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Controllable autofocusing properties of conical circular Airy beams

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ABSTRACT

In this paper, we propose a new family of circular Airy beam (CAB) through introducing a cone angle. We investigate the autofocusing properties of such conical circular Airy beam (CCAB) both analytically and numerically, particularly focusing on the novel behaviors different from the case of the conventional CAB. Under the action of positive cone angles, the autofocusing effect of CCAB is greatly strengthened, and the focal length is remarkably reduced, as compared to the case of conventional CAB. On the other hand, the autofocusing effect may be weakened or even completely eliminated, and the focal length is largely extended under the action of negative cone angles. Therefore, such dramatic enhancement or suppression of abruptly focus can be effectively controlled through adjusting the cone angle. The novel autofocusing properties will make CCAB more powerful in various applied fields including optical trapping and particle manipulation.

1. Introduction

Circular Airy beams, i.e. abruptly autofocusing beams, recently have received a considerable boost in the scientific community because of its unique abruptly autofocusing behavior unattainable with conventional Gaussian beams [1,2]. Such novel beams were theoretically introduced by Efremidis et al. in 2010 [1], afterwards were experimentally observed by Papazoglou et al. [2]. The most impressive property of such CAB may well be the ability to abruptly focuses its energy right before the focal point even in the linear media while its profile keeps almost constant along the whole propagation trajectory until the focus is reached. For this reason, such abruptly autofocusing properties of CAB highlight great potential applications in biomedical treatment and optical micromanipulation etc. In optical trapping, CAB is able to yield a greater gradient force on the particles in the focal region, when compared to conventional Gaussian beams under the same initial condition [3,4]. Therefore, the CAB has been applied to trap and guide the microparticles along the desired path. In addition, engineering CAB in the Fourier space can offer a new approach to produce an elegant paraboloid optical bottle [5,6]. Recently, CAB in the form of accelerating beams are demonstrated to be able to reshape into non-linear intense light bullets under the action of both multiphoton absorption and ionization in Kerr media [7].

Up to now, the control of the abruptly autofocusing property of the CAB has become a new and exciting field of research. For instance, several strategies have been proposed to enhance focal intensity, or control the focal pattern and trajectory of self-acceleration, such as adding different optical vortices [8,9], imposing the chirped phase factor [10], and blocking front light rings [11] and so on. Meanwhile, several other CABs with special properties have also been disclosed [12–14]. To control the propagation of CAB, some external physical mechanisms such as linear potential [15] and optical lattices [16] have been introduced by some researchers. Recently, initial angle as another new physical strategies is found to be able to drive the Airy beam perform ballistic dynamics, similar to those of projectiles moving under the action of gravity [17]. Meanwhile, the propagation dynamics of conventional Gaussian beams with initial angles have been widely studied. However, to our best knowledge, the propagation of the cone angle superimposed onto the novel CAB still remain unexplored. Therefore, it is interesting to investigate what will happen when a CAB with a cone angle is launched into a medium. The question is, how does the beam self-accelerate in this case? how do the properties of abruptly focus change under the action of the cone angle? In this paper, We will in detail reply these questions. Hence, the main aims of this paper is to present detailed investigation on abruptly focus of CCAB. In particular, we will try to disclose the novel behaviors different from those for the conventional CAB.

2. Theoretical model

Here we consider the dynamical behaviors of a radially symmetric optical beam propagating in a linear medium. Under the paraxial approximation, the normalized equation for the evolution of slowly-varying envelope E of the optical electric field can be described by

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$$\frac{\partial E}{\partial z} = \frac{i}{2}\left(\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r}\right), \tag{1}$$

where the radial coordinate r is normalized with respect to an arbitrary transverse length x_0, z is the normalized propagation distance with respect to Rayleigh lengths. The dynamical behavior of an arbitrary radially symmetric initial condition $E_0(r, 0)$ can be calculated according to the following forward Hankel transform:

$$E(r, z) = \frac{1}{2\pi}\int_0^\infty \widetilde{E_0}(k, 0)\exp(-ik^2 z/2)J_0(kr)kdk, \tag{2}$$

where $J_0(kr)$ is the zero-order Bessel function, k is the radial spatial frequency. $\widetilde{E_0}(k, 0)$ is the Hankel transform of the input envelope $E_0(r, 0)$ at z=0, and it can be obtained by applying the following backward Hankel transform

$$\widetilde{E_0}(k, 0) = 2\pi\int_0^\infty E_0(r, 0)J_0(kr)rdr, \tag{3}$$

The initial envelope of electric fields of the CAB superimposed by a cone angle v (or "velocity") in cylindrical coordinate can be written as [1,17]

$$E_0(r, 0) = Ai(r_0 - r)\exp[\alpha(r_0 - r)]\exp(ivr), \tag{4}$$

where r_0 is the initial radius of the main ring and α is the decay factor. For $r < r_0$, the energy of CCAB decays exponentially, while the slowly decaying oscillations of optical tail appear in the peripheral region. We first introduce the cone angle v into the CAB, which can provide an alternative approach to engineering the initial beams. In experiment, the actual cone angle θ is related to the normalized cone angle v through $\theta = \arcsin(v/x_0\beta)$ where $\beta = 2\pi n/\lambda_0$ is the wavenumber of the optical wave, n is the refractive index and λ_0 is the wavelength of optical wave. Therefore, in fact, the propagation properties of CCAB is controlled by the actual cone angle θ in experiment. In this paper, we will take some typical values of the parameters given as $x_0 = 100\mu m$, n=1.45, and $\lambda_0 = 600nm$. Based on the parameters, we easily obtain the actual cone angles θ belonging to different normalized cone angel v used in the following analysis, such as $(v = \pm 1) \leftrightarrow (\theta = \pm 0.0378^\circ)$ and $(v = \pm 3) \leftrightarrow (\theta = \pm 0.113^\circ)$, Therefore, from the values, one find that the propagation properties of CCAB can be adjusted by only small actual cone angles in experiment.

To explore the propagation properties of the CCAB, we will numerically compute the integral expression of CCAB in Eq. (2) in the later section since it is very difficult to obtain the accurate analytical solution to Eq. (2) under the initial condition given by Eq. (4). Here we will first derive an approximate analytical solution to help us more intuitively understand the unusual behavior of sharply autofocusing, as compared to the numerical method. As is well known, when the CAB is accelerated toward the beam axis, its radial profile over the transverse plane keeps almost constant along the whole moving trajectory until the focus is reached. Therefore, during the initial accelerated process, we take the approximation $\partial^2 E/\partial r^2 + \partial E/r\partial r \approx \partial^2 E/\partial r^2$ in Eq. (1) since the second term on the right-hand side of Eq. (1) is not significant before focusing. Thus, substituting Eq. (4) into Eq. (1), we can obtain an approximate dynamics of CCAB through the method of Fourier transform as following

$$\begin{aligned} E(r, z) = {} & Ai[-r - z^2/4 - vz + r_0 + i\alpha z]\exp[\alpha(-r + r_0) - (\alpha z^2)/2 - \alpha vz] \\ & \times \exp[-iz^3/12 + i(\alpha^2 - v^2 - r + r_0)z/2 + iv(-r + r_0) - ivz^2/2], \end{aligned} \tag{5}$$

Obviously, from Eq. (5), we easily find that CCAB accelerates in a fashion of a ballistic path in the r–z plane that is described by the following parabola

$$r = -z^2/4 - vz + r_0, \tag{6}$$

Based on Eq. (6), we can further obtain the focal length of sharply autofocus by setting r=0 as following

$$z_f = -2v + 2\sqrt{v^2 + r_0}, \tag{7}$$

Eqs. (6)–(7) is our central result of analytical expression to this research, being the closed-form approximation. Obviously, some qualitative physical insight related with the trajectory and the focal length can be directly gained from Eqs. (6)–(7) even without numerical calculation. For example, the focal length z_f decreases monotonically as increasing the value of v for $v > 0$; while it increases monotonically as increasing absolute value of v for $v < 0$. In addition, one notes that the focal length z_f will increases monotonically with the increase of r_0. Here, we need to stress that the trajectory in Eq. (6) and the focal length in Eq. (7) are somewhat rude approximations, because we omitted the first term on the right-hand side of Eq. (1) during the process of the derivation. However, the accurate numerical solution based on the Hankel transform will be employed to check the validity of our analytical prediction in the next section. In addition, here we have to mention that Eqs. (6) and (7) is only suitable for describing the dynamical behavior of CCAB before focusing. Therefore, after the main lobe of CCAB is close to the axis through self-acceleration, i.e., when the behavior of focus begins to occur, the approximate condition used in deriving Eqs. (6) and (7) is not tenable. As a consequence, our analytical model cannot predict the dynamical behavior of rapid increase of optical intensity near focal point, which will be investigated by employing the numerical method in the next section.

3. Results and discussions

In this section, we will in detail discuss the propagation properties of the CCAB beams. We note that the accurate analytical expression of Hankel transform for CAB is not derived by substituting Eqs. (3) and (4) into Eq. (2). To investigate the propagation properties of CCAB, we will employ the well-known quasi-discrete method proposed provided by Guizar-Sicairos et al. to numerically compute the integral expression in Eq. (2) [18]. This numerical approach is found to be much more efficient and accurate than other existing approaches. In the following analysis, we assume $\alpha = 0.2$ and r_0=8 unless otherwise specified.

We first discuss the properties of focal position as well as the focal trajectory during the evolution process of sharply autofocusing beam. The simulation results are shown in Fig. 1 were we plot the evolution of CCAB as the function of propagation distance z under the action of different values of v. In order to compare with the theoretical prediction more clearly, we also display the analytical accelerating trajectories of the main lobes of CCAB marked by red dashed cureve in Fig. 1, according to Eq. (6). When the CCAB is launched with a positive cone angle ($v > 0$), the direction of the initial velocity related with cone angle is in the same direction as the original self-acceleration, and so the beam still perform self-acceleration during propagation. However, the extent of self-acceleration is greatly strengthened and the focal length is remarkably reduced. When the CAB is launched with a negative cone angle ($v < 0$), the direction of the initial velocity related with cone angle is opposite to that of the original self-acceleration. Therefore, the self-acceleration of optical beam is prevented, and the CCAB may first perform deceleration and then acceleration during propagation. Accordingly, the autofocusing effect may be weakened and the focal length is largely extended. From Fig. 1, we can clearly see that the theoretical prediction is in accordance with numerical simulations. Besides, in order to analyze the accelerating process more clearly, we also present a comparison of focal length z_f obtained from analytical and numerical methods in Fig. 2. Here, the analytical focal length z_f comes from our derived Eq. (7), while the numerical focal length z_f is defined as the distance from the input point to the point at which the maximum peak intensity occurs. Obviously, the analytical and numerical focal lengths are in good qualitative agreement each other. Meanwhile, one notes that small differences between the results obtained by direct simulations as well as investigations based on Eq. (7) are also observed, especially for the case when negative incident