frac{z}{4L_{NL}}|U_1(0, T)|^2(b+3) + \frac{(-1)^{k_2}}{2}\arcsin\left\{\mathrm{sn}\left[\frac{z}{2L_{NL}}|U_1(0, T)|^2(b-1)|m_2\right]\right\} + \frac{\pi k_2}{2}, \tag{7c}$$

where $k_2$ is defined by the argument and period of the Jacobian elliptic function $\mathrm{sn}[z|U_1(0, T)|^2(b-1)/(2L_{NL})|m_2]$. In this case, the modulus $m_2(T) = [4\kappa L_{NL}/(b-1)|U_1(0, T)|^2]^2$ is a time dependent parameter as well. Note that $m_1 = 1/m_2$.

The values of $\kappa$ and $P_0$ for which $m_1 = 1$ and $m_2 = 1$ are defined as the critical coupling coefficient $\kappa_c$ and critical power $P_c$, respectively. These two parameters play an important role because they define a boundary between the two possible solutions (Solutions 1 and 2) of Eqs. (1a) and (1b). On one hand, Solution 1, which defines the Linear Regime of a TCF, applies when $\kappa_c \le \kappa < \infty$ or/and the input peak power is low ($0 < P_0 \le P_c$). It is well-known that in this case, both $P_1(z, T)$ and $P_2(z, T)$ vary sinusoidally with $z$ for any directional coupler. On the other hand, Solution 2, which defines the Nonlinear Regime of a TCF, applies when $0 < \kappa \le \kappa_c$ or/and the input peak power is high ($P_c \le P_0 < \infty$); resulting into a reduction in the power exchange efficiency, i.e., the linear coupling effect is reduced and the optical field remains primarily in the launch TCF core. In general, Solutions 1 and 2 form a complete solution of the system for all values of $\kappa$ and $P_0$.

## 3. Effect of linear coupling on nonlinear phase noise

Let us now consider an optical signal launched into one TCF core immediately after its amplification by an in-line optical amplifier. The total electric field envelope after the amplifier can be expressed mathematically as follows

$$U_1(0, T) = s_1(0, T) + n_1(0, T), \tag{8}$$

where $s_1(0, T)$ is the input signal field given by

$$s_1(0, T) = \exp\left(-\frac{1}{2}\frac{T^2}{T_0^2}\right), \tag{9}$$

and $n_1(0, T)$ is the time-dependent noise field added by the amplifier due to spontaneous emission. Typically $n_1(0, T)$ is much smaller than $s_1(0, T)$ at any time. As a result, the electric field envelope $U_1(0, T)$ is randomly varying in time, and its propagation through a TCF becomes a random process. The complex amplitude noise $n_1(0, T)$, which has both the in-phase and quadrature components, is a statistically