Optics Communications 393 (2017) 191–194

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Magnetic field control of spontaneous emission in a multi-fields driven four-level atomic system

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ARTICLE INFO

Keywords:
Spontaneous emission
Quantum interference
Coupled fields

ABSTRACT

The magnetic field control of spontaneous emission in a four-level atomic system driven by three fields is studied. Using the iterative method, an analytical expression of the spontaneous emission spectrum is obtained. Given that the dipoles of the two transitions from two upper levels to a common lower level are orthogonal, we discuss the influence of the magnetic field which is used to couple the two upper levels on the spontaneous emission spectrum, and find a conversion between destructive and constructive quantum interference by changing Larmor frequency. When introducing the interference between the two transitions, the extremely narrow spectral line is obtained in the emission spectrum, which has a potential application in the high spatial resolution sub-wavelength atom localization.

1. Introduction

The control of spontaneous emission via atomic coherence or quantum interference has attracted much attention for many years [1–7] due to its potential applications in short-wavelength lasers, atom localization, and quantum information processing. An efficient way to achieve control is to drive the atom with external fields. The quantum interference between spontaneous decay processes from two levels which are driven to another level by a coherent field was investigated by Zhu and Scully [1]. The destructive interference leads to the elimination of a spectral line and cancellation of spontaneous emission in the steady state. For the same four-level atom, Paspalakis and Knight [2] studied the phase control of spontaneous emission considering the atom driven by two laser fields with equal frequencies. Effects such as extreme spectral narrowing and total cancellation of fluorescence decay were obtained. Replacing the two laser fields by three coherent fields, Ghafoor et al.[3] studied the amplitude and phase control of spontaneous emission under three driving fields which form a loop. A wide variety of spectral behaviors can be obtained by controlling the phase and amplitude of the driving fields considering the orthogonal dipoles for two transitions from two upper levels to a common lower level.

In the four-level atomic system with three external fields, the two dipoles associated with the transitions from two upper levels to a common lower level are orthogonal, which means that there is no quantum interference between the two transitions. The question arises, therefore, how the quantum interference between the two transitions could be introduced, and how would it influence spontaneous emission? In this paper, for a four-level atom driven by three external fields with a loop, we introduce the quantum interference between the two transitions from two upper levels to a lower level into the controlling of spontaneous emission. Motivated by the magnetic field induced coherence effect [8,9], when a coherent magnetic field is used to couple the two upper levels we can obtain the parallel dipoles for the two transitions. The influence of magnetic field and the quantum interference effect between the two transitions on spontaneous emission spectrum is discussed.

2. Theoretical model and equations

We consider a system of a four-level atom interacting with three coherent fields as shown in Fig. 1(a). These fields resonantly couple the transitions, and form a closed loop. The ground level $|0\rangle$ is coupled to the excited levels $|1\rangle$ and $|2\rangle$ by two laser field Ω_1 and Ω_2, where $\Omega_2 = |\Omega_2|e^{i\phi}$, ϕ is the collective phase of the three fields. The transition $|2\rangle \rightarrow |1\rangle$ is coupled by a coherent magnetic field with the Larmor frequency $\Omega_B = \vec{\mu}_{21} \cdot \vec{B}/\hbar$, where $\vec{\mu}_{21}$ is the magnetic dipole moment. Therefore, the transition $|2\rangle \rightarrow |1\rangle$ is electric dipole forbidden while magnetic dipole allowed. The upper levels $|1\rangle$ and $|2\rangle$ decay to the lower level $|c\rangle$ by interacting with the vacuum field modes. According to dressed state theory, the levels $|0\rangle$,$|1\rangle$, $|2\rangle$ and the corresponding fields can be replaced by two groups of dressed levels $|+\rangle_1$,$|0\rangle_1$,$|-\rangle_1$ and $|+\rangle_2$,$|0\rangle_2$,$|-\rangle_2$. The level scheme in terms of dressed states is given in Fig. 1(b).

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http://dx.doi.org/10.1016/j.optcom.2017.02.054
Received 18 December 2016; Received in revised form 17 February 2017; Accepted 20 February 2017
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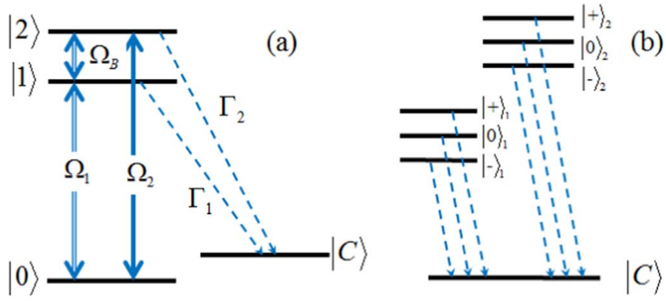

Fig. 1. Schematic diagrams, (a) four-level atomic system driven by three fields, (b) the atomic energy levels in the dressed state picture.

Under the rotating-wave approximation, the interaction Hamiltonian for the system reads($\hbar = 1$)

$$H_{\rm int} = \Omega_1|0\rangle\langle 1| + \Omega_2|0\rangle\langle 2| + \Omega_B|1\rangle\langle 2| + \sum_k g_{1k}e^{-i(\omega_k-\omega_{1c})t}|1\rangle\langle c|a_k + \sum_k g_{2k}e^{-i(\omega_k-\omega_{2c})t}|2\rangle\langle c|a_k + \text{H.c.} \quad (1)$$

where a_k is annihilation operator of the $k-$ th vacuum mode, g_{1k} and g_{2k} are the coupling constants between the $k-$ th mode and the corresponding atomic transitions. Here, ω_{1c} and ω_{2c} are the transitions frequencies from levels $|1\rangle$, $|2\rangle$ to $|c\rangle$, respectively.

At any time t, the atom-fields state vector can be written as

$$|\psi(t)\rangle = a_0(t)|0,0\rangle + a_1(t)|1,0\rangle + a_2(t)|2,0\rangle + \sum_k c_k(t)|c,1_k\rangle \quad (2)$$

where the probability amplitude $a_i(t)(i = 0, 1, 2)$ represents the state of atom at time t, $c_k(t)$ is the probability amplitude that the atom is in level $|c\rangle$ with one photon emitted spontaneously into the $k-$ th vacuum mode. Substituting the Hamiltonian and wave function into the time-dependent Schrodinger equation, the equations of motion for the probability amplitudes read

$$\dot a_0(t) = -i\Omega_1 a_1(t) - i\Omega_2 a_2(t) \quad (3a)$$

$$\dot a_1(t) = -i\Omega_1^* a_0(t) - i\Omega_B a_2(t) - \frac{\Gamma_1}{2}a_1(t) - p\frac{\sqrt{\Gamma_1\Gamma_2}}{2}e^{-i\omega_{21}t}a_2(t) \quad (3b)$$

$$\dot a_2(t) = -i\Omega_2^* a_0(t) - i\Omega_B^* a_1(t) - \frac{\Gamma_2}{2}a_2(t) - p\frac{\sqrt{\Gamma_1\Gamma_2}}{2}e^{i\omega_{21}t}a_1(t) \quad (3c)$$

$$\dot c_k(t) = -ig_{1k}e^{i(\omega_k-\omega_{1c})t}a_1(t) - ig_{2k}e^{i(\omega_k-\omega_{2c})t}a_2(t) \quad (3d)$$

where $\Omega_2 = |\Omega_2|e^{i\phi}$, $\Gamma_i = 2\pi|g_{ik}|^2D(\omega_k)(i = 1, 2)$ are the spontaneous decay rates of the two upper levels $|1\rangle$ and $|2\rangle$, and $D(\omega_k)$ is the density of modes at frequency ω_k in vacuum; p denotes the alignment of the two dipole moment matrix elements($p \equiv \mathbf{\mu}_{1c}\cdot\mathbf{\mu}_{2c}/(|\mu_{1c}||\mu_{2c}|)$). By using the Laplace transform method and considering the system initially in the ground state $|0\rangle$ we obtain

$$sa_0(s) - 1 = -i\Omega_1 a_1(s) - i\Omega_2 a_2(s) \quad (4a)$$

$$sa_1(s) = -i\Omega_1^* a_0(s) - i\Omega_B a_2(s) - \frac{\Gamma_1}{2}a_1(s) - p\frac{\sqrt{\Gamma_1\Gamma_2}}{2}a_2(s + i\omega_{21}) \quad (4b)$$

$$sa_2(s) = -i\Omega_2^* a_0(s) - i\Omega_B^* a_1(s) - \frac{\Gamma_2}{2}a_2(s) - p\frac{\sqrt{\Gamma_1\Gamma_2}}{2}a_1(s - i\omega_{21}) \quad (4c)$$

$$sb_k(s) = -ig_{1k}a_1(s - i\delta_k - i\omega_{21}/2) - ig_{2k}a_2(s - i\delta_k + i\omega_{21}/2) \quad (4d)$$

where $\delta_k = \omega_k - \omega_{1c} - \omega_{21}/2$, and a population conserving change of variable $b_k(t) = c_k(t)e^{-i\delta_k t}$ has been made. From Eq.4(a)-(c) we can get

$$a_1(s) = \frac{1}{M(s)}[A(s) + B(s)a_1(s - i\omega_{21}) + C(s)a_1(s + i\omega_{21})] \quad (5a)$$

$$a_2(s) = \frac{1}{N(s)}[E(s) + F(s)a_2(s - i\omega_{21}) + G(s)a_2(s + i\omega_{21})] \quad (5b)$$

Here.

$$M(s) = s(s + \frac{\Gamma_1}{2}) + \Omega_1^2 - \frac{(\Omega_1^*\Omega_2 + i\Omega_B s)(\Omega_1\Omega_2^* + i\Omega_B s)}{s(s + \Gamma_2/2) + \Omega_2^2} - \frac{p^2\Gamma_1\Gamma_2 s(s + i\omega_{21})/4}{(s + i\omega_{21})(s + i\omega_{21} + \Gamma_2/2) + \Omega_2^2}$$

$$A(s) = -i\Omega_1^* + \frac{i\Omega_2^*(\Omega_1^*\Omega_2 + i\Omega_B^* s)}{s(s + \Gamma_2/2) + \Omega_2^2} + \frac{i\Omega_2^* sp\sqrt{\Gamma_1\Gamma_2}/2}{(s + i\omega_{21})(s + i\omega_{21} + \Gamma_2/2) + \Omega_2^2}$$

$$B(s) = \frac{(\Omega_1^*\Omega_2 + i\Omega_B s)sp\sqrt{\Gamma_1\Gamma_2}/2}{s(s + \Gamma_2/2) + \Omega_2^2}$$

$$C(s) = \frac{[\Omega_1\Omega_2^* + i\Omega_B^*(s + i\omega_{21})]sp\sqrt{\Gamma_1\Gamma_2}/2}{(s + i\omega_{21})(s + i\omega_{21} + \Gamma_2/2) + \Omega_2^2}$$

$$N(s) = s(s + \frac{\Gamma_2}{2}) + \Omega_2^2 - \frac{(\Omega_1\Omega_2^* + i\Omega_B s)(\Omega_1^*\Omega_2 + i\Omega_B s)}{s(s + \Gamma_1/2) + \Omega_1^2} - \frac{p^2\Gamma_1\Gamma_2 s(s - i\omega_{21})/4}{(s - i\omega_{21})(s - i\omega_{21} + \Gamma_1/2) + \Omega_1^2}$$

$$E(s) = -i\Omega_2^* + \frac{i\Omega_1^*(\Omega_1\Omega_2^* + i\Omega_B^* s)}{s(s + \Gamma_1/2) + \Omega_1^2} + \frac{i\Omega_1^* sp\sqrt{\Gamma_1\Gamma_2}/2}{(s - i\omega_{21})(s - i\omega_{21} + \Gamma_1/2) + \Omega_1^2}$$

$$F(s) = \frac{[\Omega_1^*\Omega_2 + i\Omega_B(s - i\omega_{21})]sp\sqrt{\Gamma_1\Gamma_2}/2}{(s - i\omega_{21})(s - i\omega_{21} + \Gamma_1/2) + \Omega_1^2}$$

$$G(s) = \frac{(\Omega_1\Omega_2^* + i\Omega_B^* s)sp\sqrt{\Gamma_1\Gamma_2}/2}{s(s + \Gamma_1/2) + \Omega_1^2}$$

From Eq.(5) we can not express get $a_1(s)$ and $a_2(s)$ directly. However, setting $a_{10}(s) = A(s)/M(s)$ and $a_{20}(s) = E(s)/N(s)$, and using the iterative method [10,11] once in Eq. (5) we obtain

$$a_{11}(s) = a_{10}(s) + \frac{B(s)}{M(s)}a_{10}(s - i\omega_{21}) + \frac{C(s)}{M(s)}a_{10}(s + i\omega_{21}) \quad (6a)$$

$$a_{21}(s) = a_{20}(s) + \frac{F(s)}{N(s)}a_{20}(s - i\omega_{21}) + \frac{G(s)}{N(s)}a_{20}(s + i\omega_{21}) \quad (6b)$$

By utilizing the final-value theorem for $c_k(t)$ we obtain in the long-time limit:

$$c_k(t \to \infty) = b_k(s \to 0) = -ig_{1k}a_{11}(-i\delta_k - i\omega_{21}/2) - ig_{2k}a_{21}(-i\delta_k + i\omega_{21}/2) \quad (7)$$

The spontaneous emission spectrum $S(\omega_k)$ is given by

$$S(\delta_k) \propto |c_k(t \to \infty)|^2 = \Gamma_1|a_{11}(-i\delta_k - i\omega_{21}/2)|^2 + \Gamma_2|a_{21}(-i\delta_k + i\omega_{21}/2)|^2 \quad (8)$$

3. Results and discussion

Firstly, considering $p = 0$ implies that there is no quantum interference between the transitions $|1\rangle \to |c\rangle$ and $|2\rangle \to |c\rangle$. The total spontaneous emission spectrum is a non-coherent superposition of the spectrum associated with transitions $|1\rangle \to |c\rangle$ and $|2\rangle \to |c\rangle$. According to dressed state theory, each transition from a dressed state to $|c\rangle$ corresponds to a resonant peak, therefore, we can get two groups of resonant peaks in the emission spectrum, and each of the groups involves three resonant peaks. The left part of the emission spectrum is associated with the transition $|1\rangle \to |c\rangle$ while the right part is associated with the transition $|2\rangle \to |c\rangle$. From Fig. 2(a) we find that spontaneous emission is eliminated when $\phi = 0$, and the central peak is suppressed completely. In this case, only four resonant peaks exist in the emission spectrum. For $\phi = \pi/2$ the suppression of spontaneous emission disappears, and the spectrum shows six resonant peaks. As the phase ϕ varies from $0(\pi/2)$ to $\pi(3\pi/2)$, the emission spectrum is a mirror image of the spectrum of $\phi = 0(\phi = \pi/2)$.

Next, we turn our attention to the estimation of influence of the coupled magnetic field on spontaneous emission. When $\Omega_B = 0.5$, the emission spectrum associated with the transition $|1\rangle \to |c\rangle$ shows three peaks as displayed in Fig. 3(a). It is an interesting phenomenon that a dark line(the zero value in the emission spectrum)[2,12,13] occurs in the corresponding spectrum associated with the transition $|2\rangle \to |c\rangle$. The dark line means that there is a destructive interference effect between $|+\rangle_2 \to |c\rangle$, $|0\rangle_2 \to |c\rangle$ and $|-\rangle_2 \to |c\rangle$. In the following, we give an analytical condition for the appearance of the dark line. The