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Optical complexity in external cavity semiconductor laser

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ABSTRACT

In this article, the window based complexity and output modulation of a time delayed chaotic semiconductor laser (SL) model has been investigated. The window based optical complexity (OC), is measured by introducing the recurrence sample entropy (SampEn). The analysis has been done without and in the presence of external noise. The significant changes in the dynamics can be observed under induced noise with weak strength. It has also been found that there is a strong positive correlation between the output power and the complexity of the system with various sets of parameters. The laser intensity, as well as the OC can be increased with the incremental noise strength and the associated system parameters. Thus, optical complexity quantifies the system dynamics and its instabilities, since is strongly correlated with the laser outputs. This analysis can be applied to measure the laser instabilities and modulation of output power.

1. Introduction

During recent years, semiconductor lasers (SL) [1,2] have been a potential area of theoretical and experimental investigations [3,4]. A SL model with external cavities conveys various dynamical phenomena due to the changes of its key parameters like feedback strength, feedback delay, pumping current, etc. The dynamics can be described by the nature of instabilities in a delayed SL system. The theoretical model corresponding to the dynamics of a single mode SL has been introduced in 1980 [5]. One of the interesting applications of such lasers is chaos-based optical communications [6,7] in which chaotic signals can be effectively used to encrypt a message and improve privacy and security in data transmission. These systems define the dynamics, demonstrate a chaotic regime when subjected to chaotic oscillations by injection from another source [8], reflection from an external mirror [9], and also changes in the other associated parameters. It also has been reported that the output power (laser intensity) can be modulated [10] by changing the associated parameters and under the influence of an additive noise with significant noise efficiency. In [11,12], the results reveal that the size of the external cavity plays a crucial role in optimizing the output power when subjected to optical feedback. The effect of additive Gaussian noise on the dynamics of an SL has been a subject of active research for the past few decades. Gaussian noise plays a stabilizing and destabilizing role on the dynamics and also exhibits a wide variety of noise induced phenomena. Of late, it has been reported that the use of Gaussian noise is not always appropriate as it is unbounded and there exists a non-zero probability for Gaussian noise for having very large values. This causes an unexpected complicated change in the dynamics of an SL. Non-Gaussian bounded noise can play a vital role in the study of the dynamics of a model system with known parameter values causing random fluctuations. However, this kind of noise has rarely been reported due to the lack of mathematical tools so as to obtain analytical results. In our previous analysis [13], we also reported the effect of Sine-Wiener noise [14,15], a non-Gaussian bounded noise having zero mean and stationary correlation function.

The induced noise effect can be investigated well with $\frac{1}{f^{\beta}}$ noise. It can show various color spectrum as well as flicker noise with varying β. A brownian motion can also have a spectrum with β=2. The effect of $\frac{1}{f^{\beta}}$ can also be observed in several real experiments and natural phenomenon like earthquakes, experimental results on sea level and bermuda etc.

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Complexity is a measure which quantifies the amount of instability in a system. In fact, it can measure the complex agreement of state variables in a dynamical system [16–19]. It has been studied that such unpredictable behavior can be quantified by information entropy, introduced by Shanon [20]. However, various entropy measures like Permutation entropy [21–25], Sample entropy(SampEn) [26,27], KolmogorovSinai (K-S) entropy [28,29], Renyi entropy [30,31], spectral entropy (SE) [32] can be used to measure the complexity of a system. In [33] it has been observed that modified SampEn can successfully quantify the complexity of a single mode SL system. Since entropy is a statistical invariant measure, it is obvious that such invariant information does not possess similar trend in each partition of the phase space of a system. We have introduced a window based measure - optical complexity (OC) by implementing the concept of SampEn. Windowing is a kind of scaling technique which samples a signal with different time resolutions. Various methods can be used to scale a signal [34–36]. Multi-scaling is one of the promising tools in nonlinear time series analysis [34–37]. In our article, a uni-scale windowing technique [36] has been applied in windowing the phase space of the SL system. The implementation is done by modification of aforesaid SampEn with recurrence based window structure of the phase space.

It has been noted in theoretical works that the OP can be maximized when the laser is in a chaotic state [11]. In the external cavity SL, the light produced by a laser diode is directed back into its active layer upon reflections by the external mirrors. The optical delay generated by the external cavity time may result in strongly nonlinear behavior and chaos [38,39], that may eventually constructively affect the OP. In case of laser diodes, external currents also produce similar instabilities [40]. For instance, in the case of Lorenz-Haken phenomenon, a high gain and low line-width is required to achieve the instabilities in the dynamics. Haken suggested pumping the laser 20 times more than the first conventional threshold [41,42], to get a second one, above which one can achieve strong nonliterary and the Lorenz Haken chaos. The power enhancement related to the chaotic regimes, that has been theoretically predicted and numerically simulated, has been experimentally observed as well [43]. In particular, lasers diodes have been found indeed to be highly sensitive to optical feedback and to enjoy much higher output in the chaotic states than in periodic states. At the same time, the OC defined in terms of recurrence based measures, behaves similarly to the largest Lyapunov exponent [29,44], which implies that whenever the system is in periodic or multi-periodic state, it is relatively low, compared to its values in the chaotic states. In turn, the window based Sample entropy extracts information about the dynamics and its instabilities in phase space. Since the dynamics (intensity) and the instability (complexity) are associated by the information generation, they have a strong correlation.

The manuscript is organized as follows: In Section 2 we investigated the dynamics of a delayed SL-model with two external cavities without and in presence of external noise. It has been observed that the periodic structures transferred to be chaotic, in the presence of the external noise. The dynamics has been quantified by single and two parameter bifurcation diagrams followed by 0–1 test [45]. The 0–1 test is based on the measure 'mean square displacement' (MSD), from the diffusive and non diffusive part of a time series. For regular dynamics, the MSD is a bounded function of time; where as it scales linearly with time in case of chaotic state. The asymptotic growth (K_c) of MSD can be the measure to quantify the dynamics of a system or a time series. It can be applicable for a deterministic as well as stochastic dynamics [46]. The value K_c close to 1 and 0 indicates chaotic and regular dynamics respectively. Significant changes can be observed in the dynamics of the SL model under the effect of external noise. Section 3 introduces the measure OC, which is based on windowing sample entropy. The changes of the complexity with various parameters in the presence of noise are also investigated in details. The analysis of output power, its modulation and its relation to the optical complexity has also been observed. Section 5 is the conclusion.

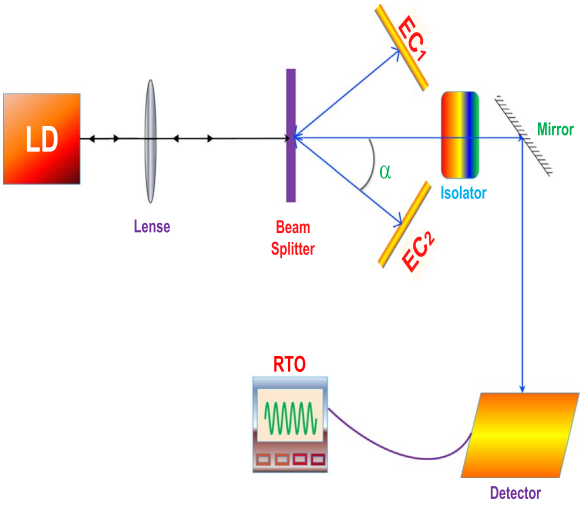

Fig. 1. (Color online) The schematic diagram of two delayed SL system. LD, EC$_1$, EC$_2$ and RTO denotes the laser diod, first extrenal cavity, second external cavity and real-time oscilloscope respectively.

2. Dynamics of two delayed semiconductor laser

2.1. Deterministic and noise induced SL model

The dynamics of a SL has been investigated by several theoretical models. Fig. 1, represents the schematic diagram of a SL with two external cavities. In the schematic diagram, the beam splitter [47,48] can be the dielectric mirror to split the incident light beam. Any partially reflecting mirror can be used for splitting light beams. The angular separation of the output beams, we can have any positive value. Different power splitting ratios can be achieved via different designs of the dielectric coating. Apart from the dielectric mirrors, there are other kind of splitters such as splitter cubes, fiber-optic beam splitters, metal coated mirrors [47,48]; to split the beams with various separation angles. The SL concerned with optical feedback can be described by a set of coupled delay differential equations-a dimensionless LK model [49,50]. We generalized the set by introducing two different delays τ_1 and τ_2, which can be written as

$$\begin{aligned}\frac{dE_0}{dt} &= 0.\,5(1+i\alpha)(n-1)+\eta_1 E_0(t-\tau_1)e^{-i\omega\tau_1}\\ &\qquad +\eta_2 E_0(t-\tau_2)e,^{-i\omega\tau_2}\\ \frac{dn}{dt} &= \frac{2}{T}(p-0.\,5(n-1)+nE^2),\end{aligned}\tag{1}$$

where $E_0(t)$ and $n(t)$ are intracavity complex electric field and carrier population, respectively; α is the line width enhancement factor; ω is frequency of the solitary laser and (η_1, η_2) are the feedback rates. For photon lifetime τ_p and carrier lifetime τ_s, T is defined as $T=\tau_s/\tau_p$. p is proportional to the pumping rate above threshold. Since the electric field amplitude E and phase ϕ holds the relation: $E_0 = Ee^{i\phi(t)}$, the eq. (1) can be written as

$$\begin{aligned}\frac{dE}{dt} &= 0.5E(n-1)+\eta_1 E(t-\tau_1)\cos(\phi-\phi(t-\tau_1)+\omega\tau_1)+\eta_2 E(t-\tau_2)\\ &\quad \cos(\phi-\phi(t-\tau_2)+\omega\tau_2),\ \frac{d\phi}{dt} = 0.5\alpha(n-1)-\eta_1\frac{E(t-\tau_1)}{E}\\ &\quad \sin(\phi-\phi(t-\tau_1)+\omega\tau_1)-\eta_2\frac{E(t-\tau_2)}{E}\sin(\phi-\phi(t-\tau_2)+\omega\tau_2),\ \frac{dn}{dt}\\ &= \frac{2}{T}(p-0.5(n-1)+n|E|^2).\end{aligned}\tag{2}$$

By perturbing the system (2) with the additive noise $\Omega(\beta)$, we get