Optics Communications 392 (2017) 86–94

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Tree structure-based bit-to-symbol mapping for multidimensional modulation format

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ARTICLE INFO

Keywords:
Optical communications
Modulation
Mapping

ABSTRACT

Bit-to-symbol mapping is one of the key issues in multidimensional modulation. In an effort to resolve this issue, a tree structure based bit-to-symbol mapping scheme is proposed. By constructing a tree structure of constellation points, any neighboring constellation points become nearest-neighbor constellation points with minimum Euclidean distance, which in turn, changes the bit-to-symbol mapping problem in multidimensional signal modulation from random to orderly. Then, through the orderly distribution of labels, the minimum Hamming distance between the nearest neighboring constellation points is ensured, eventually achieving bit-to-symbol mapping optimization for multidimensional signals. Simulation analysis indicates that, compared with random search mapping, tree mapping can effectively improve the bit error rate performance of multi-dimensional signal modulation without multiple searching, reducing the computational cost.

1. Introduction

With the continuous growth of the communications business, the demand for high-capacity, high-speed and long-distance transmission likewise grows [1–3]. Adopting a high-order modulation format can effectively increase the system transmission capacity, but its low asymptotic power efficiency (APE) causes a high symbol error rate (SER) in long-distance transmission [4,5]. To resolve this problem, multidimensional modulation has been born [6–9]. The multidimensional modulation format simultaneously applies the degrees of freedom of orthogonal states, time slots, frequency bands, and polarization states in the optical field to modulate data. The increased degrees of freedom can maximize the minimum Euclidean distance between constellation points, on the principle of not losing spectrum efficiency (SE), thus increasing the APE of the signals and resolving the conflict between SE and APE in a multidimensional modulation format.

However, while increasing the APE can effectively improve the system SER, it might not necessarily improve the system bit error rate (BER) because the system BER also depends on the bit-to-symbol mapping method. For 2D modulation formats, several mappings such as Gray mapping, Set petitioning (SP) mapping, Modified Set partitioning (MSP) mapping and Maximum Squared Euclidean Weight (MSEW) mapping are introduced in [10–12], they are applied in bit-interleaved coded modulation with Iterative Decoding (BICM-ID) system successfully and obtain good results. However, it is impossible to apply these mappings to multidimensional modulation due to the complexity of the multidimensional modulation constellation diagram. Besides, for multidimensional modulation signal mapping, an exhaustive computer search to find suitable mappings becomes intractable due to high complexity. A constellation diagram containing M points must search M! times to obtain the optimal mapping; Even for the multidimensional modulation format of a medium-level SE, the amount of computation is already overwhelming. For instance, if b=5, the number of searches will reach $2^5! > 10^{35}$. Therefore, in current multidimensional mapping, sub-optimal mapping are designed when the optimum bit mapping is not available. In [13], The optimization scheme based on the binary switching algorithm (BSA) are used in multidimensional mapping. The main idea of BSA is started with a random mapping as initial. Defining a cost functions to measure the BER boundary, for each time searching, the cost of each symbol and the total cost are calculated, and pick the symbol with the highest cost. Then, switch the bit labels of this symbol with the labels of another symbol. If no switch partner can be found for the symbol with the highest cost, the symbol with the second-highest cost will be tried to switch next. This process continues for all the symbols with decreasing costs until a symbol is found that allows a switch that lowers the total cost. The algorithm continues as described above, until no further reduction of the total cost is possible. For BSA mapping, each searching need to calculate the cost of each symbol and the total cost, with the increasing of modulation dimension, the complexity will dramatic increase. In [13], it only achieve optimized mappings for 4-dimensional 8-PSK. The case for other modulation dimensions or other orders can

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http://dx.doi.org/10.1016/j.optcom.2017.01.034
Received 17 October 2016; Received in revised form 22 January 2017; Accepted 23 January 2017
Available online 31 January 2017
0030-4018/

not be achieved. In [14], Millar applied random search (RS) algorithms in high-dimensional modulation for coherent optical communications systems. It can achieve optimized mapping for any modulation dimension and order. The main idea of RS is, for a fix Signal To Noise Ratio (SNR), it tests the BER of many randomly selected labeling, the best labeling is selected when either the improvement expected from testing additional labeling approached zero, or the maximum number of test labeling are reached. Although this method can control the amount of computation, the optimization quality of the RS algorithm is limited by the overall number of samples. The smaller the number of searches, the lower is the optimization quality; the greater the number of searches, the higher is the computational complexity. To the best of our knowledge, no general design algorithm has been proposed to solving bit mapping optimization problems of multidimensional modulation. In this paper we introduce a new bit mapping algorithm for the multidimensional modulation format.

As is known that the joint boundary of the system BER is proportional to the average Hamming distance between the nearest neighboring constellation points (i.e., the points with a minimum Euclidean distance in the constellation point set). In the literature [15–17], Simon et al. defined the average Hamming distance as the Gray penalty G_p and then used G_p as a measure of the quality of the bit mapping optimization. In this paper, G_p is precisely used as a minimization target to design a tree structure-based bit mapping optimization scheme. The main idea is to construct a tree structure that connects M nodes, in which the parent and child nodes are the nearest neighboring constellation points, to guarantee that the nearest neighboring constellation points of any constellation point S_i surround S_i as child nodes. When distributing labels, a label is first assigned to the parent node, and the next label assigned is differing in one bit position from the parent node label to the child node. Through this type of tree mapping, the labels can be distributed in an orderly way to a number M of N-dimensional constellation points, so that the labels of two nearest constellation points have a minimum Hamming distance, thus lowering G_p. Although the choice of the first parent node and the distribution order of child node labels are random, leading to differences in each tree mapping, each tree mapping constructed fully considered the Hamming distance between nearest neighboring constellation points. The resultant mapping has a lower G_p value and smaller fluctuation. Thus, a single tree mapping can achieve optimization, greatly reducing the computational cost of multiple searches. In addition, through the optimization algorithm, the tree mapping can select an optimal high-dimensional mapping scheme at the front-end, with no need for demapping, making the mapping highly reliable with a smaller amount of computation.

The organization of this paper is as follows: Section 2 describes the principle of tree mapping; Section 3 briefly introduces the high-dimensional simulation system; Section 4 gives the simulation results and compares reduction of G_p and system bit error performance between tree mapping and random search (RS mapping); and Section 5 summarizes the paper and presents the conclusions.

2. Tree structure-based bit-to-symbol mapping

2.1. design principle

We use M − ND to represent the N-dimensional modulation format, where $M = 2^b$ is the total number of constellation points, b is the number of bits contained in each constellation point, and N is the modulation dimension. The basic idea of the multidimensional modulation format is to map every b bits onto a constellation point in an N-dimensional space. The SER of the M − ND modulated signal is shown in Eq. (1) [17].

$$P_s = \frac{1}{2^b}\sum_{k=1}^{2^b}\sum_{\substack{j=1\\ j\neq k}}^{2^b} P(\hat{S} = S_k|S = S_j) \tag{1}$$

Here, $P(\hat{S} = S_k|S = S_j)$ represents the conditional probability of S_j as the transmitted symbol and S_k as the received symbol. The relationship between SER and BER of the N-dimensional modulation can be measured by the Gray penalty G_p [12], as shown by Eq. (2): Gp is defined by Eq. (3):

$$P_b = \frac{G_p}{b}\cdot P_s \tag{2}$$

$$G_p = \frac{1}{2^b}\sum_{i=1}^{2^b} G_{S_i} \tag{3}$$

where G_{S_i} is defined as $G_{s_i} = \frac{\sum_{i=1}^{N_N(S_i)} B(S_i, S_j)}{N_N(S_i)}$, $B(S_i, S_j)$ is the number of bits difference between S_i and S_j, and $N_N(S_k)$ represents the number of nearest-neighbor points of constellation point S_k. It can be seen from Eqs. (1)–(3) that if all nearest neighbors of constellation point S_i can receive a label that differs from the label of S_i by only one bit, then G_{S_i} can be minimized, as can G_p. In an N-dimensional space, how can one find the nearest constellation points of an arbitrary constellation point in a number M of order-less constellation points and further perform an orderly distribution of labels with relatively small bit difference? It is possible to construct a tree structure that connects all constellation points and makes the parent and child nodes in the tree structure into nearest-neighbor constellation points; the purpose is to guarantee that all the nearest-neighbor constellation points of any constellation point S_i surround S_i as child nodes. When distributing labels, a label is always first assigned to the parent node, and then labels that only differ from the parent node label by one bit are assigned to other child nodes. The main procedure is as follows: first, any constellation point S_{0i} can be selected as the parent node; by calculating the distance between any two points, all the constellation points that are nearest neighbors of S_{0i} are found and designated as the child nodes of S_{0i}. Then, the child nodes of S_{0i} are used as parent nodes to construct sub-trees, and their nearest neighbors are searched out as child nodes of the sub-trees. Following the same pattern, the child nodes of a point are always used as parent nodes to find their nearest neighbors as child nodes, until all constellation points are connected on the tree. When assigning labels, L_0 was first assigned to the first parent node S_{0i}, and then labels that differ from L_0 by one bit were assigned to the child nodes of S_{0i}. According to this rule, after the parent node receives a label, the labels with the smallest difference from the parent node label are always assigned to its child nodes, until all nodes have received a label. Through this type of orderly label assignment, it is possible to assign the nearest two constellation points labels with minimum Euclidean distance, thus reducing G_{S_i} and lowering G_p and eventually completing the optimization. The specific procedure can be divided into two steps. The first step is to construct a tree structure that contains all constellation points. The second step is to make an orderly assignment of labels to all nodes contained in the tree structure, starting from the first parent node. The specific procedure is as follows:

1. Construct Euclidean distance matrix $D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1M}\\ d_{21} & d_{22} & \cdots & d_{2M}\\ \vdots & & d_{ij} & \\ d_{M1} & d_{M2} & \cdots & d_{MM}\end{bmatrix}$ for a number M of multidimensional constellation points, in which, d_{ij} represents the Euclidean distance between the i-th and j-th constellation points. Initialize the numbering set of the remaining constellation points $\zeta_{po\,\mathrm{int}} = \{1, 2, 3, \ldots, M\}$ and the label number of the first parent node i_0. The first parent node i_0 is selected randomly. Set tree depth N_{layer}. Initialize remaining label set $\gamma_{po\,\mathrm{int}} = \{0, 1, 2, 3\ldots, M\}$. The elements in the set are binary bit labels.