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On the difference between single- and double-sided bandpass filtering of spatial frequencies

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ABSTRACT

It is well-known that bandpass filtering will lead to edge extraction in image processing. However, the difference between single- and double-sided bandpass filtering has never been compared and investigated in the literature. We investigate the difference between single- and double-sided bandpass spatial filtering in a 4-f coherent optical image processing system. We find that single-sided filtering can approximate the operation of a first-order derivative, while double-sided filtering gives a second-order derivative. Simulations and optical experiments confirm our findings.

1. Introduction

A standard two-lens coherent optical system with both lenses having the same focal length f but separated by 2f, is known as the 4-f spatial filtering system in coherent image processing [1–4]. The confocal plane of the optical system is called the pupil plane or the Fourier plane [5,6]. On the Fourier plane, the Fourier transform or the spectrum of the input image is displayed, where the input image is located on the front focal plane of the first lens. The spectrum of the image is a collection of different spatial frequencies of the image. Coherent image processing can be performed by inserting a pupil function on the confocal plane as the pupil function will modify the original spectrum or filter the different spatial frequencies of the input image. For example, by inserting a small hole centered along the optical axis (along z) of the optical system, we perform low-pass spatial filtering of the input image, i.e., low spatial frequencies of the image will be allowed to pass through the optical system, while the back focal plane of the second lens displays the low-pass version of the original input image. The classical paper by R. A. Phillips has illustrated clearly the interesting effect of low-pass filtering [7]. By the same token, band-pass spatial filtering can be performed by inserting an annulus on the Fourier plane. In contrast to temporal frequency (cycles per unit time) in electrical signals, spatial frequency (cycles per unit length) in optics takes on some physical meaning. Negative temporal frequencies do not exist physically, but negative spatial frequencies exist in optics. For example, transparency of $t(x, y)=\cos(2\pi ax)=\frac{1}{2}[\exp(i2\pi ax) + \exp(-i2\pi ax)]$ has spatial frequencies associated with the x-coordinate $f_x=\pm a$. When the transparency is illuminated by a plane wave normally of wavelength λ, two outgoing plane waves at angles $\theta_{\pm}=\sin^{-1}(\pm\lambda a)$ will emerge, where the angles are measured with respect to the optical axis. Hence temporal signals can be filtered at a particular “positive” frequency, while spatial images can be filtered at positive and/or negative frequencies. Indeed it is well known that by half-plane filtering in the Fourier plane, known as the Foucault test, we can detect wavefront errors of a lens or mirrors [8]. To generalize half-plane filtering a bit further, positive and negative frequencies could be filtered differently and that leads to the well-known optical implementation of the Hilbert transform for the observation of phase objects in coherent systems [9]. In passing, we want to point out that the Hilbert transform also has been investigated in the context of incoherent image processing [10]. In this paper, we want to investigate simple bandpass filtering. To be precise, we want to explore single-sided bandpass filtering and double-sided bandpass filtering, and examine their differences. The reason to investigate bandpass filtering is that it has the ability to extract edge information of an image. Edge detection is a fundamental and important basic operation in image processing, machine vision and computer vision, particularly in the area of feature extraction [11].

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2. Basic coherent optical image processing system

Fig. 1 is the standard 4-f optical coherent image processing system. On the input plane (or the object plane), we have an image in the form of a transparency, $t(x, y)$, which is assumed to be illuminated by a plane wave normally. According to Fourier optics, an ideal lens is a Fourier transformer [5,6]. The field distribution, apart from some constant, on the back focal plane of lens L1 is given by [5,6]

$$\psi_p=\mathcal{F}\{t(x, y)\}_{k_x=\frac{k_0x}{f},k_y=\frac{k_0y}{f}}=T(\frac{k_0x}{f}, \frac{k_0y}{f}). \tag{1}$$

where $T(\frac{k_0x}{f}, \frac{k_0y}{f})$ is the Fourier transform or the spectrum of $t(x, y)$, and $k_0=2\pi/\lambda$ is the wave number of the plane wave. The two-dimensional spatial Fourier transform of a signal $f(x, y)$ is given by

$$\mathcal{F}\{f(x, y)\}=F(k_x, k_y)=\iint_{-\infty}^{\infty} f(x, y)\exp(ik_xx + ik_yy)dxdy. \tag{2a}$$

and its inverse Fourier transform is

$$\mathcal{F}^{-1}\{F(k_x, k_y)\}=f(x, y)=\frac{1}{4\pi^2}\iint_{-\infty}^{\infty} F(k_x, k_y)\exp(-ik_xx - ik_yy)dk_xdk_y. \tag{2b}$$

where the transform variables are spatial variables, x, y [meter], and spatial radian frequencies, k_x, k_y [radian/meter]. We want to point out the convention used in the paper. We have used $\exp(ik_xx + ik_yy)$ as the exponent for the spatial Fourier transform, and $\exp(-i\omega t)$ as the exponent for any temporal Fourier transform, where ω and t are the temporal radian frequency variable [radian/second] and the time variable [second], respectively. This is done to be consistent with the engineering convention for a travelling plane wave. In this convention, $\mathrm{Re}\{A\exp[i(\omega t-k_0z)]\}$ denotes a plane wave travelling in the +z direction, where A is the amplitude of the wave, ω is a temporal frequency and k_0 is a propagation.

Hence the confocal plane in Fig. 1 of the optical system is often called the Fourier plane. The spectrum of the input image is now modified by pupil function $p(x, y)$, and the field immediately behind the pupil function is then $T(\frac{k_0x}{f}, \frac{k_0y}{f})p(x, y)$, which is Fourier transformed again by Lens L2, giving the field on the image plane as

$$\psi_{pi}=\mathcal{F}\left\{T(\frac{k_0x}{f}, \frac{k_0y}{f})p(x, y)\right\}_{k_x=\frac{k_0x}{f},k_y=\frac{k_0y}{f}}, \tag{3}$$

which can be evaluated, in terms of convolution, to give

$$\psi_{pi}=t(-x, -y)*\mathcal{F}\{p(x, y)\}_{k_x=\frac{k_0x}{f},k_y=\frac{k_0y}{f}}$$

$$=t(-x, -y)*P\left(\frac{k_0x}{f}, \frac{k_0y}{f}\right)=t(-x, -y)*h_c(x, y), \tag{4}$$

where P is the Fourier transform of p. The convolution integral in Eq. (4) is defined as

$$g(x, y)=g_1(x, y)*g_2(x, y)=\iint_{-\infty}^{\infty} g_1(x', y')g_2(x - x',y - y')dx'dy', \tag{5}$$

where * denotes convolution of two functions $g_1(x, y)$ and $g_2(x, y)$. The optical system under consideration is called a coherent optical system in that complex quantities are manipulated. Once we have found the complex field on the image plane given by Eq. (4), the corresponding image intensity is

$$I_i(x, y)=\psi_{pi}(x, y)\psi^*_{pi}(x, y)=|t(-x, -y)*h_c(x, y)|^2, \tag{6}$$

which is the basis for coherent image processing. From Eq. (4), we can recognize that

$$h_c(x, y) = \mathcal{F}\{p(x, y)\}_{k_x=\frac{k_0x}{f},k_y=\frac{k_0y}{f}}=P(\frac{k_0x}{f}, \frac{k_0y}{f}) \tag{7}$$

is the coherent point spread function (CPSF) of the two-lens system [5,6]. Hence, the expression given by Eq. (4) can be interpreted in that the inverted image of t(x, y) is processed by the CPSF given by Eq. (7). The CPSF, and therefore the image processing capabilities can be varied by simply changing the pupil function, p(x, y). For example, if we take $p(x, y)=1$, which means we do not modify the spectrum of the input image, $h_c(x, y)$ according to Eq. (7) becomes a delta function and the output image from Eq. (4) is $\psi_{pi}(x, y)\propto t(-x,-y)*\delta(x, y)=t(-x,-y)$. The result is an inverted image, consistent with imaging in geometrical optics. While the CPSF is given by the Fourier transform of the pupil function, by definition, the corresponding coherent transfer function (CTF) is the Fourier transform of the CPSF:

$$H_c(k_x, k_y)=\mathcal{F}\{h_c(x, y)\}=\mathcal{F}\left\{P\left(\frac{k_0x}{f}, \frac{k_0y}{f}\right)\right\}=p(\frac{-fk_x}{k_0}, \frac{-fk_y}{k_0}). \tag{8}$$

We, therefore, observe that spatial filtering is directly proportional to the functional form of the pupil function. For example, if we choose $p(x, y)=\mathrm{circ}(r/r_0)$, where $r = \sqrt{x^2 + y^2}$ and $\mathrm{circ}(r/r_0)$ denotes a value 1 within a circle of radius r_0 and 0, otherwise, from the interpretation of Eq. (8), we see that for this kind of chosen pupil, i.e., a hole or circular opening of radius on the pupil plane, filtering is of lowpass characteristic as the opening of the hole on the pupil plane only allows physically the low spatial frequencies to go through. For highpass filtering, we can, for example, choose $p(x, y)=1-\mathrm{circ}(r/r_0)$, i.e., a circular blocking of radius on the pupil plane.

3. Single- and double-sided bandpass filtering

After a brief review on coherent optical image processing in the last Section, we are now in a position to investigate the differences between single- and double-sided bandpass filtering.

3.1. Single-sided filtering

For brevity, we will perform 1-D mathematical analysis. For single-sided filtering, let us choose$p(x)=\mathrm{rect}((x-x_c)/x_0)$, where $\mathrm{rect}(x/x_0) = 1$ for $|x| < x_0/2$ and 0, otherwise. Physically, the pupil function is considered as a slit of width x_0 centered at x_c, along the y-direction. The pupil is shown is Fig. 2(a). According to Eq. (8), the CTF is

$$H_c(k_x) = p\left(\frac{-fk_x}{k_0}\right) = \mathrm{rect}\left(\left(\frac{-fk_x}{k_0} - x_C\right)/x_0\right) = \mathrm{rect}(\frac{-k_x - x_Ck_0/f}{x_0k_0/f}), \tag{9}$$

which is shown in Fig. 2(b). We see that this is band-pass filtering with the center frequency at x_Ck_0/f and the width of the passband of x_0k_0/f.

The system's CPSF, according to Eq. (7), is

$$h_c(x)=\mathcal{F}\{\mathrm{rect}((x - x_C)/x_0)\}_{k_x=\frac{k_0x}{f}}=x_0\exp\left(\frac{ik_0xx_C}{f}\right)\mathrm{sinc}(\frac{k_0xx_0}{2\pi f}), \tag{10}$$

where $\mathrm{sinc}(x)=\frac{\sin(\pi x)}{\pi x}$. This CPSF, as it turns out, can approximate a first-order derivative of the original image. We will explain this in the next Section when we report our simulation results.

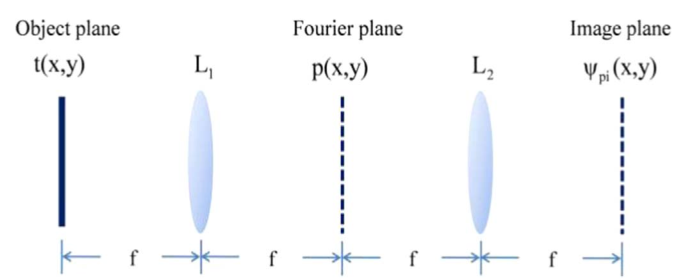

Fig. 1. Standard 4-f optical coherent image processing system.