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A simplified algorithm for digital fringe analysis in two-wave interferometry with sinusoidal phase modulation

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ABSTRACT

A compact simplified algorithm for digital detection of the amplitude and phase of the interferometric signal delivered by a two-wave interferometer with sinusoidal phase modulation is presented. The algorithm consists of simple mathematical combinations of four frames obtained by integration by a camera of the time-varying intensity in an interference pattern during the four successive quarters of the modulation period. The algorithm is invariant by circular permutation of the four image frames. Any set of four consecutive frames can be used for the calculations, which simplifies the practical implementation of the method. A numerical simulation has been carried out to evaluate the efficiency of the algorithm for fringe envelope extraction in low-coherence interferometry. A theoretical analysis of the effect of noise in phase map calculation is conducted. A comparison with the conventional sinusoidal phase-shifting algorithm is made.

1. Introduction

Phase-shifting interferometry is a powerful means of analyzing interferograms obtained from interferometric systems [1–4]. Many phase-shifting algorithms have been developed to measure the optical wavefront from several interference fringe patterns acquired with an area camera [5–16]. Several phase-shifting algorithms have also been developed for digital fringe envelope detection in white-light scanning interferometry [17–21]. Combined with a procedure for determining the position of the fringe envelope peak or associated with phase measurements, this method enables sample surface topography measurements with theoretically unlimited height range. Tomographic imaging of semi-transparent samples can also be achieved using white-light (low-coherence) interferometry. In the technique referred to as full-field optical coherence microscopy (also termed full-field optical coherence tomography), tomographic images are usually obtained by extraction of the fringe envelope using phase-shifting methods [22–26].

In all phase-shifting methods, a phase shift is introduced in the interferometer and several interferometric images are acquired. Usually, a temporal phase shift is introduced and the images are acquired sequentially. Interferometric systems have also been developed to produce and acquire several phase-shifted images simultaneously [27–31]. The most common technique to generate the required phase shift consists of displacing a reference reflector in the interferometer using a piezoelectric transducer (PZT). The phase shift can be obtained by other means such as by changing the polarization state of light [32–34], by using a photoelastic phase modulator [35], or a spatial light modulator [36].

In the phase-stepping method, the phase is stepped by a known amount between the acquisitions of the interferometric images. This method is limited in operation speed by the response time of the phase modulator to a step-function driving signal, which may be a real limitation when the phase shift is generated by a mechanical displacement.

In the so-called "integrating-bucket" method, the interferometric images are acquired while the phase is being shifted continuously. The bandwidth limitation of stepped phase-shift methods is then significantly reduced, enabling higher operation speed. In this method, the phase is usually shifted linearly in a sawtoothlike manner, and several integrated interferometric images (or "buckets") are recorded by the camera. Phase-shifting interferometry that uses sinusoidal phase modulation is less usual. An algorithm with sinusoidal phase modulation and four integrating buckets was initially proposed for phase measurements [7] and applied to surface topographic measurements [7,37]. This algorithm was extended to fringe envelope detection [35] and used in full-field optical coherence microscopy [38–40]. More recently, it was implemented in a line-scanning optical coherence microscopy system [41], and in spectral-domain optical coherence tomography for high-speed complex conjugate resolved imaging [42].

The major interest of sinusoidally-modulated phase-shifting interferometry is the high operation speed that can be reached even with a mechanically-generated modulation of the phase. This method requires the synchronization of the phase modulation with the image acquisi-

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tion. In the original and conventional algorithm, the frequency of the phase modulation is set to one quarter of the image acquisition frequency, and sequences of four different interferometric images are acquired continuously. Mathematical combinations of the four acquired images yield images of the phase and the amplitude of the interferometric signal. The calculations require sequences of four images in a specific order. A challenging technical problem arises from the calculations to be done during the continuous image acquisition. If at least one image from the continuous image flow delivered by the camera is missed, one has to wait for the next right sequence, *i.e.* the sequence with images in the right order. That is what usually happens due to calculation times, which leads to a reduction of the operation speed compared to the maximal theoretical speed. Moreover, a method has to be implemented for the identification of the right sequence of images to be considered for the calculations.

In this paper, a compact and simple algorithm for both amplitude and phase measurement of the interferometric signal delivered by a two-wave interferometer using sinusoidal phase modulation is presented. Similarly to the conventional sinusoidal phase-shifting algorithm, the algorithm proposed here is based on the combination of four frames obtained by integration of the time-varying intensity in an interference pattern during the four successive quarters of the modulation period. However, unlike the conventional algorithm, any sequence of four successive interferometric images can be considered for the calculations, regardless of what is the first image of the sequence. Analytical calculations and numerical simulations are carried out in this paper to study the performance of the algorithm for digital fringe envelope detection and phase map measurements. A comparison is made with the conventional sinusoidal phase-shifting algorithm in terms of performance.

2. Simplified algorithm

Assuming that monochromatic light is used, the optical intensity at the output of a two-wave interferometer can be written as

$$I=\bar{I}\,[1+V\cos(\phi)], \tag{1}$$

where $\bar{I}$ is the bias (mean) intensity, V the contrast (visibility) of the interferometric signal ($0 \le V \le 1$), and ϕ the optical phase. By generating a sinusoidal phase modulation in the interferometer, of amplitude ψ and period $T=2\pi/\omega$, the optical intensity at the output varies with time as

$$I(t)=\bar{I}\,\{1+V\cos[\phi+\psi\sin(\omega t+\theta)]\}, \tag{2}$$

the parameter θ being determined by the time origin. A photodetector integrates the time-varying signal $I(t)$over the four successive quarters of the modulation period T. We consider an image sensor as a photodetector, *i.e.* a two-dimensional detector array such as a CCD or CMOS camera. The time-integration of $I(i,j,t)$ is performed in parallel by all the pixels (i,j) of the camera (frame-transfer and full-frame camera). The charge storage period of the camera is set to be one-quarter of the period T of the sinusoidal phase modulation. Four frames of interferogram are thus recorded. The quantum efficiency of the detector being η, at the considered wavelength λ, the four frames are

$$E_p(i,j)=\eta\int_{(p-1)T/4}^{pT/4} I(i,j,t)\,dt \quad p=1,2,3,4. \tag{3}$$

The phase between the modulation and the periodic image acquisition is determined in this mathematical description by parameter θ. The calculation of the integral in Eq. (3) can be carried out by writing a Jacobi–Anger expansion of $I(t)$ using Bessel functions of the first kind:

$$I(t)/\bar{I}=1+V\cos\phi\left\{J_0(\psi)+2\sum_{k=1}^{\infty}J_{2k}(\psi)\cos[2k(\omega t+\theta)]\right\}-2V\sin\phi\sum_{k=0}^{\infty}J_{2k+1}(\psi)\sin\,[(2k+1)(\omega t+\theta)]. \tag{4}$$

The expression of the four frames can then be written as

$$E_1=\eta\bar{I}\frac{T}{4}[1+V\cos\phi J_0(\psi)]+\eta\bar{I}V\frac{T}{\pi}\left\{\cos\phi\sum_{k=1}^{\infty}J_{2k}(\psi)\frac{1}{2k}[-\sin(2k\theta)+\sin(2k\theta+k\pi)]+\sin\phi\sum_{k=0}^{\infty}J_{2k+1}(\psi)\frac{1}{2k+1}[-\cos(2k+1)\theta-\sin((2k+1)\theta+k\pi)]\right\}, \tag{5.a}$$

$$E_2=\eta\bar{I}\frac{T}{4}[1+V\cos\phi J_0(\psi)]+\eta\bar{I}V\frac{T}{\pi}\left\{\cos\phi\sum_{k=1}^{\infty}J_{2k}(\psi)\frac{1}{2k}[\sin(2k\theta)-\sin(2k\theta+k\pi)]+\sin\phi\sum_{k=0}^{\infty}J_{2k+1}(\psi)\frac{1}{2k+1}[-\cos(2k+1)\theta+\sin((2k+1)\theta+k\pi)]\right\}, \tag{5.b}$$

$$E_3=\eta\bar{I}\frac{T}{4}[1+V\cos\phi J_0(\psi)]+\eta\bar{I}V\frac{T}{\pi}\left\{\cos\phi\sum_{k=1}^{\infty}J_{2k}(\psi)\frac{1}{2k}[-\sin(2k\theta)+\sin(2k\theta+k\pi)]+\sin\phi\sum_{k=0}^{\infty}J_{2k+1}(\psi)\frac{1}{2k+1}[\cos(2k+1)\theta+\sin((2k+1)\theta+k\pi)]\right\}, \tag{5.c}$$

$$E_4=\eta\bar{I}\frac{T}{4}[1+V\cos\phi J_0(\psi)]+\eta\bar{I}V\frac{T}{\pi}\left\{\cos\phi\sum_{k=1}^{\infty}J_{2k}(\psi)\frac{1}{2k}[\sin(2k\theta)-\sin(2k\theta+k\pi)]+\sin\phi\sum_{k=0}^{\infty}J_{2k+1}(\psi)\frac{1}{2k+1}[\cos(2k+1)\theta-\sin((2k+1)\theta+k\pi)]\right\}. \tag{5.d}$$

One can write

$$E_1-E_2=\frac{2}{\pi}\eta T\bar{I}V(\Gamma_a\cos\phi-\Gamma_b\sin\phi), \tag{6.a}$$

and

$$E_3-E_4=\frac{2}{\pi}\eta T\bar{I}V(\Gamma_a\cos\phi+\Gamma_b\sin\phi), \tag{6.b}$$

with

$$\Gamma_a=\sum_{k=0}^{\infty}J_{4k+2}(\psi)\frac{1}{2k+1}[\sin 2(2k+1)\theta], \tag{7.a}$$

and

$$\Gamma_b=\sum_{k=0}^{\infty}J_{2k+1}(\psi)\frac{(-1)^k}{2k+1}[\sin(2k+1)\theta]. \tag{7.b}$$

Eqs. (6.a) and (6.b) can be rewritten as

$$E_1-E_2=\frac{\sqrt{2}}{\pi}\eta T\bar{I}V[(\Gamma_a+\Gamma_b)\cos(\phi+\pi/4)+(\Gamma_a-\Gamma_b)\sin(\phi+\pi/4)], \tag{8.a}$$

$$E_3-E_4=\frac{\sqrt{2}}{\pi}\eta T\bar{I}V[(\Gamma_a-\Gamma_b)\cos(\phi+\pi/4)+(\Gamma_a+\Gamma_b)\sin(\phi+\pi/4)]. \tag{8.b}$$

If $\Gamma_a=\Gamma_b=\Gamma$, the two previous equations simplify. The visibility V and phase ϕ can then be calculated according to the two following frame combinations:

$$[(E_1-E_2)^2+(E_3-E_4)^2]^{1/2}=(2\sqrt{2}\,\Gamma/\pi)(\eta T\bar{I})V, \tag{9}$$