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Quasi-absolute surface figure test with two orthogonal transverse spatial shifts

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ABSTRACT

A new zonal wavefront reconstruction algorithm with pixel-level spatial resolution and high accuracy is proposed, which is able to reconstruct the original wavefront of general aperture shape from only two difference wavefronts measured at two orthogonal shear directions with shear amounts equaling arbitrary moderate integral multiples of the sample interval. Based on this algorithm, a quasi-absolute surface figure test method is presented, which requires only two additional translational measurements with shifts of arbitrary moderate integral multiples of sample interval along x and y directions besides the original position measurement. Optical schemes of the proposed method for testing flat, spherical and cylindrical surfaces are investigated, and special considerations and challenges for calibrating spheres and cylinders are also briefly formulated theoretically. Thorough errors analysis is formulated for obtaining high accuracy test result. Simulations and experiments on a flat surface are conducted to validate the proposed algorithm and method. Compared with existing absolute test methods with Pseudo-Shear Interferometry (PSI) technique, the presented method has advantages, like, less number of measurements, arbitrary moderate shear amounts and the high signal-to-noise ratio it can reach.

1. Introduction

Absolute surface figure test is a technique to calibrate reference error and systematic error. Many excellent works have been done in this field. Basic absolute test methods comprise the liquid surface method [1,2], the traditional three-flat test and its related methods [3–8] for flats; random ball test [9], traditional two-sphere method with cat's-eye position test [10,11] for spherical surfaces. Besides these methods, shift-rotation method [12–15] is a general and effective method for both spherical and flat surfaces.

Another general method for both spherical and flat surfaces is based on Pseudo-Shear Interferometry (PSI) technique which was firstly proposed by Keenan [16]. Applying PSI technique in field of absolute surface figure test has a series of advantages over other absolute test methods, such as it requires no additional optics and no numerous measurements, it is performed in-situ, and systematic error besides the reference surface error is calibrated simultaneously. For these advantages, this method has been studied by a lot of metrologists, although this method is often called 'quasi-absolute' since the power and astigmatic parts cannot be retrieved exactly [17]. Different from the lateral shear interferometry, which can acquire difference wavefronts directly by shear optics, procedures of absolute test method with PSI technique include obtaining successive measurements with lateral shifts of the test optics, and generating difference wavefronts by a computer. Then the problem is a standard problem in wavefront reconstruction. This manuscript attempts to categorize reported absolute test methods with PSI technique based on the wavefront reconstruction algorithm it adopts.

Wavefront reconstruction algorithms can be classified into two categories: modal reconstruction and zonal reconstruction algorithms. Modal reconstruction [18–20] utilizes a set of basis functions to represent the wavefront, and high frequency parts of the wavefront are lost, hence this kind algorithm is not appropriate for absolute test. Zonal reconstruction relates the discrete sampling wavefront to the discrete sampling difference wavefronts through a coefficient matrix, thus pixel-level spatial resolution solution can be obtained by solving this inverse problem. According to the number of difference wavefronts and values of shear amounts, zonal reconstruction algorithms can be divided into three categories as following.

For one difference wavefront at each orthogonal shear direction with shear amounts equaling sample interval of the test wavefront, three typical and commonly used algorithms, which could reconstruct the test wavefront exactly apart from overall phase and tip/tilt, were proposed by Hudgin, Fried, and Southwell, respectively [21–23].

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Actually, the first implementation of absolute test method with PSI technique to our knowledge, which was fulfilled by Bloemhof in 2010 [24,25], had adopted the Hudgin's wavefront reconstruction algorithm. This is a milestone in the process of absolute test method with PSI technique. Immediately following this, another significant work was accomplished by Ma et al. who had proposed the conjugate differential method [26,27], and instead of obtaining difference wavefronts in two directions by three measurements as shown by Bloemhof, Ma et al. took two separate pairs of measurements for the two shifting directions, thus the precision of the slope approximation was enhanced by reducing coupling between multi-step tests. Despite the pioneering contributions Bloemhof and Ma et al. have made, the signal-to-noise ratio is very low for shear amounts both equaling sample interval of the test wavefront, and it is not convenient to shift the test optics by as small as the sample interval.

For at least four difference wavefronts, i.e., two difference wavefronts along each shear direction, the shear amounts are not restricted by one pixel. Typical algorithms which could reconstruct the test wavefront exactly were proposed by Elster *et al.*, Yin and Nomura, respectively [28–30]. Based on these off-the-shelf algorithms, Morin et al.[31] and Vidal et al.[32] studied absolute test method with PSI technique by five translational position measurements. More significantly, Vidal *et al.* studied the method to estimate pitch and roll errors existing in each measurement and retrieved power and astigmatism parts approximately. However, the increased number of measurements (five measurements) may introduce more translational movement errors, guidance errors (pitch and roll errors), and unavoidable noises thus decreasing accuracy of the absolute test result. Moreover, the shear amounts choices are required to meet certain requirements.

In case that difference wavefronts number is less than four and the shear amount is not restricted by one pixel, some approximate algorithms are developed. The generalized zonal reconstruction (GZR) algorithm [33] and the combined zonal and modal reconstruction (CZMR) algorithm [34] proposed by Dai can obtain reconstruction result with only two difference wavefronts for two orthogonal shear directions. These seem to be the most appropriate algorithms for absolute test method with PSI technique because only as less as three measurements are required, and the shear amounts can equal arbitrary moderate integral multiples of the sample interval. However, applying these algorithms to absolute test is not capable, because the accuracy depends heavily on the initial values it chooses, i.e., the accuracy of GZR is high only when phase values at selected points of the subgrid exhibit good linear property, and so is CZMR when phase values at selected points can be represented by Zernike polynomials very well. To avoid selecting initial values for solving the inverse problem, Song utilized the PSI technique in absolute test by wisely adding a 90° rotational measurement besides the original position measurement and the two translational measurements [35]. However, this method introduced a rotational position measurement which complicated the experiment setup and introduced more errors for this additional measurement.

In this paper, a new zonal wavefront reconstruction algorithm with pixel-level spatial resolution and high accuracy is firstly proposed, which is able to reconstruct the original wavefront from only two difference wavefronts measured in two orthogonal shear directions with shear amounts equaling arbitrary moderate integral multiples of the sample interval. Secondly, based on the proposed algorithm, a quasi-absolute test method is presented, which requires only two additional translational measurements with shear amounts equaling arbitrary moderate integral multiples of the sample interval besides the original position measurement. Optical schemes of the proposed method for testing flat, spherical and cylindrical surfaces are investigated, and special operations and challenges with regard to calibrate spherical and cylindrical surfaces utilizing this quasi-absolute method are briefly formulated theoretically. Thirdly, simulations on a flat surface are conducted to demonstrate that the high reconstruction accuracy of the proposed algorithm can meet requirements of absolute test. Moreover, errors that affect the proposed method are thoroughly analyzed. Finally, preliminary experiments on a flat surface are conducted to verify the proposed method.

2. Wavefront reconstruction algorithm from two difference wavefronts

2.1. Mathematical model

The original wavefront is denoted by $\mathbf{W}$, which is sampled by a rectangle grid with size $n_y \times n_x$. Define shear amounts in x and y shear directions as s_x and s_y, respectively, which are both integral multiples of the sample interval. $\Delta\mathbf{W}_x$ and $\Delta\mathbf{W}_y$ denote the difference wavefronts with shear amounts s_x and s_y, respectively. Define the phase value of the original wavefront $\mathbf{W}$ at the ith row and jth column as $w(i, j)$ and ranges of i and j are $1 \le i \le n_y$ and $1 \le j \le n_x$, respectively. The data structure of $\mathbf{W}$ is shown in Eq. (1). The relations of $\mathbf{W}$ with $\Delta\mathbf{W}_x$ and $\Delta\mathbf{W}_y$ are shown in Eq. (2) and Eq. (3), respectively.

$$\mathbf{W} = \begin{bmatrix} w(1,1) & w(1,2) & \cdots & w(1,n_x) \\ w(2,1) & w(2,2) & \cdots & w(2,n_x) \\ \vdots & \vdots & \ddots & \vdots \\ w(n_y,1) & w(n_y,2) & \cdots & w(n_y,n_x) \end{bmatrix}. \tag{1}$$

$$\Delta\mathbf{W}_x = \begin{bmatrix} w(1,s_x+1) & w(1,s_x+2) & \cdots & w(1,n_x) \\ w(2,s_x+1) & w(2,s_x+2) & \cdots & w(2,n_x) \\ \vdots & \vdots & \ddots & \vdots \\ w(n_y,s_x+1) & w(n_y,s_x+2) & \cdots & w(n_y,n_x) \end{bmatrix} - \begin{bmatrix} w(1,1) & w(1,2) & \cdots & w(1,n_x-s_x) \\ w(2,1) & w(2,2) & \cdots & w(2,n_x-s_x) \\ \vdots & \vdots & \ddots & \vdots \\ w(n_y,1) & w(n_y,2) & \cdots & w(n_y,n_x-s_x) \end{bmatrix}. \tag{2}$$

$$\Delta\mathbf{W}_y = \begin{bmatrix} w(s_y+1,1) & w(s_y+1,2) & \cdots & w(s_y+1,n_x) \\ w(s_y+2,1) & w(s_y+2,2) & \cdots & w(s_y+2,n_x) \\ \vdots & \vdots & \ddots & \vdots \\ w(n_y,1) & w(n_y,2) & \cdots & w(n_y,n_x) \end{bmatrix} - \begin{bmatrix} w(1,1) & w(1,2) & \cdots & w(1,n_x) \\ w(2,1) & w(2,2) & \cdots & w(2,n_x) \\ \vdots & \vdots & \ddots & \vdots \\ w(n_y-s_y,1) & w(n_y-s_y,2) & \cdots & w(n_y-s_y,n_x) \end{bmatrix}. \tag{3}$$

Straighten the matrix $\mathbf{W}$, $\Delta\mathbf{W}_x$ and $\Delta\mathbf{W}_y$ row by row, and the corresponding produced vectors are denoted by $\overrightarrow{\mathbf{W}}$, $\overrightarrow{\Delta\mathbf{W}_x}$ and $\overrightarrow{\Delta\mathbf{W}_y}$, respectively. According to Eq. (2), the vectorized original wavefront $\overrightarrow{\mathbf{W}}$ and vectorized difference wavefront $\overrightarrow{\Delta\mathbf{W}_x}$ are related by

$$\overrightarrow{\Delta\mathbf{W}_x} = \mathbf{M}_x \overrightarrow{\mathbf{W}}, \tag{4}$$

where $\mathbf{M}_x$ is a matrix with the dimension $n_y(n_x - s_x) \times n_x n_y$, and it is given by

$$\mathbf{M}_x = \mathrm{diag}(\mathbf{D}_x \;\; \mathbf{D}_x \;\; \cdots \;\; \mathbf{D}_x), \tag{5}$$

where $\mathbf{D}_x$ is a matrix with the dimension $(n_x - s_x) \times n_x$, and it is given by

$$\mathbf{D}_x(u, v) = \begin{cases} -1 & u = v \\ 1 & u = v - s_x \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

where $1 \le u \le n_x - s_x$, $1 \le v \le n_x$.

Similarly, according to Eq. (3), the vectorized original wavefront $\overrightarrow{\mathbf{W}}$ and vectorized difference wavefront $\overrightarrow{\Delta\mathbf{W}_y}$ are related by

$$\overrightarrow{\Delta\mathbf{W}_y} = \mathbf{M}_y \overrightarrow{\mathbf{W}}, \tag{7}$$

where $\mathbf{M}_y$ is a matrix with the dimension of $n_x(n_y - s_y) \times n_x n_y$, and it is