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The optical Anderson localization in three-dimensional percolation system

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ABSTRACT

We study the optical Anderson localization associated with the properties of three-dimensional (3D) disordered percolation system, where the percolating clusters are filled by active media composed by light noncoherent emitters. In such a non-uniformly spatial structure the radiating and scattering of field occur by incoherent way. We numerically study 3D field structures where the wave localization takes place and propose the criterion of field localization based on conception of a mean photon free path in such system. The analysis of a mean free path and the Inverse participation ratio (IPR) shows that the localization arises closely to the threshold of 3D percolation phase transition.

1. Introduction

Disordered photonic materials can diffuse and localize light through random multiple scattering, offering opportunities to study mesoscopic phenomena, control light–matter interactions, and provide new strategies for photonic applications [1]. Wave transport in disordered systems is a fast developed topic, one of that is the optical wave in random dielectrics [2]. In disordered optical materials, the multiple scattering of light and the interferences between propagating waves lead to the formation of electromagnetic modes with varying spatial extent, depending on scattering strength, structural correlations, and dimensionality [3] of the system [4,5]. This combination leads to a series of interesting physical effects and also creates large potential for new disorder-based optical applications [6].

It is well-known that at the optical Anderson localization (OAL) the counter-propagating waves form closed loops; interference along these loops lead to randomly shaped standing-wave patterns confining the light [6,7]. In one and two dimensions, the chance of coming back to the same region a necessary requirement to form a loop is much higher than in three dimensions. For this reason, observing OAL in three dimensions is extremely difficult for light waves [6]. Three-dimensional disordered structures have been studied recently for investigation of complex optical phenomena, including light localization [6–11], and random lasing [12–14]. In most lasing random materials, the intensity is spread throughout the sample and, in general, there are several lasing modes. In certain cases interference of different modes can lead to light localization [15,16].

The study for Anderson transition in 3-D optical systems still has not been conclusive despite considerable efforts. The localization transition may be difficult to reach for the light waves due to various effects in dense disordered media required to achieve strong scattering [see e.g. [17] and references therein]. The experimental observation of OAL [18] just below the Anderson transition in an open 3D disordered medium shows strong fluctuations of the wave function that leads to nontrivial length-scale dependence of the intensity distribution (multifractality). Such behavior can be specified by varying the system size and quantified deeper by using the generalized inverse participation ratios (IPR).

Various schemes can be proposed to study OAL, e.g. the use a coherently prepared three-level atomic medium provides a disordered scheme for realizing the Anderson localization (see [19] and references therein).

Other perspective 3D disordered systems where the optical transport was studied belong to percolating crystals. In such materials the optical transparency assisted by disordered porous clusters was observed [20]. Also it was studied some interesting properties of optical nanoemitters incorporated into 3D spanning cluster [21]. In supercritical state, the field intensity is large enough to produce a dynamic high-density coherent field. For material with small losses the long-term coherence arises in the area close to the percolation threshold. It is found also the random lasing assisted by nanoemitters incorporated into such a disordered structure [22]. In this system the spanning cluster produces a global percolation that results in qualitatively modification of its spatial properties. One can argues that already in a vicinity of the percolating phase transition the fractal dimension of such system $D_H \simeq 2.54$ considerably differs from the dimension of the embedded space D=3 (multifractality). One of important question is whether the optical Anderson localization [23] can still be archived for

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a non-integer dimension case with a fractal (Hausdorff) dimension of $D_H < 3$, where the strong randomness for properties of the system is expected.

In this paper we study the optical localization associated with the structure of three-dimensional (3D) disordered percolation system, where the percolating clusters are filled by active media composed by light noncoherent emitters. In such a non-uniformly spatial structure the radiating and scattering of field occur by incoherent way. We have studied the range of parameters where the wave localization can take place. The Fermat principle and Monte Carlo approach are applied to characterize the optimal optical path and interconnection between the radiating emitters. This allows to consider the average free path of light as a mean that in a simplest case is a ratio of the length of path to number of emitters. This leads to formulating of condition when the localization can occur (similarly to Ioffe-Regel criterion) for a percolating model. Our 3D simulations allow calculating of such free path to study the situation where such condition can be satisfied. FDTD numerical simulations have shown that in such a system the localized optical modes arise closely to the percolation threshold. We also studied the properties of associated field inverse participating ratio (IPR) and found that mean free path, IPR and the percolation order parameter show well-pronounced critical behavior near the percolating threshold. Considered in this paper random percolating system is specially challenging due to its spatial inhomogeneity and also multifractality in 3D.

The paper is organized as follows. In Section 2 we formulate the main equations. In Section 3 we present the numerical results on the field distribution generated by the emitters in percolating medium. Section 4 contains the approach and formulation the conditions of optical localization in percolating system. In Section 5 we study the properties of the inverse participation ratio (IPR) for percolating systems, and the last sections contain the discussion and conclusions.

2. Basic equations

We study the integral emission of electromagnetic energy from a cubical sample $(x, y, z) \in [0, l_0]$. The output flux of energy can be written as

$$I = \oint_S (\mathbf{K}\cdot\mathbf{n})dS = I_x + I_y + I_z, \tag{1}$$

where $\mathbf{K}$ is the Pointing vector, $\mathbf{n}$ is the normal unit vector to the surface S of cube, and $I_{x,y,z}$ indicate the fluxes from two faces of the cube perpendicular to a particular direction. To find the emission from the system we solve numerically the equation that couples the polarization density $\mathbf{P}$, the electric field $\mathbf{E}$, and occupations of the levels of emitters. In the case of uncoupled emitters this equation is [24]

$$\frac{\partial^2\mathbf{P}}{\partial t^2} + \Delta\omega_a\frac{\partial\mathbf{P}}{\partial t} + \omega_a^2\mathbf{P} = \frac{6\pi\varepsilon_0 c^3}{\tau_{21}\omega_a^2}(N_1 - N_2)\mathbf{E}. \tag{2}$$

Here $\Delta\omega_a = \tau_{21}^{-1} + 2T_2^{-1}$, where T_2 is the mean time between dephasing events, τ_{21} is the decay time from the second atomic level to the first one, and ω_a is the frequency of radiation. The electric and magnetic fields, $\mathbf{E}$ and $\mathbf{H}$, and the current $\mathbf{j} = \partial\mathbf{P}/\partial t$ are found from the Maxwell equations, together with the equations for the densities $N_i(\mathbf{r}, t)$ of atoms residing in $i-$ th level. In the case of four level laser $i = 0, 1, 2, 3$ these rate equations read (see [25] and references therein)

$$\frac{\partial N_3}{\partial t} = A_r N_0 - \frac{N_3}{\tau_{32}},\ \frac{\partial N_2}{\partial t} = \frac{N_3(t)}{\tau_{32}} + \frac{(\mathbf{j}\cdot\mathbf{E})}{\hbar\omega_a} - \frac{N_2}{\tau_{21}}, \tag{3a}$$

$$\frac{\partial N_1}{\partial t} = \frac{N_2(t)}{\tau_{21}} - \frac{(\mathbf{j}\cdot\mathbf{E})}{\hbar\omega_a} - \frac{N_1}{\tau_{10}},\ \frac{\partial N_0}{\partial t} = \frac{N_1}{\tau_{10}} - A_r N_0. \tag{3b}$$

An external source excites emitters from the ground level (i=0) to third level at a certain rate A_r, which is proportional to the pumping intensity in experiments. After a short lifetime τ_{32}, the emitters transfer nonradiatively to the second level. The second level and the first level are the upper and the lower lasing levels, respectively. Emitters can decay from the upper to the lower level by both spontaneous and stimulated emission, and $(\mathbf{j}\cdot\mathbf{E})/\hbar\omega_a$ is the stimulated radiation rate. Finally, emitters can decay nonradiatively from the first level back to the ground level. The lifetimes and energies of upper and lower lasing levels are τ_{21}, E_2 and τ_{10}, E_1, respectively. The individual frequency of radiation of each emitter is then $\omega_a = (E_2 - E_1)/\hbar$.

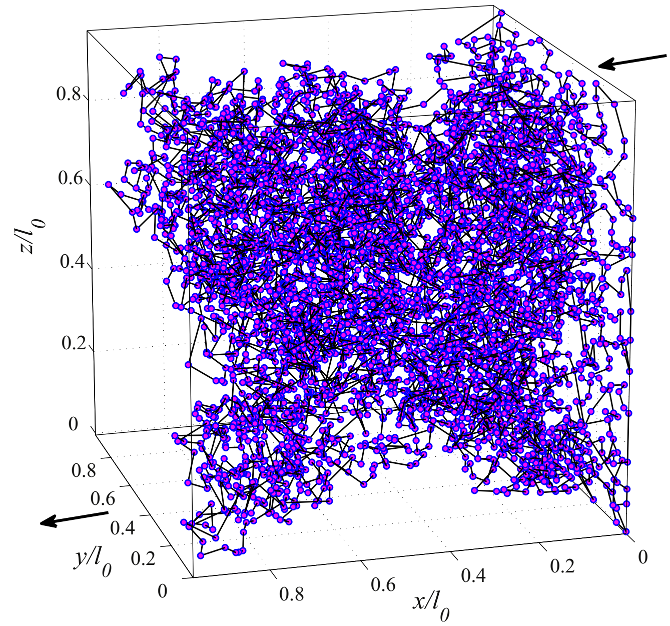

Fig. 1. Typical spatial structure of the incipient percolating cluster near the percolation threshold at p=0.32 in the cube $l_0 \times l_0 \times l_0$, where $l_0 = 10^{-4}m$. The cluster is shown for $50 \times 50 \times 50$ numerical grid. Only clusters coupled to the spanning cluster are shown, while all the internal clusters unconnected to the entry side are not displayed. In this configuration considerable quantity of the emitters are incorporated closely to the entry side (indicated by incoming arrow) of the crystal. The solid line connects the nodes jointed with the use of the variational Fermat's principle. See details in Section 4.

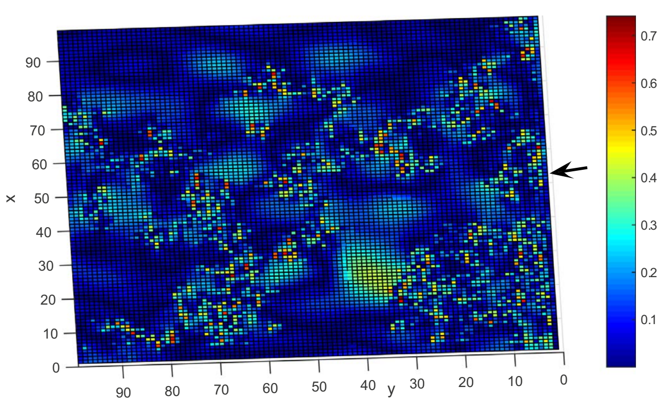

Fig. 2. The typical field distribution (in central intersection) $|E_x|$ in the percolating system with p=0.316 nearly p_c for cub with L=100. In this case the number of emitters N is about 10^5. Small color squares display the radiated field in position of emitters inside the cluster. However the spot around $x = 20$, $y = 40$ exhibits the amplitude of localized mode beyond the cluster. Other modes are generated too, but with lesser amplitudes.

Below we consider the situation when incipient percolating cluster is completely filled with light sources. Such cluster (simple cubic lattice) is shown in Fig. 1, where all internal uncoupled clusters have been omitted. We indicate that the percolation cluster in Fig. 1 has a typical dendrite shape that, however, depends on the actual random sampling. Re-running simulation with another random seed value will lead to percolation cluster with somewhat different geometry, which will also have similar sponge structure. The cluster is grown in the x-