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A PID controller with desired closed-loop time response and stability margin

Xinxin Wang[1], Xiaoqiang Yan[1], Donghai Li[2]

School of mechanical engineering, university of science and technology Beijing, Beijing 100083
E-mail: wangxinxin404@163.com yxqzhw@263.net

State Key Lab of Power Systems, Department of Thermal Engineering, Tsinghua University, Beijing 100084
E-mail: lidongh@mail.tsinghua.edu.cn

Abstract: In this article, we propose a PID tuning method which can simultaneously guarantee desired closed-loop time response and stability margin. For systems without time-delay, dominant pole placement method is used to guarantee the desired closed-loop time response and generalized-frequency method is adopted to give systems the desired stability margin. For systems with time-delay, however, the desired closed-loop time response and the stability margin are obtained through dominant pole placement method and gain-phase margin test method respectively. To demonstrate the effectiveness and confirm the validity of the proposed methodology, examples are provided for illustration.

Key Words: PID controller, the desired closed-loop time response, stability margin

1. Introduction

The PID controllers developed at the earliest is by far the most common control algorithm for many industrial process control applications due to its simple structure, resulting easily to be implemented, and strongly robustness, making it more available apply to industrial environment [1]. Although the PID controller has only three parameters, it is not easy to tune them to obtain the desired goals of the closed-loop dynamics. there exist many methods for finding parameters of PID controllers, for instants, Z-N method, Cohen-Coon method, rule-based empirical tuning, pole placement, gain and phase margin method, internal model control (IMC), optimization methods, robust loop shaping and so on[2].

Pole placement method is popular to design PID controller parameters. But it is impossible to arbitrarily allocate all poles for complicated models which have a number of poles to be constrained necessarily. One effective method to overcome this difficulty is that the complicated model is simplified down to a first- or second-order plus delay model [3]. Dominant pole placement is another method to deal with the difficulty and proved to be an effective PID parameters tuning method, owing to the fact that the closed-loop time response can be expressed by its dominant poles, to give a system the desired closed-loop dynamics. In Ref. [4], dominant pole placement method was detailed stated. But this method don't always guarantee exact dominant pole, that is to say, the specified dominant pole may be fail out dominance as a result of closing to other poles. The problem has been solved in Ref. [5] which guarantee dominant pole placement using root locus and Nyquist plot. But it should be noted that the specifications cannot always be met exactly owing to the closed-loop zeros introduced by PID controllers. Based on Ref. [5], Ref. [6] has deeply analyzed the influence of closed-loop zeros on closed-loop time response.

Not only is set-point response the major issue, but robustness to process uncertainties also is the key specification. There is a large number of setting PID controller parameters methods, which have been published in literature, taking robustness into account. In some cases the methods considered only robustness. Only is gain and phase margin met [7, 8] and only attenuation index is guaranteed by generalized- frequency method [9]. However the most interesting methods are the ones that combine performance and robustness. In Ref. [10], authors combined the Maximum Sensitivity with the IAE index in the set-point and in the load-disturbance. In Ref. [11], authors, using genetic algorithms, searched one point which minimizes the H2 tracking performance under the condition that the H∞ constraint (robustness constraint or disturbance attenuation constraint) is satisfied. Ref. [12] proposed a method that simultaneously met closed-loop time response and phase margin. It is obvious that the drawback is that the method cannot guarantee gain margin. A virtual gain-phase margin tester compensator, in order to meet specified gain margin and phase margin, was proposed [13]. Based on Ref. [13], an approach of setting PID parameters with desired closed-loop time response and guaranteed gain and phase margins for time-delay systems was discussed [14]. But it needs to previously determine derivative gain k_d using other methods, such as integral of square error (ISE), integral of absolute error (IAE), etc.

In this paper, a PID tuning method which can simultaneously guarantee desired closed-loop time response and stability margin are proposed. The rest of the paper is organized as follows. Section 2 respectively discuss how to tune PID controller parameters for systems without time-delay, using dominant pole placement and generalized-frequency method, and for systems with time-delay, using dominant pole placement and gain-phase margin test method. Illustrating examples are shown in section 3. Section 4 is the conclusion.

2. PID Controller Design

In this section, the related methods are described. Closed-loop time response is expressed by dominant poles and generalized-frequency method and gain-phase margins test method are used in order to guarantee robustness. Two simple procedures are proposed to find PID controllers' parameters respectively for systems without and with time-delay meeting closed-loop time response and stability margin.

A. PID parameters for systems without time-delay

1. Range of k_p via dominant pole placement

Consider a plant given by

$$G(\mathrm{s})=\frac{N(s)}{D(s)} \tag{1}$$

where $G(s)$ is a proper and co-prime rational function. The transfer function of PID controller is $C(s)=k_p+\frac{k_i}{s}+k_d s$. For a conventional unity negative feedback system, the closed-loop characteristic equation is

$$1+C(s)G(s)=0\,. \tag{2}$$

Suppose that the dominant poles we choose are $\rho_{1,2}=-a\pm bj$, what it needs is that the real part of other non-dominant poles must less than or equal to $-ka$, where k is usually 3~5.

Substitute the dominant pole, ρ_1, into (2), and we get a complex equation

$$k_p+\frac{k_i}{-a+bj}+k_d\left(-a+bj\right)=-\frac{1}{G\left(-a+bj\right)}\,. \tag{3}$$

Comparing real and imaginary of equation (3), we can get

$$\begin{cases} k_i \quad \dfrac{a^2+b^2}{2a}k_p-\left(a^2+b^2\right)x_1 \\ k_d \qquad \dfrac{1}{2a}k_p+x_2 \end{cases}, \tag{4}$$

where $x_1=\frac{1}{2b}\mathrm{Im}\left[\frac{-1}{G(-a+bj)}\right]+\frac{1}{2a}\mathrm{Re}\left[\frac{-1}{G(-a+bj)}\right]$,

$x_2=\frac{1}{2b}\mathrm{Im}\left[\frac{-1}{G(-a+bj)}\right]-\frac{1}{2a}\mathrm{Re}\left[\frac{-1}{G(-a+bj)}\right]$.

Substitute (4) into (2), we can obtain

$$\frac{1}{k_p}+\overline{G}(s)=0\,, \tag{5}$$

where

$$\overline{G}(s)=\frac{N(s)\left(s^2+2as+a^2+b^2\right)}{2aD(s)s+2ax_2N(s)s^2-2a\left(a^2+b^2\right)x_1N(s)}$$

Using Nyquist criterion [15], the net number of the modified Nyquist plot of $\overline{G}(-ka+j\omega)$ encircling the point $\left(-1/k_p,0\right)$ in the clockwise direction equal to 2, the number of the closed-loop poles of $\overline{G}(s)$ whose real part larger than $-ka$, minus the number of the open loop poles of $\overline{G}(s)$ whose real part larger than $-ka$. This condition can determine the range of k_p guaranteeing dominant poles.

2. Range of k_p via generalized-frequency method

In order to give systems some stability margin, it is common that the dashed fold line is used to distinguish if all closed-loop poles lie in the sector area shown in fig.1, where the equation of the dashed fold line is $s=-|m\omega|+j\omega, m>0, \mathrm{m}=\tan\alpha$. m , known as attenuation index and be a constant, represents stability margin.

In order to allocate all closed-loop poles of $\overline{G}(s)$ in the sector area, according to Nyquist criterion, the net number of the modified Nyquist plot of $\overline{G}(-m\omega+j\omega)$ encircling the point $\left(-1/k_p,0\right)$ in the anticlockwise direction equals to the number of the open loop poles of $\overline{G}(s)$ which lie in the right side of the dashed fold line. This condition can ensure attenuation index larger than m, that is, give system a certain extent stability margin. Then the interval of k_p is obtained.

Finally, combining the interval of k_p from dominant poles constraint with the interval of k_p from stability margin constraint, the parameters of PID controllers are determined and the final positon of closed-loop poles are shown in fig.1. The dominant poles lie in dashed line triangle and other poles shadow area.

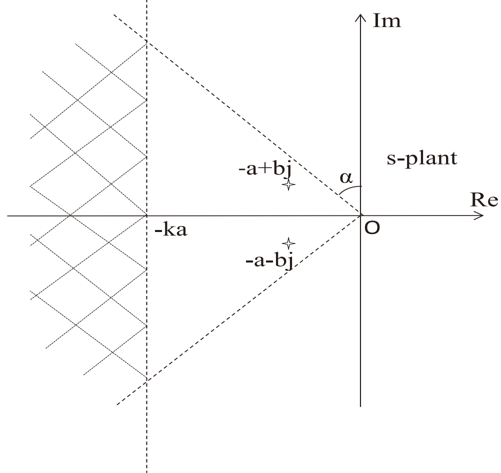

Fig.1.desired placement of poles

B. PID parameters for systems with time-delay

1. Range of k_p via dominant pole placement

For the systems with time-delay, following the procedure outlined in the first subpart in part A of section Ⅱ, the $\overline{G}(s)$ becomes

$$\overline{G}(s)=\frac{N(s)\left(s^2+2as+a^2+b^2\right)e^{-Ls}}{2aD(s)s+2ax_2N(s)s^2e^{-Ls}-2a\left(a^2+b^2\right)x_1N(s)e^{-Ls}} \tag{6}$$

It is impossible to directly obtain the number of the open loop poles of $\overline{G}(s)$ whose real part larger than $-ka$. Therefor the transformation of the denominator of $\overline{G}(s)$ is necessary. It becomes