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Diffraction of a Gaussian beam by a four-sector binary grating with a shift between adjacent sectors

Lj. Janicijevic[a], S. Topuzoski[a,*], L. Stoyanov[b], A. Dreischuh[b]

[a] Faculty of Natural Sciences and Mathematics, Institute of Physics, University "Ss. Cyril and Methodius", Skopje 1000, Republic of Macedonia
[b] Department of Quantum Electronics, Faculty of Physics, Sofia University "St. Kliment Ohridski", Sofia 1164, Bulgaria

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ABSTRACT

In this article as a diffractive optical element we consider a composed four-sector binary grating under Gaussian laser beam illumination. The angular sectors are bounded by the directions $y = x$ and $y = -x$, and consist of parts of a binary rectilinear grating; thereby, two neighboring parts are shifted by a half spatial rectilinear grating period. The diffracted wave field amplitude is calculated, showing that the straight-through, zeroth-diffraction-order beam is an amplitude-reduced Gaussian beam, and the higher-diffraction-order beams, deviated with respect to the propagation axis, are non-vortex beams described by modified Bessel functions. The transverse intensity profiles of the higher-diffraction-order beams, numerically and experimentally obtained, have form of a four-leaf clover; they are similar to the Laguerre-Gaussian LG(0,2) beam (with radial mode number $n = 0$ and azimuthal mode number $l = 2$) described by circular cosine function, in a paraxial, far-field approximation.

1. Introduction

Besides the fundamental mode (Gaussian beam), the Hermite–Gaussian (HG) and Laguerre–Gaussian (LG) beams are also solutions of the paraxial wave equation [1]. Lot of research has been done to analyze their theoretical and experimental properties, and to investigate their applications in the basic optical sciences and in other scientific fields (see e.g. Ref. 2 and references therein). Linearly polarized LG beams with nonzero azimuthal mode number are carriers of screw dislocations and possess orbital angular momentum (OAM) [3]. They are optical vortex beams. In the field of singular optics the mostly used are LG beams with zero radial mode number. The family of LG beams covers the cases of the equiaxial linear combinations (addition or subtraction) of two LG beams with equal azimuthal mode number value l, but with opposite signs of l (opposite orientations of their OAMs), as well. As a result, the two coupled vortex beams create a beam without OAM (no topological charge), possessing profiles described by circular functions $\cos(l\varphi)$ or $\sin(l\varphi)$.

The linearly polarized HG(m,n) beams have degenerate edge dislocations in their wavefronts and do not possess OAM [3].

Nye and Berry have described, classified and analyzed the wavefronts defects in wave trains and monochromatic waves [4]. In laser beams having structure of transverse cavity modes, edge dislocations occur as black lines between π-shifted in phase mode spots; the simplest is the TEM_{01}, where the zeroth-value intensity line divides the beam into two parts corresponding to phase shift of π. In [5] the authors showed that, an edge dislocation of the wavefront can be produced experimentally by using two binary periodic gratings, shifted by half a period on a line of zero amplitude. Then, in the process of diffraction, the incident Gaussian laser beam is divided with a dark line into two bright spots. Whereas, a binary fork-shaped grating with an edge dislocation in direction $\theta = 0$ produces mixed screw-edge dislocation, as shown experimentally in [5].

The vortex beams are created in laser resonators, or by using diffractive optical elements which transform the Gaussian beam into a vortex one, such as spiral phase plate [6–8], helical axicon [9–12], helical lens [13], computer-generated holograms [14,15], fork-shaped gratings [16,17] etc. The computer-generated gratings (CGGs) accompanied with the photo-reduction methods have an advantage over the expensive lithographic methods. Except the simple, fast and cheap production, they make possible the creation of combined gratings, which can substitute the laser resonators in making new interesting laser modes. Liquid crystal spatial light modulators make this procedure even more flexible ensuring high efficiency and fast reconfiguration.

In this article we consider a CGG constructed by inserting parts of a binary rectilinear grating into the four equal angular sectors, bounded by the directions $y = x$ and $y = -x$ (Fig. 1). Thus, two neighboring parts

* Corresponding author.
E-mail address: suzana_topuzoski@yahoo.com (S. Topuzoski).

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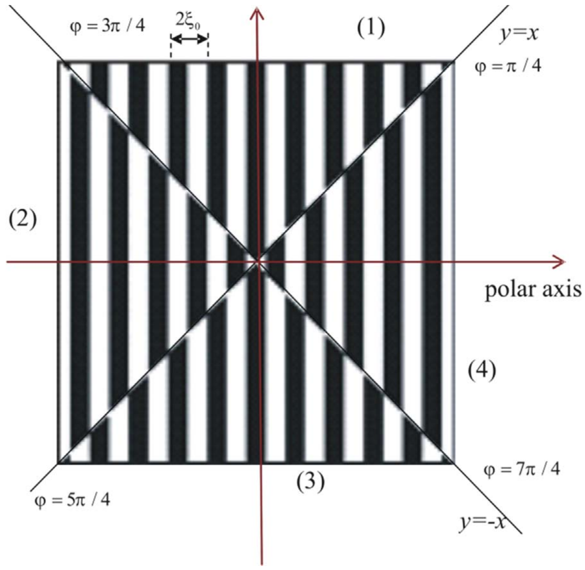

Fig. 1. The computer-generated four-sector grating.

of the grating are shifted by a half spatial grating period along x axis. We analytically calculate the diffraction pattern obtained by illuminating the grating with a Gaussian laser beam, which enters into the grating plane with its waist and intersects the grating plane centre with its axis. The far-field diffraction patterns of the higher-diffraction-order (HDO) beams, in a paraxial approximation are similar to Hermite-Gaussian HG(1,1) or cosine-LG(n=0,l=2) laser mode: four bright spots are nested in four quadrants divided by crossed one-dimensional phase dislocations. With this method we create in the HDOs beams with coupled optical vortices [18] and crossed dark lines, which are of interest for many applications as optical trapping, optical communication, angular alignment etc.

2. Construction and transmission function of the grating

The computer generation of this grating consists in inserting parts of a binary rectilinear grating into the four equal angular sectors, bounded by the directions $y = x$ and $y = -x$. The area of each of the sectors, numbered by n=(1), (2), (3) and (4), is successively covered by a negative (in (1) and (3)) and positive (in (2) and (4)) gratings, both possessing the same period $d = 2\xi_0$ (Fig. 1). In a rectangular coordinate system, whose ordinate is the axis of symmetry of both types of gratings, their transmission functions are expressed by the cosine Fourier series as

$$t_g^{\pm}(x) = \frac{1}{2} \pm \sum_{m=1}^{\infty} \text{sinc}\left((2m-1)\frac{\pi}{2}\right)\cos\left((2m-1)\frac{\pi}{\xi_0}x\right). \tag{1}$$

Since we will treat the problem of diffraction of a Gaussian laser beam by the computer-generated gratings in cylindrical coordinate system, we will use the polar coordinates (r, φ) for the grating's plane. The pole is situated in the intersection point of the $y = x$ and $y = -x$ lines. Then, the transmission functions $t_g^{+}(r, \varphi)$ for the positive (with white central line) and $t_g^{-}(r, \varphi)$ for the negative (with dark central line) gratings, are defined as

$$t_g^{\pm}(r, \varphi) = \frac{1}{2} \pm \frac{1}{2}\sum_{m=1}^{\infty} \text{sinc}\left((2m-1)\frac{\pi}{2}\right)\left[\exp\left(+i(2m-1)\frac{\pi}{\xi_0}r\cos\varphi\right) + \exp\left(-i(2m-1)\frac{\pi}{\xi_0}r\cos\varphi\right)\right]. \tag{2}$$

In Eq. (1) and Eq. (2) the transmission coefficients are $\text{sinc}((2m-1)\pi/2) = 2(-1)^{(m-1)}/(\pi(2m-1))(m = 1, 2, 3, ...)$, while the sign "+" in front of the sum stands for the positive grating.

As it is seen in Fig. 1, the n-th quadrant is occupied by one of the upper mentioned gratings. Each of them is an angular sector of π/2 rad, which in absence of the grating, is a completely transparent aperture between the directions $\varphi = (2n-1)\pi/4$ and $\varphi = (2n+1)\pi/4$. Using the Heaviside unite step function $E(\varphi - \varphi_0) = \begin{cases} 1 & \varphi > \varphi_0 \\ 0 & \text{otherwise} \end{cases} (0 < \varphi < 2\pi)$, we define the n-th sector aperture transmission function as

$$t_a^{(n)}(\varphi) = E(\varphi - (2n-1)\pi/4) - E(\varphi - (2n+1)\pi/4) = \begin{cases} 1 & (2n-1)\pi/4 < \varphi < (2n+1)\pi/4 \\ 0 & \text{otherwise} \end{cases} \quad (n = 1, 2, 3, 4) \tag{3}$$

The transmission function of the composed grating in Fig. 1 is a sum of the four sector transmission functions $t(r, \varphi) = t_a^{(n)}(\varphi)t_g^{\pm}(r, \varphi)$ (for n=1,2,3,4) and is defined by

$$t(r, \varphi) = \sum_{n=1}^{4} \{E(\varphi - (2n-1)\pi/4) - E(\varphi - (2n+1)\pi/4)\} \times \left\{\frac{1}{2} + (-1)^n \sum_{m=1}^{\infty} \text{sinc}\left((2m-1)\frac{\pi}{2}\right)\cos\left((2m-1)\frac{\pi}{\xi_0}r\cos\varphi\right)\right\}. \tag{4}$$

The grating whose transmission function is given by expression Eq. (4) will be used as an optical diffracting device in our further investigation.

3. Diffraction of a Gaussian laser beam by the composed four-sector grating

The Gaussian beam is normally incident on the plane of the grating, with its propagation axis (z axis of the cylindrical coordinate system) passing through its centre, and its waist located in the plane of the grating. Thus, the incident beam is defined by: $U^i(r, \varphi, z = 0) = \exp(-r^2/w_0^2) = \exp(-ikr^2/2q(0))$ where w_0 is the beam waist radius, $k = 2\pi/\lambda$ is the propagation constant and $q(0)$ is the beam complex parameter in the waist plane. If the grating is absent, at distance z from the origin the beam has a complex parameter $q(z) = z + ikw_0^2/2$, with $q(0) = ikw_0^2/2 = iz_0$, and z_0 being the beam Rayleigh distance. The field of the diffracted light is defined by the Fresnel-Kirchhoff integral

$$U(\rho, \theta, z) = \frac{ik}{2\pi z}\exp\left[-ik\left(z + \frac{\rho^2}{2z}\right)\right]\int_0^{\infty}\int_0^{2\pi} t(r, \varphi)U^{(i)}(r, \varphi)\exp \times \left[-\frac{ik}{2z}(r^2 - 2r\rho\cos(\varphi - \theta))\right] r\,\mathrm{d}r\,\mathrm{d}\varphi. \tag{5}$$

The polar coordinates (ρ, θ) characterize the observation plane Π situated at distance z from the grating. Substitution of the incident beam and the transmission function Eq. (4) in the diffraction integral gives

$$U(\rho, \theta, z) = \sum_{n=1}^{4}\left[U_{m=0}(\rho, \theta, z) + \sum_{m=1}^{\infty}(U_{+(2m-1)}(\rho, \theta, z) + U_{-(2m-1)}(\rho, \theta, z))\right]. \tag{6}$$

The part of the solution $\sum_{n=1}^{4} U_{m=0}(\rho, \theta, z)$ defines the zeroth diffraction order