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Multipole expansion of circularly symmetric Bessel beams of arbitrary order for scattering calculations

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ABSTRACT

A rigorous, simple and efficient approach is derived in this paper for multipole expansion of a circularly symmetric Bessel beam. Different from the existing rigorous methods which are based on the plane wave spectrum of a Bessel beam, a straight-forward integral procedure is presented in a traditional way to obtain the analytical expressions of the expansion coefficients, also called beam shape coefficients (BSCs). The convergence and correctness of the BSCs are verified numerically in detail for both on-axis and off-axis cases. The results in this paper are useful in various analytical scattering theories, such as the generalized Lorenz-Mie theory and the Null-field method, when a Bessel beam is considered.

1. Introduction

Analyses of interactions between shaped beams and small particles became more and more important in recent years due to their essential roles in optical characterization, optical trapping and manipulation, remote sensing and others [1,2]. Concerning shaped beams, there has been an increasing interest in Bessel beams [3,4], which is mainly due to its special properties, including propagation invariance, self-reconstruction, and the transfer of orbital angular momentum as well as spin angular momentum to matter. Prospective applications of Bessel beams can be found in wide range of fields, such as optical communication, biomedicine, optical manipulation, and imaging [5–7].

When describing a shaped beam for use in analytical scattering theories, such as the generalized Lorenz-Mie theories (GLMTs) [8] and the Null-field method [9], electric and magnetic fields are required to be expanded in terms of proper wave harmonics, e.g. vector spherical wave functions (VSWFs) for isotropic medium, or quasi-VSWFs for anisotropic medium [10]. The calculation of expansion coefficients, or the sub-coefficients which are called as beam shape coefficients (BSCs), is one of the key issues when dealing with any type of shaped beam. With decades of efforts devoted to the description of an arbitrary shaped beam, the BSCs of an arbitrary shaped beam can be evaluated by several methods [11], including quadratures, finite series, localized approximations (LA), and the integral localized approximation (ILA) [12]. In the case of Gaussian beams, whose field expressions are not exact solutions to Maxwell's equations, the most efficient method for evaluating the BSCs has been the LA method [13]. The reconstructed fields based on LA BSCs are exact solutions to Maxwell's equations, which provide good approximations to the origin fields. The LA method has also been applied to the scattering of Bessel beams in several studies [14–16]. Application of LA for a Bessel beam is valid when the half-cone angle of the Bessel beam is relatively small [15]. However, significant errors occur when the half-cone angle is sufficiently large [17,18]. Therefore, a rigorous and efficient way for the calculation of BSCs of a Bessel beam is needed.

Accurate BSCs of a Bessel beam can be obtained by a double quadrature over spherical coordinates, which is the original method used in the GLMTs [19,20]. Numerical evaluation of double quadrature for a zero-order Bessel beam was used by Preston et al. [21], and was also applied to a high-order Bessel beam by Mitri [22]. Although it is very time-consuming and complex in the numerical evaluation, this method provides accurate BSCs which can be used for validation of BSCs obtained using other approximate methods [14]. Double quadrature can be reduced to single quadrature for a zero-order Bessel beam, as shown by Cizmar et al. [23]. This reduction of quadrature was achieved based on an angular spectrum representation (ASR) of a Bessel beam, where the Bessel beam is regarded as a superposition of partial plane waves with delta distribution in polar angle $\delta(\alpha - \alpha_0)$. Based on the ASR description of a zero-order Bessel beam, the results were further improved by Taylor and Love [24] who derived an

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analytical expression of BSCs without integral. The BSCs were obtained by a superposition of the expansion coefficients of partial plane waves which consist of the plane wave spectrum of a Bessel beam. The calculation of BSCs of Bessel beams based on the angular spectrum representation was also analyzed by Lock [25], where a general zero-order Bessel beam was considered. The same procedure was also extended to the case of polarized Bessel beams of arbitrary order by Chen et al. [26], and was applied by Ma and Li [27] to a study of an unpolarized Bessel beam.

BSCs in analytical form allow scattering calculations to be carried out considerably more efficiently and accurately. This is of advantage for problems where a large number of scattering calculations must be performed, e.g. forces and torque prediction in an optical tweezers [28]. The existing approaches for calculating BSCs analytically are based on the ASR description of a Bessel beam. In these approaches, a Bessel beam is required to be represented as a plane wave spectrum using the ASR, and then analytical expressions of BSCs are obtained by a superposition of the expansion coefficients of each partial plane wave. In this paper, we show that the analytical expressions of BSCs of a circularly symmetric Bessel beam (whose energy density and Poynting vector component along its propagation direction are circularly symmetric in the transverse plane) can be obtained in a straightforward way by performing the integrals directly, it is a simpler approach than the existing methods.

The other parts of this paper is organized as follows. Derivations of analytical expressions of BSCs is presented in Section 2 for a circularly symmetric Bessel beam. The correctness and convergence of the BSCs are verified numerically in detail in Section 3. Conclusions are given in Section 4.

2. Derivations of beam shape coefficients

A geometry of a spherical particle illuminated by an off-axis Bessel beam is shown in Fig. 1. Two Cartesian coordinate systems, $Oxyz$ and $O_b uvw$, are used. The $Oxyz$ is attached to the particle and the $O_b uvw$ is attached to the Bessel beam. The axes $O_b u$, $O_b v$ and $O_b w$ are parallel to the axes Ox, Oy and Oz, respectively. The coordinates of O_b in $Oxyz$ are denoted as $(x_0,\ y_0,\ z_0)$. In the description of an ideal Bessel beam, two different procedures are commonly applied to obtain the fields of an l-order Bessel beam: (a) the ASR procedure which obtains the fields by a superposition of partial plane waves, and (b) the Davis procedure which obtains the fields from a polarized vector potential. Although the two different procedures give two seemingly different answers for the fields, it turns out that the functional dependence of the two answers is identical for circularly symmetric Bessel beams. A general description for circularly symmetric Bessel beams was derived recently [29,30]. This generalization of the description makes the Davis type Bessel beam and the ASR type Bessel beam merely the two simplest cases of an infinite number of possible circularly symmetric Bessel beams, corresponding to different values of the arbitrary function $g(\alpha_0)$. The electric field components of a general circularly symmetric Bessel beam with its beam center locating at an arbitrary point $(x_0,\ y_0,\ z_0)$ are

$$\begin{aligned} E_x^{(1,0)} &= E_0 g(\alpha_0) e^{-ik_z(z-z_0)}\Big\{(1 + cos\alpha_0)(-i)^l e^{il\phi_G} J_l(\sigma_G) - \frac{1}{2}(1 - cos\alpha_0)\times \\ &\quad [(-i)^{l-2} e^{i(l-2)\phi_G} J_{l-2}(\sigma_G) + (-i)^{l+2} e^{i(l+2)\phi_G} J_{l+2}(\sigma_G)]\Big\} E_y^{(1,0)} \\ &= E_0 g(\alpha_0) e^{-ik_z(z-z_0)} \frac{1}{2i}(1 - \cos\alpha_0) \\ &\quad [(-i)^{l-2} e^{i(l-2)\phi_G} J_{l-2}(\sigma_G) - (-i)^{l+2} e^{i(l+2)\phi_G} J_{l+2}(\sigma_G)] E_z^{(1,0)} \\ &= -E_0 g(\alpha_0) e^{-ik_z(z-z_0)} \sin\alpha_0 [(-i)^{l-1} e^{i(l-1)\phi_G} J_{l-1}(\sigma_G) \\ &\quad + (-i)^{l+1} e^{i(l+1)\phi_G} J_{l+1}(\sigma_G)], \end{aligned} \tag{1}$$

where superscript $(1,\ 0)$ which is reminiscent of x-polarization is used, and $\sigma_G = k_t\rho_G$, $\rho_G = [(x - x_0)^2 + (y - y_0)^2]^{1/2}$, $\phi_G = \tan^{-1}[(y - y_0)/(x - x_0)]$. The transverse and longitudinal wave numbers are $k_t = k \sin\alpha_0$ and $k_z = k\cos\alpha_0$, respectively. The l-order Bessel function of the first kind is denoted as $J_l(\cdot)$. The wavenumber is k, and α_0 is the half-cone angle of the Bessel beam which is defined with respect to the axis of wave propagation. When $g(\alpha_0) = (1 + cos\alpha_0)/4$, the expressions in Eq. (1) reduce to those of a Davis circularly symmetric Bessel beam used in [25,31]. When $g(\alpha_0) = 1/2$, they reduce to those of an ASR Bessel beam used in [23,24,26]. The expressions for magnetic fields are not presented for the sake of brevity, since they can be obtained from electric fields in Eq. (1) by the relation $\mathbf{B}(\mathbf{r}) = (i/\omega)\nabla \times \mathbf{E}(\mathbf{r})$. The time dependence $\exp(i\omega t)$ is assumed in this paper.

Following the theoretical treatments in the GLMT, the radial electric and magnetic field components derived using the Bromwich scalar potentials are (Sec. III.3 in [8])

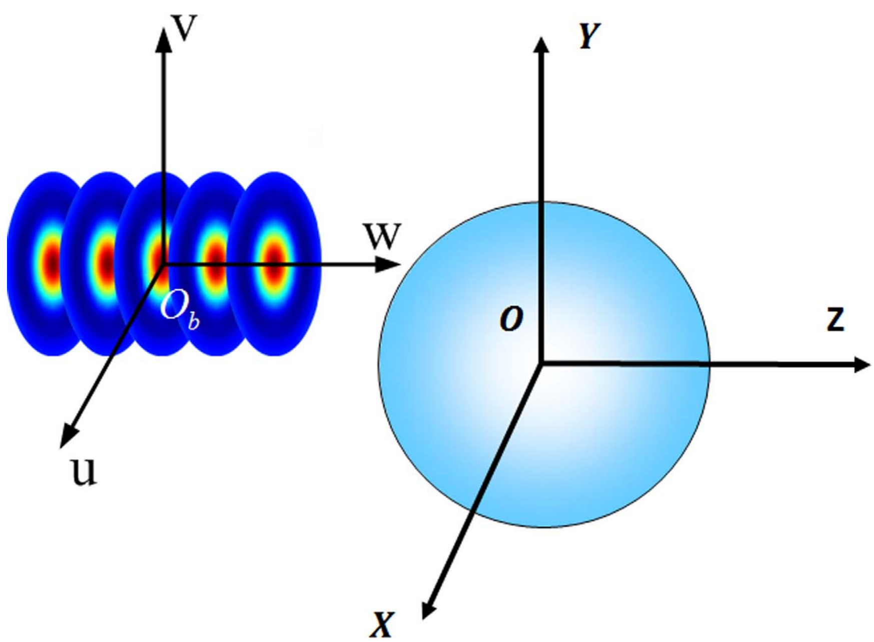

Fig. 1. Geometry of a spherical particle illuminated by an off-axis Bessel beam. A Cartesian coordinate system $OXYZ$ is attached to the particle, and a Cartesian coordinate system $O_b uvw$ is attached to the Bessel beam.