Optics Communications 393 (2017) 185–190

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Reflection statistics of weakly disordered optical medium when its mean refractive index is different from an outside medium

Prabhakar Pradhan[a,*], Daniel John Park[b], Ilker Capoglu[b], Hariharan Subramanian[b], Dhwanil Damania[b], Lusik Cherkezyan[b], Allen Taflove[c], Vadim Backman[b]

[a] Department of Physics and Materials Science, BioNanoPhotonics Laboratory, University of Memphis, Memphis, TN 38152, United States
[b] Biomedical Engineering Department, Northwestern University, Evanston, IL 60208, United States
[c] Electrical Engineering and Computer Science Department, Northwestern University, Evanston, IL 60208, United States

ARTICLE INFO

Keywords:
Scattering theory
Backscattering
Weakly disordered biological media
Cancer detection

ABSTRACT

Statistical properties of light waves reflected from a one-dimensional (1D) disordered optical medium [$n(x) = n_O + dn(x)$, $<dn(x)>=0$] have been well studied, however, most of the studies have focused on the situation when the mean refractive index of the optical medium matched with the outside medium, i.e., $n_O = n_{out} = 1$. Further, considering $dn(x)$ as a Gaussian color noise refractive index medium with exponential spatial correlation decay length l_c and k as the incident wave vector, it has been shown that for smaller correlation length limit, i.e., $kl_c < < 1$, both the mean reflection coefficient $<r>$ and *std* of r, $\sigma(r)$, have same value, and they follow the relation $<r> = \sigma(r) \propto <dn^2> l_c$. However, when the refractive index of the sample medium is different from the outside medium, the reflection statistics may have interesting features, which has not been well studied or understood. We studied the reflection statistics of a 1D weakly disordered optical medium with the mean background refractive index n_0 being different from the outside medium n_{out} ($\neq n_0$), to see the effect of mismatching (i.e., value of $n_O - n_{out}$) on the reflection statistics. In the mismatched case, the results show that the mean reflection coefficient $<r>$ follows a form similar to that of the matched refractive-index case, i.e., $<r(dn, l_c)> \propto <dn^2> l_c$, with a linear increased shift, which is due to 1D uniform background reflection from a slab. However, $\sigma(r)$ is shown to be $\sigma(r) \propto (<dn^2> l_c)^{1/2}$, which is different from the matched case. This change in *std* of r is attributed to the interference between the mismatched-crerated edge mediated multiple scattering that are coupled with the random scattering. Applications to light scattering from random layered media and biological cells are discussed.

1. Introduction

The statistical transport properties of one-dimensional (1D) mesoscopic disordered optical and electronic media are now well studied [1–6]. The Schrödinger equation and Maxwell's wave equation are similar in the sense that they can be projected to the Helmholtz equation; therefore, the formalisms are the same for corresponding scalar waves in both cases [7–10]. After the Landauer formalism showed that the reflection coefficient is related to the resistance/conductance of the sample, the outer scattering parameters such as the reflection and transmission coefficients became important for the study of localization and conductance fluctuations in the electronics case [7,8]. Similarly, extending the idea from the electronic systems, in most of the previous studies of light scattering and localization properties of different optical disordered media, the fluctuation part of the refractive index is primarily considered while the sample's mean refractive index is the same as the outside medium [7,9–11]. The results show that both the average reflection and the fluctuations have the same form for the mesoscopic optical sample. However, the mismatch of the refractive index between the sample and the outside medium and its effect upon the reflection statistics remains poorly understood.

In this paper, we study reflection statistics in the context of the synergistic effects between refractive index mismatched values and the fluctuation of the refractive index. For a biological medium, for example a biological cell, the spatial fluctuation of the refractive index is relatively weak (~.01) and buried in a higher uniform mean background refractive index (~1.38). Enhancement of the backscattering signals from the weakly fluctuating refractive index, as mediated by the refractive index mismatching, is also addressed. Finally, applications of the method for light scattering from biological cells are discussed in terms of the enhancement of the scattering signal from the spatial refractive index nanoscale fluctuations of a biological cell that is

* Corresponding author.
E-mail address: ppradhan@memphis.edu (P. Pradhan).

http://dx.doi.org/10.1016/j.optcom.2017.02.041
Received 9 December 2016; Received in revised form 13 February 2017; Accepted 15 February 2017

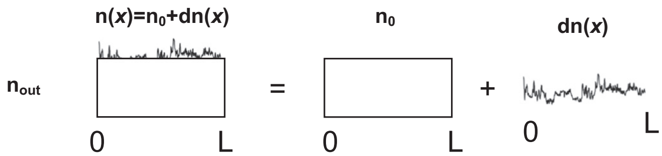

Fig. 1. Schematic of a mismatched case where outside refractive index is n_{out} and sample refractive index is $n(x) = n_0 + dn\,(x)$, with n_0 as the average refractive index of the sample and $dn(x)$ as the spatial refractive index fluctuation of the sample $<dn(x)>=0$. For matched case, $n_0=n_{out}$.

associated with the progress of cancer. This will improve the sensitivity of cancer detection.

2. Reflection statistics of matched disordered media

Consider a 1D sample of length L with refractive index inside the sample $n(x) = n_0 + dn(x)$ (for $0 < x < L$), where the average refractive index of the samples is $n_0 = <n(x)>$, $dn(x)$ is the fluctuation part of the refractive index with its average $<dn(x)>=0$, and n_{out} is the refractive index of the outside medium as shown schematically in Fig. 1. The 'matched' case can be defined as *equality* between the mean refractive index of the sample and the outside medium, i.e., $n_0 = n_{\text{out}}$, whereas the 'mismatched' case can be defined as $n_0 \neq n_{\text{out}}$. Since we are interested in the reflection statistics, let us define $R(L)$ as the complex reflection amplitude of a sample of length L which is illuminated by a plane wave of wave vector k. Then, the mean and standard deviation of the reflection coefficient ($r = RR^*$) are the primary concerns of this work. For example, we will prove below that the mean reflection coefficient for the mismatched case, $<r_{mismatched}>$, can be written in terms of the reflection coefficient of a slab ($n_0 \neq n_{\text{out}}$) and the mean reflection coefficient of the matched case, r_{slab} and $<r_{matched}>$, as:

$$<r_{mismatched}> \;=\; r_{slab} + \;<\; r_{matched} > \times F(k,\, dn,\, l_c,\, \text{n}_0,\, L,\, r_{slab}). \quad (1)$$

In the literature, reflection statistics from disordered optical media are primarily studied assuming the matched case; however, the mismatched case, as defined above, is not well studied. Therefore, we first briefly review the results of a 1D matched case ($r_{matched}$) before describing the results of the mismatched case ($r_{mismatched}$). The statistics of spatial random refractive index fluctuation, $dn(x)$, generally represented by Gaussian color noise, i.e., $<dn(x)>=0$ and $<dn(x)\times dn(x')>\; = \;<dn^2>exp(-|x-x'|/l_c)$, where l_c is the exponential spatial correlation decay length of the spatial refractive index fluctuation $dn(x)$. Then, using the Fokker-Planck approach, the above $<r_{matched}>$ can be solved analytically in the weakly disordered limit (i.e., $dn \ll n_0$) [7,9]. The mean value of the reflection coefficient and its standard deviation both have the same value, $<r_{matched}> = \sigma(r_{matched}) = L/\xi$, where the inverse of the localization length has the form $\xi^{-1} = 2k^2<dn^2>\times l_c/[1+(2kl_c)^2]$. This is true for a weakly disordered sample where $\xi > L$, which can also be defined as a Born approximation limit of the scattering from the weakly disordered part of the refractive index.

3. Reflection statistics of mismatched disordered media

However, index mismatched weakly disordered samples are quite common for optical scattering experiments. For example, biological cells and tissues have refractive indices $n_0 \sim 1.3 - 1.5$ and $dn \sim 0.01 - 0.1$ with the outside air medium $n_{\text{out}} = 1$. In the case of weak refractive index fluctuations, the backscattering light transport properties of such biological cells can be decomposed into a multiple-transport 1D channel or a quasi-1D parallel multichannel problem [12]. It was recently shown that quasi-1D multichannel backscattering would provide sensitivity to changes in the nanoscale signal relative to a three-dimensional (3D) bulk for weakly disordered media such as biological cells. Furthermore, the quasi-1D analysis approach has been proven to be useful for early pre-cancer screening by detecting changes in the nanoscale refractive index fluctuations of cells related to the progress of carcinogenesis in different types of cancers [13–16].

To derive the form of Eq. (1), we start from a stochastic Langevin equation (here, stochasticity enters into the equation through the $dn\,(x)$ term) for the index mismatched case ($n_{\text{out}} \neq n_0$) which gives the reflection amplitude R_t. For simplicity, we will consider that the sample is kept in air, i.e., $n_{out} = 1$ and $n_0 > 1$. Substituting these terms (n_{out}, n_0, and dn) with color noise, the Langevin equation for the mismatched case can be derived following the invariant imbedding approach [7]:

$$\frac{dR_t(L)}{dL} = 2ikR_t(L) + \frac{ik}{2}[(n_0^2 - 1) + 2\,n_0 dn(L)] \times [1 + R_t(L)]^2. \quad (2)$$

The complex total reflection amplitude $R_t(L)$ from a weakly disordered medium can be considered as a combination of: (i) a deterministic sinusoidal oscillation component R_{slab} based on the pure background of a thin-film slab of length L without any stochastic $dn(x)$ terms, and (ii) a R component that contains $dn(x)$ terms. Therefore, we may write $R_t = R_{slab} + R$. Each term can then be easily derived from Eq. (2) as follows:

$$R_t(L) = R_{slab}(L) + R(L), \quad (3a)$$

$$\frac{dR_{slab}(L)}{dL} = 2ikR_{slab} + \frac{ik}{2}(n_0^2 - 1) \times [1 + R_{slab}]^2\,, \quad (3b)$$

$$\frac{dR(L)}{dL} = 2ikR + \frac{ik}{2}(2n_0 dn(L)) \times [1 + R_{slab} + R]^2 + \frac{i}{2}k(n_0^2 - 1) \times [2R(1 + R_{slab}) + R^2]. \quad (3c)$$

In Eq. (3a-c), the perturbative contribution by the stochastic term $dn(x)$ has many cross-terms between R_{slab} and R. We will assume that R is in the first order in $dn(x)$. By performing a phase transformation as below in Eq. (3b-c), we can further simplify and assimilate the $R_{slab} - R$ cross-terms in the equation. For this, we introduce new variables, $Q(L)$ and $\alpha(L)$, which are derived from $R(L)$ by a phase transformation as follows:

$$R_{slab}(L) = Q_{slab}(L)\cdot e^{2ik\alpha(L)}\,, \quad (4a)$$

$$R(L) = Q(L)\cdot e^{2ik\alpha(L)}. \quad (4b)$$

This yields a new set of simplified equations for $Q(L)$ and $\alpha(L)$ which further simplifies to:

$$\frac{d\alpha(L)}{dL} = 1 + \frac{(n_0^2 - 1)}{2}(1 + R_{slab}), \quad (5a)$$

$$\frac{dQ}{dL} = \frac{i}{2}k(2n_0 dn(L))e^{-2ik\alpha}[1 + R_{slab} + Qe^{2ik\alpha}]^2 + \frac{i}{2}k(n_0^2 - 1)(Q)^2 e^{2ik\alpha}. \quad (5b)$$

With the new representation above Eq. (5), the mean $<r_t>$ and the standard deviation $\sigma(r_t)$ of the reflectance for mismatched case $r_{mismatched} = r_t \equiv R_t R_t^*$ can be derived. By using Eq. (3a) and performing a realization averaging over the disordered samples, we obtain:

$$<r_t(L)> \;=\; <R_t R_t^*> \;=\; <(Q_{slab} + Q) \times (Q_{slab} + Q)^*>\,, \\ = r_{slab} + Q_{slab}^* <Q> + c.\,c. + <|Q|^2>\,, \quad (6a)$$

where $r_t = |R_t|^2$ and

$$r_{slab} = |R_{slab}|^2 = |Q_{slab}|^2 = \frac{(n_0^2 - 1)^2 \sin^2(n_0 kL)}{4n_0^2 + (n_0^2 - 1)^2 \sin^2(n_0 kL)}. \quad (6b)$$

In Eq. (6a), we have separated the slab's pure reflection/interference contribution, r_{slab}, (when $dn=0$) and the disorder contribution. The pure slab solution, that is for $\text{n}_0 - 1 > 0$ and dn=0, is presented in Eq. (6b). This also confirms the validation of the invariant imbedding Langevin Eq. (2) with mismatched situation. In particular, the