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Relationship between height and width of resonance peaks in a whispering gallery mode resonator immersed in water and sucrose solutions

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A B S T R A C T

We employed a recently developed whispering gallery mode (WGM) dip sensor made of silica to obtain spectra for many resonance peaks in water and solutions of sucrose at different concentrations and thus having different refractive indices (RI). The apparent Q factor was estimated by fitting each peak profile in the busy resonance spectrum by a Lorentzian or a sum of Lorentzians. A plot of the Q factor as a function the peak height for all the peaks analyzed indicates a straight line with a negative slope as the upper limit, for each of water and the solutions. A coupling model for a resonator and a pair of fiber tapers to feed and pick up light, developed here, supports the presence of the upper limit. We also found that the round-trip attenuation of WGM was greater than the one estimated from light absorption by water, and the difference increased with the concentration of sucrose.

1. Introduction

Miniature resonators that house whispering gallery modes (WGM) have been intensively studied in the past two decades [1–5]. A high quality factor Q of the resonance mode is critical for narrow-band filtering [6,7], laser emission [8,9], and sensing applications [10–13]. The Q value is determined by different factors: (1) absorption and scattering of the traveling light wave by the materials of resonator and surroundings; (2) roughness of the interface between the resonator and the surroundings; and (3) closeness of the effective refractive index of WGM to the surrounding's refractive index (RI). The last factor affects the radiation loss due to reflection by a curved surface of the resonator as well as the coupling of light between the WGM and the mode of light propagating in the linear waveguide that feeds light into the resonator and picks up the WGM. It is often the case that the coupling is a dominant factor in determining the Q observed [14].

The number of modes observed in a given range of wavelength can decrease by adopting a toroid or cylinder for the resonator and employing a smaller resonator [15]. For sensing purposes, especially in water and organic solvents, a busy spectrum in a spherical resonator as large as 0.5 mm diameter is rather preferred, since it allows seeing resonance lines in every range of wavelength as narrow as 1 ppm. A narrow range of wavelength scan is a prerequisite for high-sensitivity sensing. On top of this advantage, a large sphere facilitates the coupling between the resonator and the tapers.

We recently developed a pre-assembled, all-silica dip sensor that consists of a spherical resonator formed at the tip of a fiber stem and a pair of single-ended tapers running parallel to the stem [12]. One of the tapers feeds light into the resonator, and the other picks up light. A 90° bend of the stem at a point close to the resonator allows all three fibers to be contained in a cylinder just a few millimeters across. In this configuration of taper–resonator coupling, a WGM is observed as a peak on zero baseline, when the wavelength of monochromatic light is scanned in the feed taper. Nearly all past studies on WGM were conducted using a different configuration [16]. Namely, a thinned through fiber touches the side of a resonator, and a WGM shows up as a dip from a finite power of light in the receiving end of the fiber. Strong coupling is usually employed to facilitate observation of dips in the wavelength scan. Often, the coupling is not far from the so-called critical coupling condition. In this situation, the line width is determined primarily by the coupling [17]. A weak coupling would allow one to see resonance in a narrow line, but that would result in a shallow dip, thus making it difficult to see it on a large offset that fluctuates. In contrast, the inherently weak coupling in the resonator touching the tips of two single-ended tapers in the dip sensor allows us to observe resonance lines with widths mostly determined by the power loss of WGM traveling in a lossy medium such as water.

The resonance spectrum is busy in our dip sensor in water. We see ~1000 lines in the so-called free-spectral range (=wavelength divided by the number of waves in the circular orbit of WGM). The densely

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populated lines are the result of a high multiplicity of modes, caused by different radial orders and different meridional indices. Each mode has its own peak wavelength, peak height, and peak width. It may appear that there is no relationship between the Q value and the peak height among those modes. However, a plot of the Q value as a function of the peak height for all the peaks observed in a given range of wavelength shows that there is a relationship, and that is not without a reason: We can describe the relationship using a coupling theory for the relevant resonator configuration.

In the present report, we first present the theory for Q factors in a resonator coupled to a pair of single-ended tapers. We derive a formula for the apparent Q factor as seen in the resonance spectrum of light power in the pick-up fiber, expressed as a function of the coupling coefficient and the round-trip loss. After a brief description of the experimental procedure, we will show the results of the Q factor for a dip sensor in water and solutions of sucrose at different concentrations. Since the RI of the solution increases with the concentration, a concentrated solution provides the resonator with a stronger coupling to broaden resonance peaks. We also estimate the round-trip loss from the plot of the Q factor as a function of the peak height. Here we do not use the dip sensor for sensing, but rather utilize its mechanical robustness. By the robustness we mean that the coupling is held unchanged in repeated dip into the same solution. Dipping into a different solution changes the RI environment only, not the spatial arrangement of the tapers relative to the resonator.

2. Theory

2.1. Theory of coupling for a resonator touching a pair of single-ended tapers

Fig. 1 illustrates placement of a resonator of a circular cross section and two single-ended tapers coupled to the resonator at points 1 and 2. Monochromatic light from a laser reaches the tip of one of the tapers (field amplitude is E_1 at point 1). A part of the light transfers to the resonator at point 1, and starts to circulate as a WGM around the resonator near its surface. After one cycle (E_3), a part of the light may transfer to the taper tip. The fields after the coupling are E_2 within the taper tip, and E_4 within the resonator. The light that reaches the tip dissipates into the surroundings. For simplicity, we assume that there is no reflection from the tip. At point 2, the resonator is coupled to the other taper that picks up light (E_7) to deliver it to a photodetector. The fields before and after the second coupling are E_5 and E_6, respectively. We assume that the transfer of light between the resonator and the taper is reciprocal and the transfer coefficients are shared by the two contacts. Then, the fields are related to each other as [18]

$$E_4 = rE_3 + \mathrm{i}tE_1 \tag{1}$$

$$E_2 = rE_1 + \mathrm{i}tE_3 \tag{2}$$

$$E_6 = rE_5 \tag{3}$$

$$E_7 = \mathrm{i}tE_5 \tag{4}$$

where t and r (both being positive) are the transfer and reflection coefficients:

$$r^2 + t^2 = 1 \tag{5}$$

A large t or small r indicates a strong coupling. As the light travels within the resonator, the complex amplitude of the field changes. We relate E_5 and E_3 to E_4 and E_6, respectively, as

$$E_5 = h_{12}E_4 \tag{6}$$

$$E_3 = h_{21}E_6 \tag{7}$$

From Eqs. (1)–(7), we can derive relationships between fields. For example,

$$\frac{E_3}{E_1} = \frac{\mathrm{i}hrt}{1-hr^2} \tag{8}$$

$$\frac{E_7}{E_1} = -\frac{h_{12}t^2}{1-hr^2} \tag{9}$$

where $h=h_{12}h_{21}$ represents the round-trip amplitude change. The h and h_{12} may be expressed as

$$h = \tau \exp(\mathrm{i}\phi) \tag{10}$$

$$h_{12} = \tau_{12} \exp(\mathrm{i}\phi_{12}) \tag{11}$$

where τ and τ_{12} represent amplitude attenuation in a round trip and in a trip from 1 to 2, respectively ($0 < \tau < \tau_{12} < 1$). The phase changes in these trips are ϕ and ϕ_{12}. We consider weak attenuations only, $1-\tau \ll 1$.

A wavelength scan causes ϕ to change. At resonance, $\phi=\phi_0=2\pi l$, where l is the number of waves in the circular orbit of length $2\pi a$, and a is the radius of the resonator. With $2\pi a = l\lambda_0/n_{\mathrm{eff}}$, where n_{eff} is the effective RI of WGM for light of vacuum wavelength λ_0, we obtain

$$\phi_0 = \frac{4\pi^2 a n_{\mathrm{eff}}}{\lambda_0} \tag{12}$$

The ratio of the light power in the pick-up fiber to the one in the feed fiber is

$$\left|\frac{E_7}{E_1}\right|^2 = \frac{\tau_{12}{}^2(1-r^2)^2}{1-2\tau r^2\cos\phi + \tau^2 r^4} \tag{13}$$

The power relationships satisfy the conservation of energy:

$$|E_1|^2 + |E_3|^2 = |E_2|^2 + |E_4|^2 \tag{14}$$

$$|E_5|^2 = |E_6|^2 + |E_7|^2 \tag{15}$$

Below, we consider the peak profile at near the resonance wavelength λ_0 (phase ϕ_0). For a small difference $\Delta\phi$ of the phase from ϕ_0, the wavelength change $\Delta\lambda$ has the opposite sign:

$$\phi = \phi_0 + \Delta\phi = \phi_0 - \frac{\Delta\lambda}{\lambda_0}\phi_0 \tag{16}$$

Then,

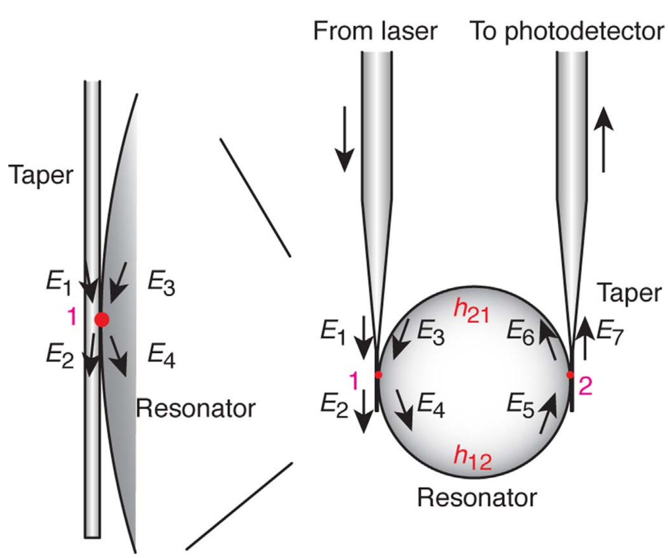

Fig. 1. Light propagation in a pre-assembled dip sensor. A pair of single-ended tapers formed at the ends of single-mode fibers (feed fiber and pick-up fiber) touch a resonator at points 1 and 2. Electric fields E_1 through E_7 are defined at the points of contact. The complex amplitude of the field changes by h_{12} as light travels from point 1 to point 2, and by h_{21} for a travel from 2 back to 1. A zoomed view of the taper–resonator contact is shown to the left.