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Sub- and super-luminal light propagation using a Rydberg state

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ABSTRACT

We present a theoretical study to investigate sub- and super-luminal light propagation in a rubidium atomic system consisting of a Rydberg state by using density matrix formalism. The analysis is performed in a 4-level vee+ladder system interacting with a weak probe, and strong control and switching fields. The dispersion and absorption profiles are shown for stationary atoms as well as for moving atoms by carrying out Doppler averaging at room temperature. We also present the group index variation with control Rabi frequency and observe that a transparent medium can be switched from sub- to super-luminal propagation in the presence of switching field. Finally, the transient response of the medium is discussed, which shows that the considered 4-level scheme has potential applications in absorptive optical switching.

1. Introduction

Over the last two decades, the phenomenon of electromagnetically induced transparency (EIT), in which an initially absorbing medium for a weak probe field becomes nearly transparent in the presence of a strong control field, has been extensively studied in 3-level configurations—lambda (Λ), vee (V), and ladder (Ξ) [1–3]. There are parallel developments to demonstrate Rydberg EIT by using the excited level as Rydberg state in ladder scheme. These studies have been utilized for the measurement of fine and hyperfine splitting of Rydberg states in rubidium [4,5], electro-optic control of atom-light interactions [6], determination of atom-wall-induced light shifts [7], and microwave dressing of Rydberg states [8].

The presence of additional energy levels and laser fields from the usual 3-level EIT systems leads to the modification of transparency window and allows the possibility of electromagnetically induced absorption (EIA)—a phenomenon in which a transparent medium shows enhanced absorption at line center. EIA has been studied—both theoretically as well as experimentally—mainly in 4-level N-type (Λ+V) systems [9–12]. Recently, our group has theoretically shown that EIA resonances are also possible in a new kind of 4-level system in vee +ladder configuration (V+Ξ) [13]. We extended this study to investigate the wavelength mismatch effect in EIA and reported that EIA resonances can be studied using a Rydberg state excited with diode lasers [14].

Just like the imaginary part of the induced coherence on the probe transition can lead to anomalous absorption, the real part can lead to anomalous dispersion. This can be used for applications such as sub- and super-luminal light propagation [15–19]. Enhanced transmission leads to sub-luminal propagation; therefore an EIT medium can only be used for this. On the other hand, enhanced absorption results in super-luminal propagation, and the presence of four levels in an EIA configuration allows for switching between enhanced transmission and enhanced absorption by varying the strengths of the two control fields. Therefore, an EIA medium can be switched between sub- and super-luminal propagation. This kind of switching between the two has also been studied (theoretically) in N-type systems [20–23].

In this work, we present for the first time details of sub- and super-luminal light propagation in a 4-level system *comprising a Rydberg state for the uppermost level*. Rydberg states are important for applications in quantum-information processing, but a scheme involving Rydberg states is not possible in the widely studied N-type systems. Furthermore, in contrast to our work in Ref. [14] where we study only Rydberg EIA, we concentrate here on the dispersion curve (its slope) and refractive index of the medium. We also present the transient response of our system, which shows that the medium can be used for absorptive optical switching.

2. Theoretical considerations

2.1. Density matrix analysis

The 4-level vee+ladder system is shown in Fig. 1. The ground state $|g\rangle$ is coupled with state $|e\rangle$ with a weak probe field. A strong control field is applied between levels $|e\rangle$ and $|r\rangle$ which allows the formation of ladder system. The vee configuration is formed by coupling levels $|g\rangle$

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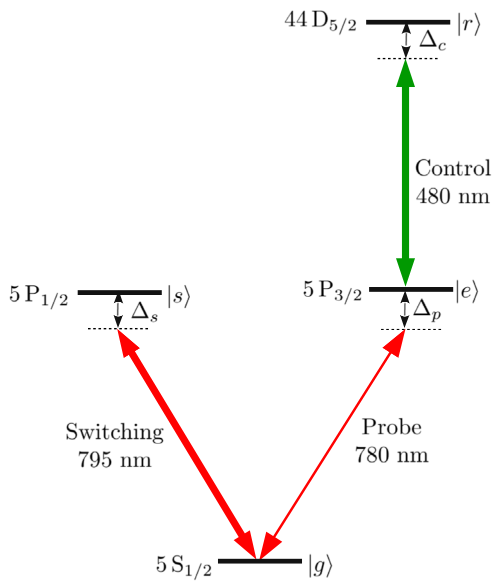

Fig. 1. 4-level vee + ladder system under consideration.

and $|s\rangle$ with a strong switching field. The detunings of respective fields are defined as $\Delta_p = \omega_p - \omega_{eg}$, $\Delta_c = \omega_c - \omega_{re}$ and $\Delta_s = \omega_s - \omega_{sg}$, and their Rabi frequencies and wavelengths are denoted by Ω and λ.

For specificity, we have considered the energy levels of rubidium atom—level $|g\rangle$ is the $5S_{1/2}$ ground state; levels $|e\rangle$ and $|s\rangle$ are $5P_{3/2}$ and $5P_{1/2}$ excited states; and level $|r\rangle$ is the $44D_{5/2}$ Rydberg state. In this case, the wavelengths of applied fields are: $\lambda_p = 780$ nm , $\lambda_s = 795$ nm and $\lambda_c = 480$ nm. Also, the decay rates of these states are: $\Gamma_g = 0$, $\Gamma_e/2\pi = 6.1$ MHz, $\Gamma_s/2\pi = 5.9$ MHz and $\Gamma_r/2\pi = 0.3$ MHz. Since the probe field is weak, it ensures that levels $|e\rangle$ and $|r\rangle$ are always unpopulated and population cycles between levels $|g\rangle$ and $|s\rangle$. This level scheme has been utilized for the realization of EIA using a Rydberg state [14].

Using the density matrix formalism, the optical Bloch equations—by incorporating the decay of atoms from each level and repopulation from excited levels—for the populations are written by

$$\begin{aligned}
\dot{\rho}_{gg} &= \Gamma_e\rho_{ee} + \Gamma_s\rho_{ss} + \tfrac{i}{2}\Omega_p\left(\rho_{eg}-\rho_{ge}\right) + \tfrac{i}{2}\Omega_s\left(\rho_{sg}-\rho_{gs}\right)\\
\dot{\rho}_{ee} &= -\Gamma_e\rho_{ee} + \Gamma_r\rho_{rr} + \tfrac{i}{2}\Omega_p\left(\rho_{ge}-\rho_{eg}\right) + \tfrac{i}{2}\Omega_c(\rho_{re}-\rho_{er})\\
\dot{\rho}_{ss} &= -\Gamma_s\rho_{ss} + \tfrac{i}{2}\Omega_s\left(\rho_{gs}-\rho_{sg}\right)\\
\dot{\rho}_{rr} &= -\Gamma_r\rho_{rr} + \tfrac{i}{2}\Omega_c(\rho_{er}-\rho_{re})
\end{aligned} \tag{1}$$

and the equations for the coherences can be written as follows

$$\begin{aligned}
\dot{\rho}_{eg} &= \gamma_{eg}\rho_{eg} - \tfrac{i}{2}\Omega_p\left(\rho_{ee}-\rho_{gg}\right) - \tfrac{i}{2}\left(\Omega_s\rho_{es}-\Omega_c\rho_{rg}\right)\\
\dot{\rho}_{sg} &= \gamma_{sg}\rho_{sg} - \tfrac{i}{2}\Omega_s\left(\rho_{ss}-\rho_{gg}\right) - \tfrac{i}{2}\Omega_p\rho_{se}\\
\dot{\rho}_{rg} &= \gamma_{rg}\rho_{rg} + \tfrac{i}{2}\left(\Omega_c\rho_{eg}-\Omega_p\rho_{re}-\Omega_s\rho_{rs}\right)\\
\dot{\rho}_{se} &= \gamma_{se}\rho_{se} + \tfrac{i}{2}\left(\Omega_s\rho_{ge}-\Omega_p\rho_{sg}-\Omega_c\rho_{sr}\right)\\
\dot{\rho}_{re} &= \gamma_{re}\rho_{re} - \tfrac{i}{2}\Omega_c(\rho_{rr}-\rho_{ee}) - \tfrac{i}{2}\Omega_p\rho_{rg}\\
\dot{\rho}_{rs} &= \gamma_{rs}\rho_{rs} + \tfrac{i}{2}\left(\Omega_c\rho_{es}-\Omega_s\rho_{rg}\right)
\end{aligned} \tag{2}$$

The γ's are defined as

$$\begin{aligned}
&\gamma_{eg} = -\tfrac{\Gamma_e}{2} + i\Delta_p; \quad \gamma_{sg} = -\tfrac{\Gamma_s}{2} + i\Delta_s;\\
&\gamma_{rg} = -\tfrac{\Gamma_r}{2} + i(\Delta_p + \Delta_c);\ \gamma_{se} = -\tfrac{\Gamma_s+\Gamma_e}{2} + i(\Delta_s-\Delta_p);\\
&\gamma_{re} = -\tfrac{\Gamma_r+\Gamma_e}{2} + i\Delta_c;\ \ \gamma_{rs} = -\tfrac{\Gamma_r+\Gamma_s}{2} + i(\Delta_p + \Delta_c-\Delta_s).
\end{aligned}$$

The steady state solution of coupled density matrix-equations for ρ_{eg} under weak probe conditions is given as

$$\rho_{eg} = -\frac{i\Omega_p\rho_{gg}}{2\gamma_{eg}\beta} + \frac{i\Omega_p\Omega_s^2(\rho_{gg}-\rho_{ss})}{8\gamma_{eg}\gamma_{gs}\gamma_{es}\beta}\left(1-\frac{\Omega_c^2}{4\gamma_{es}\gamma_{rs}\alpha}-\frac{\Omega_c^2}{4\gamma_{rg}\gamma_{rs}\alpha}\right) \tag{3}$$

where

$$\rho_{gg}-\rho_{ss} = \left[1 + \frac{\Omega_s^2}{2\left(\frac{\Gamma_s^2}{4}+\Delta_s^2\right)}\right]^{-1}$$

$$\beta = 1 + \frac{\Omega_s^2}{4\gamma_{eg}\gamma_{es}} + \frac{\Omega_c^2}{4\gamma_{eg}\gamma_{rg}} - \frac{\Omega_s^2\Omega_c^2}{16\gamma_{eg}\gamma_{rs}\alpha}\left[\frac{1}{\gamma_{es}} + \frac{1}{\gamma_{rg}}\right]^2$$

$$\alpha = 1 + \frac{\Omega_s^2}{4\gamma_{rg}\gamma_{rs}} + \frac{\Omega_c^2}{4\gamma_{es}\gamma_{rs}}$$

In the present work, the observable is the response of atoms to the weak probe field. The dispersion (η) and absorption ($\mathcal{A}$) of probe field is proportional to real and imaginary parts of ρ_{eg}, and are given by [24]

$$\eta = \mathrm{Re}\left\{\frac{\rho_{eg}\Gamma_e}{\Omega_p}\right\} \quad \text{and} \quad \mathcal{A} = \mathrm{Im}\left\{\frac{\rho_{eg}\Gamma_e}{\Omega_p}\right\} \tag{4}$$

2.2. Group index and group velocity

The group index of the probe field can be evaluated in terms of susceptibility (χ) through the relation

$$n_g = 1 + \frac{\mathrm{Re}\{\chi\}}{2} + \frac{\omega_p}{2}\frac{\partial \mathrm{Re}\{\chi\}}{\partial\omega_p} \tag{5}$$

where

$$\chi = \frac{N|\mu_{eg}|^2}{\hbar\varepsilon_0\Omega_p}\rho_{eg}$$

and corresponding group velocity is

$$v_g = \frac{c}{n_g} \tag{6}$$

Here, c is the velocity of light in vacuum, ω_p is probe frequency, N is the atom number density in the medium, and $|\mu_{eg}|$ is the magnitude of dipole matrix element between levels $|e\rangle$ and $|g\rangle$. The above equation shows that v_g is inversely proportional to the slope of dispersion curve. Large group index ($n_g \gg 1$) due to steep positive dispersion leads to sub-luminal ($v_g \ll c$) propagation. On the other hand, small ($n_g < 1$) or negative group index due to negative dispersion shows super-luminal ($v_g > c$) propagation.

2.3. Doppler averaging

For an atomic vapor at room temperature, we have to account for thermal velocity distribution by carrying out Doppler averaging, which is the typical condition used for an experimental realization. We consider an atom with velocity v interacting with co-propagating probe and switching fields, and a counter-propagating control field [13,14]. Since the wavelengths of probe and control fields are mismatched, the two photon absorption for the ladder sub-system is non-Doppler free, i.e. $k_p v - k_c v \neq 0$, and probe absorption spreads over different velocity classes [14].

We perform Doppler averaging using the one-dimensional Maxwell-Boltzmann velocity distribution. For an atom of mass M at temperature T, the Maxwell-Boltzmann distribution of velocities is given by

$$f(v)dv = \sqrt{\frac{M}{2\pi k_B T}}\exp\left(-\frac{Mv^2}{2k_B T}\right)dv \tag{7}$$