Optics Communications 398 (2017) 67–76

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

# Bilaterally asymmetric reflection and transmission of light by a grating structure containing a topological insulator

Annunziata Diovisalvi[a,b], Akhlesh Lakhtakia[a], Vincenzo Fiumara[b,*], Francesco Chiadini[c]

[a] NanoMM–Nanoengineered Metamaterials Group, Department of Engineering Science and Mechanics, Pennsylvania State University, University Park, PA 16802-6812, USA
[b] School of Engineering, University of Basilicata, Viale dell'Ateneo Lucano 10, 85100 Potenza, Italy
[c] Department of Industrial Engineering, University of Salerno, via Giovanni Paolo II 132, 84084 Fisciano, SA, Italy

ARTICLE INFO

*Keywords:*
Columnar thin film
Floquet harmonics
Rigorous coupled–wave approach
Superlattice
Topological insulator

ABSTRACT

A boundary-value problem was formulated to investigate the reflection and transmission of light by a device consisting of an orthorhombic dielectric material that sits atop a 1D grating and is coated with a 3D topological insulator. In view of the periodicity of the grating, the electromagnetic field phasors were represented in terms of Floquet harmonics and the analysis was conducted by using the rigorous coupled-wave approach. We found that the device can exhibit bilaterally asymmetric reflection and transmission in the mid-infrared wavelength regime, provided that the surface admittance of the topological insulator is sufficiently high. This bilateral asymmetry is exhibited in narrow regimes for both the free-space wavelength and the angle of incidence. Bilateral asymmetry is exhibited more significantly by the specular components than by the nonspecular components of the reflected and transmitted plane waves.

## 1. Introduction

As free space (or air, practically) is a Lorentz-reciprocal medium [1,2], no change occurs when the positions of a transmitter and a receiver facing each other are interchanged in an experiment [3,4]. No change will occur in that experiment even if a bounded scatterer made of a Lorentz-reciprocal medium were to be present [5–8]. When that scatterer is a slab of finite thickness and infinite transverse extent, both reflection and transmission of an incident plane wave are therefore bilaterally symmetric processes. This is most easily seen with a homogeneous slab, reflection and transmission then being entirely specular [9], and that situation does not change when the slab is nonhomogeneous in the thickness direction [10].

However, a significant change will occur if the slab is made of a material that is not Lorentz reciprocal [2]. Reflection and transmission can then be bilaterally asymmetric [9], which characteristic can be very useful in one-way devices that are required to make energy flow in specific channels in specific directions, much like water flow in a watershed, and therefore are the mainstays of electronic circuits [11]. Miniature one-way optical devices will function, in essence, as electromagnetic diodes or isolators. Coupled to polarization filters and/or bandpass/bandstop filters, these devices could be used to reduce back-scattering noise [12], and thus have a transformative impact on everyday life.

Lorentz nonreciprocity is exploited in the microwave regime by the application of a quasistatic magnetic field on a ferrite [13], but a decent substitute is not available at higher frequencies. That situation may change in the terahertz regime with the emergence of topological insulators such as $Bi_2Se_3$ and $Bi_2Te_3$ [14–16]. A topological insulator is an isotropic substance which possesses protected surface states. A topological insulator may be modeled classically as an achiral biisotropic material whose Lorentz nonreciprocity is captured by a magnetoelectric pseudoscalar $\gamma_{TI}$ [17,18]. Alternatively, a topological insulator may be regarded as an isotropic dielectric–magnetic material with surface states described by a surface admittance $\gamma_{TI}$ [19], these surface states differing from ordinary surface conducting states [20]. These two different models give rise to identical results in terms of scattering [19,20]. As topological insulation is a surface phenomenon manifesting as protected conducting states that exist at the surface, but not in the bulk, of a topological insulator, we prefer to adopt the second viewpoint in this paper.

In order to assess the promise of topological insulators for bilaterally asymmetric reflection and transmission, we have theoretically formulated the boundary-value problem for the mid-wavelength infrared (MWIR) response of an orthorhombic dielectric material that sits atop a 1D grating and is coated with a 3D topological insulator

* Corresponding author.
*E-mail addresses:* annunziata.diovisalvi@unibas.it (A. Diovisalvi), akhlesh@psu.edu (A. Lakhtakia), vincenzo.fiumara@unibas.it (V. Fiumara), fchiadini@unisa.it (F. Chiadini).

http://dx.doi.org/10.1016/j.optcom.2017.04.017
Received 14 March 2017; Accepted 7 April 2017

(with a bandgap around 300 meV). The 1D grating is responsible for nonspecular reflection and transmission [21,22].

The plan of this paper is as follows. The formulation of the boundary-value problem is presented in Section 2. Numerical results illustrating the desired bilateral asymmetry are presented and discussed in Section 3, and the paper concludes with a collation of significant conclusions in Section 4. Vectors are in boldface, dyadics are double-underlined, column vectors are in boldface and enclosed with square brackets, and matrixes are underlined twice and enclosed with square brackets. Cartesian unit vectors are denoted by $\mathbf{u}_x$, $\mathbf{u}_y$, and $\mathbf{u}_z$. An $\exp(-i\omega t)$ time-dependence is implicit, with $\omega$ as the angular frequency, $i=\sqrt{-1}$, and $t$ as time. The free-space wavenumber, the free-space wavelength, and the intrinsic impedance of free space are denoted by $k_0=\omega\sqrt{\varepsilon_0\mu_0}$, $\lambda_0=2\pi/k_0$, and $\eta_0=\sqrt{\mu_0/\varepsilon_0}$, respectively, with $\mu_0$ and $\varepsilon_0$ being the permeability and permittivity of free space. We denote the fine structure constant by $\tilde{\alpha}=(q_e^2\eta_0)/2\tilde{h}$, where $q_e$ is the quantum of charge and $\tilde{h}$ is the Planck constant.

## 2. Boundary–value problem

### 2.1. Description

The boundary-value problem is shown in Fig. 1 where a plane wave is obliquely incident from the vacuous half-space $z<-h$ on a device occupying the region $z\in[-h, h_3]$, whereas transmission occurs in the vacuous half-space $z>h_3$. The upper surface of a topographic substrate is decorated periodically with rectangular grooves of width $q_1L$ and height $\Delta h_2$, resulting in the substrate having maximum and minimum thicknesses $\Delta h_2+\Delta h_3$ and $\Delta h_3$, respectively. The grooves are spaced $L$ apart along the $x$ axis, the substrate being of infinite length along the $y$ axis and $q_1\in(0,1)$. The relative permittivity scalar of the substrate material is denoted by $\varepsilon_g$.

The orthorhombic dielectric material chosen is a columnar thin film (CTF) [23,24]. In order to fabricate the CTF, a collimated vapor flux was supposedly directed at the topographic substrate at an angle $\chi_v\in(0,\pi/2]$ with respect to the $x$ axis in the $xz$ plane [25], inducing the growth of columns oriented at an angle $\chi\geq\chi_v$ with respect to the $x$ axis in the $xz$ plane [23]. The $xz$ plane is therefore the morphologically significant plane of the CTF.

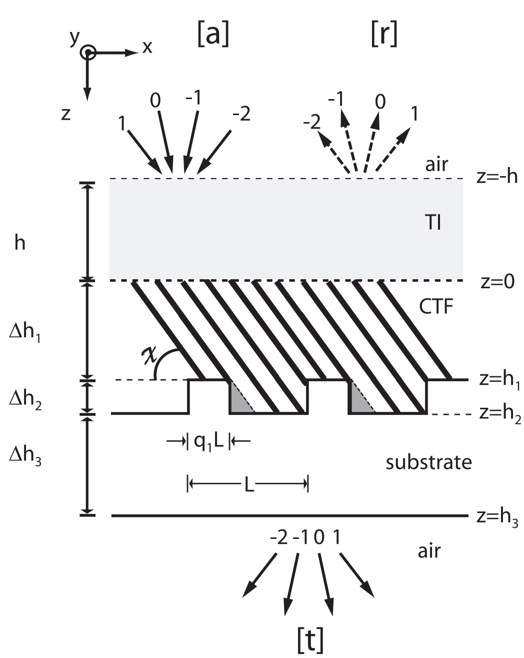


**Fig. 1.** Schematic of the boundary-value problem solved here. The four-region device occupies the region $z\in[-h, h_3]$. All materials are supposed to have the same relative permeability as free space.

If

$$\frac{\Delta h_2}{q_1L}>\tan\chi_v\,, \tag{1}$$

the columns do not cover all the region between the adjacent grooves due to shadowing of the collimated vapor flux by the grooves [25]. A roughly triangular prism of air (equivalently, free space) is trapped at the right edge of each rectangular groove (the grey triangles in Fig. 1) [26,27]. The columns were supposedly allowed to grow a distance $\Delta h_1$ above the rectangular grooves to the plane $z$=0.

The relative permittivity dyadic of the CTF is expressed as

$$\underline{\underline{\varepsilon}}_{CTF}=\underline{\underline{S}}_y(\chi)\bullet(n_1^2\,\mathbf{u}_x\mathbf{u}_x+n_3^2\,\mathbf{u}_y\mathbf{u}_y+n_2^2\,\mathbf{u}_z\mathbf{u}_z)\bullet\underline{\underline{S}}_y^{-1}(\chi), \tag{2}$$

where the tilt dyadic

$$\underline{\underline{S}}_y(\chi)=\mathbf{u}_y\mathbf{u}_y+(\mathbf{u}_x\mathbf{u}_x+\mathbf{u}_z\mathbf{u}_z)\cos\chi+(\mathbf{u}_z\mathbf{u}_x-\mathbf{u}_x\mathbf{u}_z)\sin\chi \tag{3}$$

is unitary. The angle $\chi$ as well as the principal refractive indexes $n_1$, $n_2$, and $n_3$ are functions of $\chi_v$ and also depend on the material evaporated to fabricate the CTF [24,28].

On top of the CTF, a homogeneous slab of a topological insulator [14–16] is present, the thickness of this slab being denoted by $h$ and its relative permittivity scalar by $\varepsilon_{TI}$. The surface states of the topological insulator are characterized by an admittance scalar $\gamma_{TI}$ [9,19]. Accordingly, the boundary conditions prevailing on the plane $z=-h$ are [19]

$$\left.\begin{aligned}&\mathbf{u}_z\times[\mathbf{E}(x,z^-)-\mathbf{E}(x,z^+)]=\mathbf{0}\\&\mathbf{u}_z\times[\mathbf{H}(x,z^-)-\mathbf{H}(x,z^+)]=-\gamma_{TI}\mathbf{u}_z\times\mathbf{E}(x,z^+)\end{aligned}\right\},\quad z=-h \tag{4}$$

and those on the plane $z$=0 are

$$\left.\begin{aligned}&\mathbf{u}_z\times[\mathbf{E}(x,z^+)-\mathbf{E}(x,z^-)]=\mathbf{0}\\&\mathbf{u}_z\times[\mathbf{H}(x,z^+)-\mathbf{H}(x,z^-)]=-\gamma_{TI}\mathbf{u}_z\times\mathbf{E}(x,z^-)\end{aligned}\right\},\quad z=0, \tag{5}$$

where $\mathbf{E}(x,z^{\pm})=\lim_{\nu\to 0}\mathbf{E}(x,z\pm\nu)$ with $\nu\nleq 0$.

A linearly polarized plane wave traveling in the half space $z<-h$ is incident on the device. The wave vector of the incident plane wave lies wholly in the $xz$ plane. The polarization state of the plane wave is either $p$ (i.e., transverse magnetic) or $s$ (i.e., transverse electric) [23,29]. Our objective is to analyze the reflected plane waves in the region $z<-h$ and the transmitted plane waves in the region $z>h_3$.

### 2.2. Fourier representation of relative permittivity

Due to the periodicity of the four-region device along the $x$ axis in the superlattice region $h_1<z<h_2$, its relative permittivity dyadic $\underline{\underline{\varepsilon}}(x,z)$ can be represented as the Fourier series

$$\underline{\underline{\varepsilon}}(x,z)=\sum_{n\in\mathbb{Z}}\underline{\underline{\varepsilon}}^{(n)}(z)\exp(in\kappa_x x),\qquad z\in(-h,h_3),\qquad x\in(-\infty,\infty), \tag{6}$$

where $\kappa_x=2\pi/L$ and $\mathbb{Z}=\{0,\pm1,\pm2,\ldots\}$. The Fourier amplitudes $\underline{\underline{\varepsilon}}^{(n)}(z)$, $z\in(-h,h_3)$, are given by

$$\underline{\underline{\varepsilon}}^{(n)}(z)=\begin{cases}\frac{i}{2\pi n}[(\varepsilon_g-1)e^{-i2\pi nq_1}\underline{\underline{I}}\\\quad+(\underline{\underline{I}}-\underline{\underline{\varepsilon}}_{CTF})e^{-i2\pi nq_2(z)}+(\underline{\underline{\varepsilon}}_{CTF}-\varepsilon_g\underline{\underline{I}})]\,, & z\in(h_1,h_2)\\0\,, & z\notin(h_1,h_2)\end{cases} \tag{7}$$

$\forall\ n\neq 0$, but

$$\underline{\underline{\varepsilon}}^{(0)}(z)=\begin{cases}\varepsilon_{TI}\underline{\underline{I}}\,, & z\in(-h,0)\\\underline{\underline{\varepsilon}}_{CTF}\,, & z\in(0,h_1)\\q_1(\varepsilon_g-1)\underline{\underline{I}}+q_2(z)(\underline{\underline{I}}-\underline{\underline{\varepsilon}}_{CTF})+\underline{\underline{\varepsilon}}_{CTF}\,, & z\in(h_1,h_2)\\\varepsilon_g\underline{\underline{I}}\,, & z\in(h_2,h_3)\end{cases}. \tag{8}$$

where we have used the function